Practical Alternating Currents

AND

Alternating Current Testing.

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PREFACE

This volume is an attempt to introduce the student to the main principles of alternating currents and alternating-current machinery from an experimental, rather than from an abstract theoretical, standpoint.

It is hoped that it may prove of use to students in technical schools and colleges in connection with their laboratory work, and to young engineers engaged in handling or testing alternating-current machinery.

The writer hopes that the book may also appeal to a wider range of readers who wish to gain some practical insight into alternating-current working. He believes that the experimental method of treatment is for this purpose the one best fitted to convey exact and concrete ideas, even to the student who is unable himself to carry out the measurements described. He has thus aimed at producing a text-book on the principles of alternating-current working, as well as a companion to the laboratory or test house. The experiments are described in great detail, so as to enable the conclusions derived from them to be followed by the reader who has not the means for obtaining experimental results of his own.

It will be seen that the use of the higher mathematics has been avoided, except in the final chapter on curve analysis, and, as far as possible, a treatment based an actual results and examples has been employed.

With regard to the selection of the experiments, some of the earlier ones have been given mainly for the sake of the principles involved, or as exercises in calculation. In the later part of the book, while the most general practical measurements are given in full, many other experiments are only alluded to, and left to the student to carry out with such modifications as his experience may suggest.

The experiments may be taken as a general outline for a course of practical work in the laboratory; but, as the student

makes progress, he should learn to depart more and more from the exact form given in the book, and endeavour by different ways to familiarise himself with the behaviour of the machines he is handling. He must not forget that a large part of the "experiment" is actually to be done on paper after the readings have been taken.

The experimental results given in the various tables and curves were, with few exceptions, obtained expressly for the purpose of this book. The greater part of this arduous work was actually carried out by the writer's friend, Mr. U. A. Oschwald, B.A., without whose assistance the book could hardly have appeared in its present form.

Additions and minor alterations have been made as each new issue of the book has been called for; in the present issue a more modern form of circle diagram for the induction motor has been introduced occupying its logical position as a simplified form of the original Heyland diagram.

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CHAPTER I

Alternating Electromotive Force and Current.

General Laws.—Two experimental laws may be said to form the foundation of the actions which take place in electrical generators and motors, whether for alternating or continuous currents.

Law of the Generator.—When a conductor moves in such a way as to cut magnetic lines of force, there will be an electromotive force generated in the conductor.

Law of the Motor.—When a current flows along a conductor situated in the field. A magnetic field is usually spoken of as con-way as to cut the lines of force of the field.

In order to make clear the application of these laws to the case of alternating-current machinery it will be advisable to consider them more fully.

Production of Electromotive Force.—The strength of a magnetic field is expressed numerically by the pull, measured in dynes*, which would be experienced by a unit magnetic pole† if situated in a field. A magnetic field is usually spoken of as consisting of a number of lines of force. The density of these mathematical lines is, for convenience, so chosen that the number of lines which pass through a square centimetre of a surface which is at right angles to their direction is also the number representing the strength of the field. Thus the number of lines of force per square centimetre gives the force which would act on a unit magnetic pole at the point considered.

Whenever an electric conductor moves in such a magnetic field so as to cut the lines of force, an electromotive force, or E.M.F., is produced‡ in the conductor. The numerical value (in C.G.S., or "absolute" units) of this electromotive force is the number of lines cut through by the conductor in 1 sec.

Thus

$$\text{E.M.F.} = \text{Rate of cutting magnetic lines.}$$

Expressed in volts (the "practical" unit of electromotive force), the electromotive force will be the above number of units divided by 10^8 (since one volt is equal to 10^8 absolute units of electromotive force), so that

$$\text{E.M.F. (in volts)} = \text{Magnetic lines cut per second} \times 10^{-8}.$$

* A dyne is the unit of force in the absolute or C.G.S. system of units. It is the force necessary to give the unit acceleration (1 cm. per second per second) to unit mass (1 gramme). A force of 1 dyne is roughly equal to the weight of 1 milligramme.

† Unit magnetic pole is defined to be a pole which, if placed 1 cm. from an equal and similar pole, would be repelled with a force of 1 dyne, the medium being air.

‡ This electromotive force must be considered to be an experimental fact. The numerical relation existing between the value of the electromotive force and the rate at which the conductor cuts the lines is a result of the definition given to the unit of electromotive force.

If the conductor moves in a direction parallel to the lines of force, it cannot be said to cut them. In such a case no electromotive force results.

If there are a number of conductors, instead of one only, in each of which is induced an electromotive force, these electromotive forces will produce a resultant electromotive force if the conductors are connected together. This will be equal to their sum, if the conductors are connected together in such a manner that the electromotive forces are all directed towards the same end of the composite conductor, which they then form.

The direction in which this electromotive force acts depends upon the relative direction of the lines of force and the movement of the conductor. If the conductor moves at right angles to the lines of force there will be three directions mutually perpendicular to each other, viz.: (1) The direction of the lines of force (from the north to the south pole), (2) the direction of motion of the conductor, and (3) the direction in which the electromotive force is induced, *i.e.*, the direction along the conductor in which it tends to produce a current (from + to —). The relation between these three quantities is most conveniently remembered by Fleming's rule, as illustrated in the annexed Fig. 1.

Using the right hand, and holding the thumb and first two fingers so that each is perpendicular to the plane containing the other two, the three directions are given as follows:—

Thumb.—Direction of motion of conductor.

First finger.—Direction of lines of force.

Second finger.—Direction of electromotive force* induced.

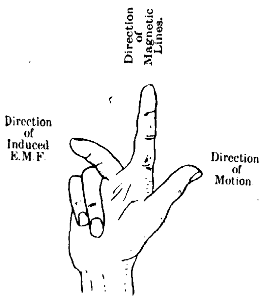


FIG. 1.—Fleming's Rule for a Generator.

* The following mnemonics may help the student in remembering Fleming's rule:

Thu *M* b .... *M* otion.

F irst finger .... *F* orce.

Se *C* ond finger............ *C* urrent.

The Magnetic Effects of a Current.—A current flowing in a straight conductor gives rise to a field composed of circular lines of force, the plane of the circles being perpendicular to the conductor, and their common centre being the centre of the conductor. The direction of these lines of force is related to the direction of the current in the manner indicated in the following rule:—

Place the thumb of the right hand along the conductor so as to point in the direction of flow of the current; if the fingers then grasp the conductor, they will point in the direction in which a north pole would be urged by the magnetic field round the conductor.

In machines, conductors are generally grouped together in windings, forming coils or solenoids. In such cases the same relative direction of current and magnetic field must exist, but usually in these cases it is the field formed *within* the coil which is the most important. The rule for determining the direction of this field may be stated as follows:—

Place the fingers of the right hand so as to follow the direction of the current round the coil, and the extended thumb will indicate the end of the solenoid which will be a north pole.

Action of a Current on a Magnetic Field.—As already stated, when a conductor is moved across lines of force, it has an electromotive force induced in it, tending to produce a current. Conversely, if a conductor situated in a magnetic field carries a current, it will tend to move across the lines of force. In other words, the field will exert a force upon a conductor carrying a current. The direction of the force is such that if the conductor is free to move, it will do so in a direction perpendicular to the lines of force, so as to cut them in the sense necessary to produce an electromotive force opposing the current. Thus a "back electromotive force" is set up in any conductor moving under the mutual action of its current and a magnetic field, opposing the current which causes the motion. In this case, which is that of the conductors in the armature of a motor, the relation between the direction of motion, lines of force, and current is given by Fleming's rule as previously stated (see Fig. 1), except that the *left* hand must be used, instead of the right, the fingers indicating the directions of the same quantities as before.

The magnitude of the force between a conductor carrying current and the field in which it is situated is given by the following formula, absolute C.G.S. units being employed:—

$$\underset{\text{(in dynes)}}{\text{Force acting on conductor}} = \underset{\text{(in absolute units)}}{\text{Current}} \times \underset{\text{(in cm.)}}{\text{Length of conductor}} \times \underset{\text{(in lines per cm.}^2\text{)}}{\text{Strength of field.}}$$

In this form the statement is only true when the conductor is at right angles to the lines of force, and the direction of the force is then perpendicular to the plane, parallel to the lines forming the field, which contains the conductor.

If the conductor is inclined at an angle θ to the lines of force, the pull = current $\times$ length $\times$ field strength $\times \sin\theta$.

When converted into more convenient units, the force is given by the following equations:—

$$f \text{ (lbs.)} = \frac{I.\,l.\,H.}{9{,}810{,}000} \qquad \begin{array}{l} l'' \text{ in inches.} \\ H \text{ in lines per sq. cm.} \end{array}$$

$$f \text{ (lbs.)} = \frac{I.\,l''\,H.''}{1{,}755{,}000} \qquad \begin{array}{l} l'' \text{ in inches.} \\ H'' \text{ in lines per sq. inch.} \end{array}$$

$$f \text{ (kilogrammes)} = \frac{I.\,l''.\,H.}{11{,}303{,}000} \qquad \begin{array}{l} I = \text{current in amps.} \\ l = \text{length of conductor in cm.} \\ H = \text{lines per sq. cm.} \end{array}$$

Production of an Alternating Electromotive Force.—An electrical generator is a machine in which conductors are made to cut lines of force, in order that an electromotive force may be generated in them. The most convenient way of causing conductors to cut lines of force is to make either the conductors or the magnetic field rotate. The conductors usually pass alternately across poles of north and south polarity, and in doing so, the direction of the electromotive force induced in them is reversed as the conductor passes from pole to pole. In both direct and alternating-current generators the electromotive force (and also the current) produced in the armature conductors is an *alternating* one. The difference between a direct and alternating-current generator lies in the method of collecting the current from the armature conductors rather than in the nature of the currents induced. In the direct-current machine the current is rectified at the commutator, while in the alternator the current in the external circuit has the same direction and variations as it has in the armature conductors.

Nature of Alternating Electromotive Force.—In order to study the manner in which the alternating electromotive force of a generator varies, we may take the case of a single conductor travelling in a circular path at a uniform speed in a uniform field between a pair of opposite poles. (See Fig. 2.)

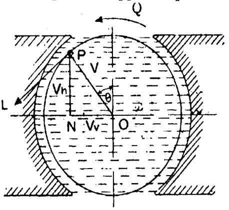


Fig. 2.

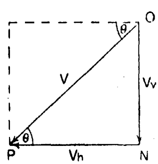


Fig. 3.

Variation in E.M.F. Induced in Rotating Conductor.

The electromotive force generated in the conductor is numerically equal to the rate at which it cuts the lines of force divided by 10^8. Evidently this rate varies with the position of the con-

ductor. Referring to Fig. 2, the electromotive force generated in the conductor P will be zero at the moment when the conductor moves horizontally at the top and bottom of its path, since in this position it moves *along* and not across the lines. The maximum rate of cutting lines occurs when the conductor moves vertically, *i.e.*, perpendicularly to the lines. Any movement horizontally, or parallel with the lines, has no effect in producing electromotive force, whereas the vertical velocity determines the amount of electromotive force generated. It therefore becomes important to consider separately the vertical velocity of the conductor, since this is the factor affecting the voltage generated.

Now a velocity, when represented graphically in magnitude and direction by a straight line, may be resolved into two components in any direction. These components may then be represented by the sides of a parallelogram of which the line representing the velocity forms the diagonal. Thus if $O\ P$ (Fig. 3) represents to scale the constant velocity of the conductor, $N\ P$ and $O\ N$ will represent to the same scale the horizontal and vertical velocities of the conductor when travelling in the direction of $O\ P$.

Thus the sides of the triangle $O\ P\ N$ represent to the same scale the actual velocity of the conductor P and the vertical and horizontal components of this velocity respectively,

$O\ P$ representing the actual velocity.
$O\ N$ representing the vertical velocity.
$N\ P$ representing the horizontal velocity.

Now the triangle $O\ P\ N$, obtained by dropping a perpendicular $P\ N$ on the horizontal centre-line drawn through O, in Fig. 2 is similar to the triangle similarly lettered in Fig. 3, but is turned through a right angle. Thus, if in Fig. 2 the line $O\ P$ is taken to represent in magnitude the velocity of the conductor in a direction at right angles to $O\ P$, $O\ N$ will represent the vertical velocity, and $N\ P$ will represent its horizontal velocity in magnitude. This will be true whatever position the conductor P may have.

Let the electromotive force induced in the conductor when moving vertically have the value V volts. The electromotive force induced when the conductor is moving at the same speed in any other direction, such as the direction $P\ L$ (Fig. 2) is less than V, since the vertical velocity of the conductor is then less, and its actual value will be given by

$$\text{Volts} = V \times \frac{\text{vertical velocity at time considered}}{\text{vertical velocity when conductor moves vertically}}$$

$$= V \times \frac{O\ N}{O\ P} = V \sin \theta,$$

if θ is the angle $P\ O\ Q$ through which the conductor has moved from the vertical axis.

Thus the variations of the electromotive force induced in the conductor as it moves uniformly in a circular path are proportional to the changes undergone by the sine of an angle which passes through all values from 0° to 360°.

The values of the electromotive force may be readily plotted

graphically from Fig. 2, by drawing $O\ P$ in a number of different positions. For each position of $O\ P$ the length of $O\ N$ may be taken to represent the electromotive force on the same scale of volts upon which the radius $O\ P$ represents the maximum voltage. This has been done in Fig. 4, where the path of the conductor P is indicated by a circle, and for each value of the angle $Q\ O\ P$, measured horizontally in degrees, the length $O\ N$ is set up vertically to represent the value of the electromotive force for that position of the conductor, a curve being drawn through the points thus obtained.

It will be seen that the curve obtained in this way is a sine curve. The change of direction of the electromotive force, which occurs when the conductor begins to cut the lines in the reverse direction, is shown by the curve crossing the zero line. Ordinates

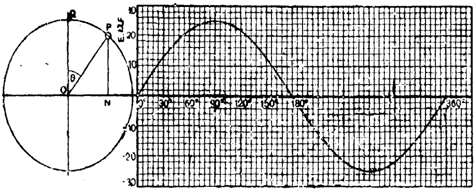


FIG. 4.—Graphic Representation of Variation of Voltage.

above the line indicate electromotive forces in one direction along the conductor. Ordinates below the line represent those in an opposite direction. For the sake of uniformity it is usual to call electromotive forces measured below the line *Negative*.

The curve shown in Fig. 4 may be taken to represent the variations of voltage given by an alternator. For example, the maximum voltage on the vertical scale of the diagram is 25. It would be possible to obtain from the curve the exact voltage given out by the machine for any position of the armature if the position of maximum or minimum voltage is known.

The complete curve shown for a variation from 0° to 360° corresponds to a rotation of the armature or field of the alternator equal to the angular displacement between the centre of one N pole and the centre of the next N pole. In a 2-pole machine this is one complete revolution.

Definitions.—The complete series of changes represented in Fig. 4 is called a *Cycle*; the time taken for the electromotive force to pass through these values is termed a *Period*. The *Periodicity* of an alternating circuit is the number of periods in one second. Thus an alternator giving a periodicity of 50 would impress on the circuit the series of changes shown in the figure, 50 times per second. *Frequency* is another term for periodicity.

The *Amplitude* is the maximum value of the variable quantity which passes through the changes shown above. Thus the length $O\ P$ is the amplitude in Fig. 4, corresponding to 25 volts.

The angle measured from the starting point Q to any particular point P on the curve is called the *phase* of this point; thus in Fig. 4 the phase of P is measured by the angle $Q\ O\ P$.

Example.—An alternator gives a maximum of 100 volts. If its frequency is 50, what will be the phase and value of its voltage after $3\frac{2}{3}$ secs., starting from the point of zero electromotive force? In three seconds it will have passed through 360° 150 times, and in $\frac{2}{3}$ secs. it will further pass through $\frac{2}{3}\times 50$ of 360°, *i.e.*, $33\frac{1}{3}$ times 360°. Hence at the end of $3\frac{2}{3}$ secs. it will have passed through the complete cycle of changes 183 times, and its phase will be $\frac{360}{3}$ or 120°. The value of the electromotive force will then be V sin θ, or

$$100 \sin 120^\circ = 100 \times \cdot 866 = 86\cdot 6 \text{ volts.}$$

Effect of Increase in Number of Poles.—If the alternator has more than two poles, its armature will probably be provided with at least as many conductors as there are poles. Assuming this minimum number of conductors to be joined together in series, they will all contribute to the total voltage of the armature winding. If, further, the conductors have exactly the same pitch as the poles of the alternator, they will enter and leave the polar flux simultaneously, so that the voltages induced in all of them are identical in phase. The voltage of the complete winding is then numerically equal to the sum of the individual voltages, and has the same phase as they have.

In the case of a multipolar machine, a period will be the time during which a conductor moves from one pole to a similar position under the next pole of the same polarity. Thus in a 12-pole alternator there will be six periods during each revolution of the armature. In general, if

n = revs. per minute of armature, or field,
f = periodicity,
p = number of pairs of poles,

$$f = \frac{n\ p}{60} \text{ cycles per sec.}$$

Large machines which run at comparatively low speeds have a large number of poles, in order to supply current of the frequency usually required.

In an electrical distribution system the periodicity is a fixed quantity, and the number of poles and speed of the generators have to be adjusted to suit this condition.

A cycle is often said to have 360 "electrical" degrees, in order that a distinction may be made between the phase angle of induced electromotive force and the angle of displacement in space of the field. Thus, in a 12-pole machine, $\frac{1}{6}$ of a revolution of the field system, *i.e.*, an actual rotation in space through 60°, would be equivalent to a phase angle of 360 "electrical" degrees.

Increase in Number of Conductors.—In order to increase the voltage generated in the armature, the number of conductors connected in series is usually increased.

The total voltage generated in an armature composed of a great number of conductors will not be equal to the sum of the voltages induced in the individual conductors unless the following conditions are fulfilled: (1) Conductors under opposite poles must be joined together alternately at the front and back of the armature, so that the electromotive force which is induced in successive conductors in opposite directions is made to act in the same sequence in the complete winding. (2) The conductors passing under a single pole at any time must be grouped together sufficiently closely to enter and leave the pole nearly simultaneously.*

The connections of such a winding are shown diagrammatically in Fig. 5.

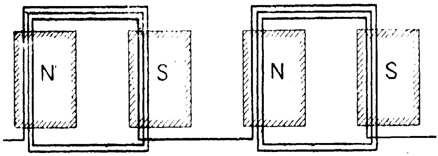


FIG. 5.—Connection of Conductors in Multipolar Armature.

Production of Current by an Alternator.—If the armature winding of an alternator is connected to a closed circuit, the electromotive force generated in the armature will send a current through the circuit. The strength of this current will depend on the ratio between the armature voltage and the resistance of the circuit. Since the armature voltage is a constantly varying quantity, the current in the circuit will undergo the same variations.

Thus at any instant † $I = \frac{E}{R}$

Where I value of current,
E „ „ voltage,
R resistance of circuit.

Since the current will go through similar variations to the voltage, it may be represented by a curve in the same way as the voltage in Fig. 4.

E.M.F. of Self-induction in a Winding.—A current flowing through a winding always produces a magnetic field. The magnitude and direction of the field will depend upon the magnitude and direction of the current. Consequently, an alternating current is always accompanied by a rapidly *changing* field. The effect

* The effect of a distributed winding is discussed later.
† The effect of self-induction is, of course, not considered in this example.

of the varying field upon the circuit in which it is produced must next be considered.

Imagine a coil of wire through which an alternating current is flowing.

The number of lines of force which are produced within the coil will be proportional to the amount of current flowing*; consequently, every variation in current produces a proportionate change in the number of lines passing through the coil. The formation of lines of force passing through a coil is equivalent to the movement of that number of lines from the outside to the inside of the coil. That is to say, the formation of 1,000 lines of force passing through the coil will produce the same effect as if the movement of a magnet caused 1,000 lines of force to be cut by the conductors forming the coil.

As stated on page 13, an electromotive force is produced in a conductor when it is made to cut lines of force. Hence the change of field produced in a coil by a change of the current in it will also produce an electromotive force. This electromotive force is called the *electromotive force of self-induction.* The magnitude of the electromotive force is governed by the same rule as already given on page 13 for the electromotive force induced by motion in a magnetic field. The voltage induced in each conductor of the coil is numerically equal to the number of lines cut per second divided by 10^8. The direction of the electromotive force thus produced is always such as to oppose the change of the current to which the electromotive force is due.

Thus a current started in a conductor will produce a magnetic field, which will in turn produce an electromotive force in the conductor opposite to the direction of the current. Similarly, the stoppage of a current will cause a disappearance of magnetic lines, and consequently an electromotive force, in the direction which tends to maintain the flow of current. Since an alternating current is constantly changing both its magnitude and direction, there will be electromotive forces constantly induced, and always tending to oppose the change of current occurring at the time. Because the induced electromotive force opposes the changes occurring in the current, it is often called the "back electromotive force" or "counter electromotive force" due to self-induction. The electromotive force of self-induction depends upon the rate at which the current changes, and not directly on the *amount* of current or the *number* of lines of force in the circuit.

In order to make clear this relation between the electromotive force of self-induction and the current in a circuit, it will be best to draw two curves representing respectively the successive values of the current and the corresponding values of the rate at which the current changes. This is shown in Fig. 6, where Curve I. shows the values of the current whose maximum is 12·5 amperes, and the

* Except in circuits containing iron, when this is only approximately true.

dotted curve No. II. shows the change of current measured in amperes per cycle.

To obtain the second curve from the first, draw at a number of points tangents to curve No. I., as shown by the line P M, which is the tangent at P. From P draw a horizontal line P N corresponding in length to $\frac{1}{4}$ cycle, and a vertical line through N to meet the tangent at M. The length N M then shows the change in current which would occur in $\frac{1}{4}$ cycle if the rate of increase or decrease remained the same as it actually is at P.

Four times the length of M N will consequently give the height of the "rate of change" curve at the point corresponding to the

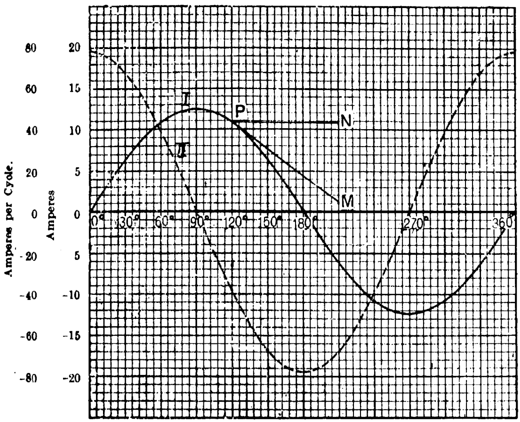


FIG. 6.—Curves of Current and Rate of Change of Current.

same angle as P. The scale of amperes per cycle for the dotted curve (Fig. 6) is taken four times as great as the scale of amperes for the original curve, so that, when the length N M is plotted vertically, it corresponds to four times the change of current in $\frac{1}{4}$ cycle, *i.e.*, to the rate of change of current for a whole cycle.

When N M is drawn downwards, as in the case shown, its slope indicates that the current is decreasing, and that the rate of change of current is negative. Accordingly, it is plotted below the base-line in Curve II.

It will be noticed that the Curves I. and II. in Fig. 6 are similar in shape, but that the second curve is displaced by one-quarter of a period to the left of the Curve I. From this it is apparent that the *change* of current is most rapid at the point where the current

curve crosses the axis and changes its direction. In other words the *rate of change*, is greatest when the *current* is zero. The rate of change is zero, on the other hand, at the moment when the current has attained its maximum value in either direction.

The maximum value of the "rate of change of current" curve is seen to correspond to a change of 78·5 amperes per cycle, *i.e.*, the maximum rate of change of current measured in amperes per cycle is $\frac{78 \cdot 5}{12 \cdot 5} = 6 \cdot 3$ times the maximum value of the current. More accurately, the maximum rate of change per cycle is 2π times the maximum current. If the current passes through f cycles per second, the maximum rate of change of current measured in *amperes per second* will be $2\pi f \times$ the maximum value of the current.*

Consequently, a curve showing rate of change of current *per second* would be a curve similar to Curve II., but having each ordinate f times as great.

Since the number of magnetic lines in the circuit is proportional to the current, Curve I., which represents the current, will also represent these lines to some scale. Again, because the electromotive force of self-induction is proportional to the rate of change of the lines, Curve II. will represent this electromotive force to some scale. Curves I. and II. will thus represent (on some scale) the corresponding values of the current and of the electromotive force of self-induction.

The actual value of this electromotive force must now be investigated.

Coefficient of Self-induction.—Let i = value of current in the coil at any moment;

T = number of turns composing the coil;

F_1 = number of lines of force produced in the coil when $i = 1$ ampere.

Then, when the current changes at the rate of 1 ampere per second, the change produced in the number of lines entering the coil will be F_1 per second. The total voltage induced in the coil by this change will be

$$\frac{\text{lines cut per second by each coil} \times \text{number of coils}}{10^8}$$

or in symbols—

$$\text{Voltage} = E = \frac{F_1 . T.}{10^8}$$

The fraction $\frac{F_1 . T.}{10^8}$ is called the coefficient of self-induction of the coil and is generally denoted by L.

* The reason for this exact relation between current and rate of change of current is that the rate of change of the sine of an angle is equal to its cosine when the angle is measured in units of circular measure or radians. Since there are 2π radians in one cycle, or 360°, the rate of change per cycle is $2\pi \times$ the cosine, also the maximum value of the cosine is equal to the maximum value of the sine. The two curves have a relative displacement of 90°. If points on Curve I. are represented by $I \sin \theta$, the corresponding points on Curve II. will be given by the expression $2\pi I \cos. \theta$.

Definition.—The coefficient of self-induction of a coil is numerically equal to the electromotive force induced in it by a change of current of 1 ampere per second,

or

The coefficient of self-induction is numerically equal to the number of magnetic lines formed in a coil by a current of 1 ampere multiplied by the number of turns of the coil through which these lines are threaded, divided by 10^8.

This coefficient is measured in terms of the *henry*. The henry is thus the unit of the coefficient of self-induction.

The coefficient of self-induction of a circuit is frequently called the *inductance* of the circuit.

A milli-henry $= \frac{1}{1{,}000}$ henry, and is generally used in stating the properties of a circuit, because the inductance of a circuit is usually a small part of one henry.

The coefficient of self-induction of a coil carrying a current of I amperes, which produces in it a flux of F lines, may be conveniently expressed as follows :—

$$L = \frac{\text{linkages of flux with turns}}{\text{current} \times 10^8} = \frac{F \times T}{I \times 10^8}$$

Relation between Current and Voltage of Circuit.—It follows from our definition of the coefficient of self-induction that the electromotive force of the self-induction in the circuit is

$$E = L,$$

when the current changes at the rate of 1 ampere per second.

If the current changes at the rate of $\dot{c}$ amperes per second, then the induced voltage will be

$$E = \dot{c}\, L.$$

Since the dotted curve in Fig. 6 shows the rate of change of current *per cycle*, it would give the rate of change *per second* if its ordinates were multiplied by f. It would then be the curve of $\dot{c}$.

Hence we might obtain a curve showing the fluctuations of the electromotive force of self-induction by drawing a curve in which each ordinate of Curve II. (Fig. 6) is multiplied by $f\ L$. The curve would have to be plotted to a vertical scale of volts, the horizontal scale being the same as for the other curves.

Since the maximum rate of change of current per second = maximum value of $\dot{c} = 2\ \pi\ f\ I$, where I = maximum value of current, we may write as the maximum value of the electromotive force of self-induction

$$E = 2\ \pi\, f\ I\ L.$$

From what has just been stated, it appears that when an alternating current flows in a circuit, it will give rise to an alternating electromotive force opposing the change of current at every moment. Also, if the Curve I in Fig. 6 is taken to represent the values of such a current, a curve similar to Curve II. (but drawn to scale $f\ L$ times as great) will show the values of the voltage which must be applied to the circuit in order to overcome the back electromotive

force of self-induction caused by the changing field which the change of current produces.

Such a curve of voltage would be $\frac{1}{4}$ period *in advance* of the current curve in phase, as shown by the fact that the voltage curve would reach its maximum and minimum values always 90° earlier than the curve of current.

We are now in a position to apply the results shown in the curves in Fig. 6 to explain the conditions governing the production of a current.

From Ohm's law we know that, when a current flows in a circuit having resistance, an electromotive force must be applied to the circuit having a value at any instant equal to the product of the current multiplied by the resistance of the circuit. This will evidently be an electromotive force which varies with the current, and is *in phase* with it, *i.e.*, passes through its maximum and minimum values at the same time as the current.

In Fig. 7 is represented a current having a maximum value of 25 amperes (Curve C). This current is supposed to be flowing in a circuit having a resistance of 1·2 ohm. The dotted curve E, represents the electromotive force which must be applied to the circuit in order to overcome this resistance. This curve is obtained by multiplying each ordinate of the current curve by 1·2. The maximum value of this electromotive force is $25 \times 1{\cdot}2 = 30$ volts. In Fig. 7 current and volts are represented to the same scale.

The circuit in which the current flows has a coefficient of self-induction of 5 milli-henries = ·005 henries.

The change of current will produce an electromotive force of self-induction in the circuit, which must also be overcome by the alternator supplying the circuit, in order that the current C may be maintained. This applied voltage is $\frac{1}{4}$ period in advance of the current. The curve representing it might be obtained by drawing a curve similar to Curve II. in Fig. 6, and then multiplying each ordinate by f L. A more direct way is the following :—

It has already been pointed out that the maximum value of the electromotive force of self-induction $= 2\,\pi\,f\,I\,L$, where I is the maximum value of the current, and that the curve representing this electromotive force is $\frac{1}{4}$ period in advance of the current in phase. Consequently, we may directly draw a since curve having a maximum value of $2\,\pi\,f\,L\,I$ and placed $\frac{1}{4}$ period in advance of the curve of current.

This has been done in Fig. 7, and is shown as the dotted curve E_1 whose maximum value is

$$2\,\pi\,f\,L\,I = 2\,\pi \times 50 \times {\cdot}005 \times 25 = 39{\cdot}3 \text{ volts,}$$

where f = periodicity = 50 cycles per second.

The curves drawn in Fig. 7 represent completely the conditions which usually exist in an alternating circuit. These may be briefly recapitulated as follows :—

A current I (Curve C) to produce which required the application of an electromotive force varying in the same manner as, and simultaneously with, the current in order to overcome the resistance

of the circuit. This electromotive force (Curve E_r) is always numerically equal to the product of $I \times R$ where R is the resistance of the circuit.

Further, an electromotive force overcoming the electromotive force produced by, and opposing the variation of the current (Curve E_i). This electromotive force is numerically equal to $\dot{c}\,L$, where $\dot{c}$ is the rate at which the current varies (measured in amperes per second), and L is the coefficient of self-induction of the circuit. The curve representing this voltage is similar in character to the curve of current, but a quarter of a period earlier in phase.

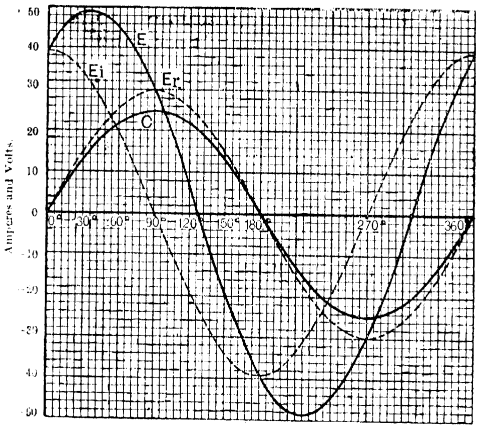


Fig. 7.—Curves of Current and Component and Total E.M.F.

Resultant Electromotive Force.—In the previous paragraph it was explained that two electromotive forces are required to maintain an alternating current, viz., one electromotive force in phase with the current to produce the *passage* of the current. This may be called the resistance electromotive force, and is the electromotive force which would be required in a continuous-current circuit to maintain the flow of the current.

A second electromotive force produces the *variation* of the current, and may be called the inductance electromotive force. This is only required in a circuit in which the current changes, and constitutes the main difference between alternating and continuous current problems. The inductance electromotive force

is a quarter of a period earlier in phase than the resistance electromotive force, that is, the curve is displaced to the left in the diagrams.

An alternator must supply sufficient voltage to a circuit to be equivalent to both of these electromotive forces if an alternating current is to be maintained. The actual voltage supplied by the alternator at any instant must be equal to the sum of the values of the two individual voltages at the same instant, and may be plotted as a curve, the height of which at each point is equal to the sum of the ordinates of the voltages E and E. This curve has been obtained in Fig. 7, where the resultant voltage of the alternator (Curve E) has at each point a height equal to the sum of the heights of these curves.

It is to be observed that (1) this curve is again similar in character to the previous curves; (2) its maximum value (49 volts) is greater than that of either of the component electromotive forces, but is less than the sum of the two; (3) the resultant electromotive force is intermediate in phase between the two component electromotive forces.

Angle of Lag.—A further most important result is to be noted from an inspection of the curves just given, viz., that the curves representing the current and the resultant voltage (*i.e.*, the current and total voltage applied to the circuit) are not in phase with each other. In the curves given in Fig. 7, it will be seen that the curves of current and total voltage pass through zero at points situated 0·145 period or 52¼° apart.* The angle between these curves, which is most conveniently measured as the angle on the horizontal scale of the curve between the points at which they pass through their zero value, is the *angle of phase difference* between current and voltage. The angle is called the *angle of lag* of the circuit if the current passes through zero *after* the voltage, or the *angle of lead* if the current passes through zero *before* the voltage. The current always lags behind the applied voltage, except in cases where the circuit possesses electrostatic capacity or supplies over-excited synchronous motors—conditions discussed later on.

The angle of lag of the current in a circuit is of great practical importance, and depends upon the nature of the resistances and other apparatus forming the circuit.

The student cannot be too careful to familiarise himself with the reasons for the fact that in all alternating circuits possessing self-induction there is a difference of phase between current and voltage.

He should carefully work the following examples, and, if necessary, read again the foregoing pages.

Example.—A circuit has a resistance of ·5 ohm., and an inductance of 20 millihenries. It carries an alternating current whose

* A complete period is always taken as corresponding to 360°, or an angle of 2π radians in circular measure.

maximum value is 24 amperes. Plot curves on square paper to represent the following quantities:—

(*a*) The current. (*b*) The rate of change of current per cycle. On a separate sheet plot the following electromotive forces, making use of the curves (*a*) and (*b*). (*c*) Electromotive force necessary to overcome resistance. (*d*) Electromotive force necessary to overcome self-induction. From (*c*) and (*d*) obtain the curve (*e*) representing the total electromotive force necessary to maintain the current.

Cautions.—It may not be out of place to mention one or two points which frequently cause confusion at first in connection with curves similar to those just discussed.

(1) A curve displaced towards the *right* is *later* in phase. This is the contrary of what one might expect on considering the matter carelessly, but is evident from the construction given at first for obtaining the curve.

(2) The voltage applied to a circuit is opposite in direction to the voltages set up in the circuit due to resistance of self-induction. This may appear obvious, but neglect of this distinction will cause confusion. In the diagram Fig. 7 the electromotive forces shown are those supplied from the external source to the circuit, and not those set up in the circuit. They are on the same side of the horizontal axis, and therefore in the *same* direction, as the current, whereas the opposing voltages must obviously be in the opposite direction, *i.e.*, on the opposite side of the zero line.

Thus the curve marked E in Fig. 7 is the voltage overcoming (and, therefore, oppositely directed to) the voltage due to the resistance of the circuit which opposes the flow of current. The lattter might be represented by a similar curve to E at equal distances on the opposite side of the base line.

The distinction here alluded to has been carefully observed in the text, but the difference between the electromotive forces *due* to and *overcoming* resistance or inductance is sometimes confusing to the student, on account of the loose use of the terms often applied to them. In general, it is only the voltages *overcoming* resistance, reactance, &c., which are spoken of, or represented in diagrams.

Vector Diagrams.—The method of representing variable quantities such as currents and electromotive forces by curves is exceedingly laborious, and consequently a more rapid but equally complete method of representation is usually employed.

On page 18 was explained the method of obtaining points on the curve of electromotive force. A point on the curve was found corresponding to each position of the radius $O\ P$ in Fig. 2 by marking off a height equal to the horizontal projection $O\ N$ of $O\ P$. The complete curve was determined by imagining that P made a complete revolution of the circle, and thus gave points forming one complete period of the curve.

By drawing the line $O\ P$ of such a length that it represents some number of volts to a fixed scale, and imagining it to rotate about the end O, it may be taken to represent the varying electromotive force, without the curve obtained from its successive

positions being actually drawn. It must then be remembered that the actual value of the variable quantity represented by $O\ P$ may always be obtained by measuring the length of the horizontal line drawn from O to meet a vertical through P. The length of this horizontal line is usually called the horizontal projection of $O\ P$. The length of $O\ P$ itself will be the *maximum* value of the variable quantity represented by it. Its phase is represented by the angle between $O\ P$ and the vertical through O, while the actual value of the quantity when in this phase is represented by the horizontal projection $O\ P$ for the position in which $O\ P$ is drawn.

When drawing such a line to represent a varying quantity, an arrow-head is shown at one end to indicate that this end is to be taken as rotating about the other. Further, a convention must be adopted with regard to the direction in which rotation occurs. It is now universally agreed that this rotation shall be anti-clockwise, although the opposite direction of rotation was adopted by a considerable number of writers in the past. The rate of the imaginary revolution of the line will be equal to the frequency of the current or electromotive force represented. On a complete

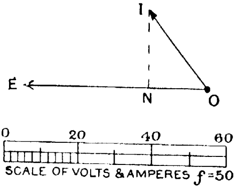


FIG. 8.—Vector Diagram of Voltage and Current.

diagram this should be indicated as in Fig. 8, thus $f = 50$, including a frequency of 50 cycles per second, so that the vector $O\ I$ must be taken to rotate 50 times per second.

By means of the convention just described, a single line is sufficient to indicate the maximum value and manner of variation of a periodic quantity, and also to indicate it's phase and instantaneous value at any moment. This method further enables us to show the relation between two such quantities which have the same periodicity, but which may have any relative magnitude and phase. The quantities need not necessarily be of the same nature. Thus, for instance, the relation between a current and a voltage may easily be shown on the same diagram by two different lines. We then have a simple example of a vector diagram.

The scale to which the lines have been drawn should be indicated whenever use is made of vector diagrams.

The difference in phase between the various quantities is measured directly as degrees of angle between the lines representing them in the vector diagram, 360° being one complete period.

A vector diagram thus consists of a number of rotating vectors, which, although drawn as fixed lines, are to be imagined as rotating at the speed of f revolutions per second. The instantaneous value of any one of the quantities is obtained by projecting its vector on a fixed axis (according to our assumption, the horizontal axis).

By way of illustration, Fig. 8 has been drawn to represent the current and voltage of the circuit given by the curves in Fig. 7. Consequently the vector $O\ I$ represents 25 amperes on the scale of amperes, this being the maximum value of the current. Similarly $O\ E$ represents 49 volts, the maximum value of the voltage. The angle $I\ O\ E$ between the vectors is $52\frac{1}{2}°$, which was found to be the angle of lag of the circuit.

In order to distinguish between lengths representing electromotive forces and those representing currents, the following distinction will be observed in all figures subsequently given, where the meaning of the vectors is not otherwise evident. Vectors indicating electromotive forces are shown with thin arrow-heads. Vectors representing currents have thick black arrow-heads. Also vectors referring to magnetic flux have arrow-heads with double barb.

The diagram Fig. 8 represents the instantaneous condition of current and voltage at the instant when the voltage has its maximum value, since the line $O\ E$ is shown horizontal; it will thus have its maximum horizontal projection at this moment. The line $O\ I$, representing the current, is drawn $52\frac{1}{2}°$ later in phase, the angle $I\ O\ E$ being $52\frac{1}{2}°$, and the direction of the rotation being anti-clockwise. Thus the current will reach its maximum value $\frac{52\frac{1}{2}}{360}$ of a cycle later than the instant for which Fig. 8 is drawn. The instantaneous value of the current in Fig. 8 is obtained by drawing a vertical line $I\ N$ and measuring the length of $O\ N$ on the scale. It is seen to be 15·3 amperes, which corresponds with the value measured on the curve in Fig. 7 for the same phase.

Resultant of Two Vectors.—Fig. 9 shows by means of vectors the component and resultant electromotive forces of Fig. 7; the instant for which the diagram is drawn is not the same as in Fig. 8. The two voltages E_R and E_X are drawn at right angles, since they differ in phase by 90° The lettering will make obvious the identity of the vectors. The scale is the same as for Fig. 8.

As before, the instantaneous values could be found by drawing verticals through the rotating end of each line. The student should identify the phase and instantaneous values of the vectors of Fig. 9 in Fig. 7.

In this diagram it will be found that if the two component voltages $E_R\ E_X$ are taken to be two sides of a parallelogram, the completed parallelogram would have the vector E as its diagonal. That is, the resultant electromotive force would be represented in magnitude and phase by the diagonal of a parallelogram of which the two components formed the sides.

That this must always be true follows from the following consideration. Since the instantaneous value of the resultant electromotive force is the sum of the values at each instant of the component electromotive forces, the horizontal projection of the vector representing the resultant must always equal the sum of the projections of the components, because the horizontal projections are instantaneous values of the quantities.

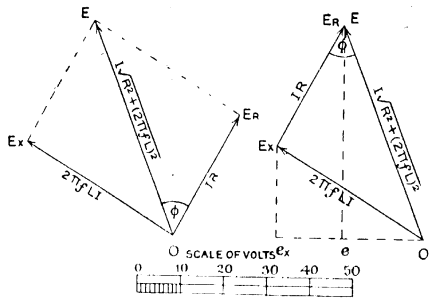


FIG. 9. FIG. 10.

Resultant and Component Electromotive Forces.

The easiest way to obtain the sum of the projections of two such lines as E_R E_X is to draw one of them (say E_R) from the end of the other (E_X) (see Fig. 10), so that the arrows of the two lines point in the same direction. The horizontal projection of E_X is then Oe_X and the horizontal projection of E_R is $e_X\ e$. Since the projection of E_R is measured from left to right, instead of from right to left, it must be considered negative, and thus the sum of the two projections is Oe. But Oe is the same as the projection of the line $O\ E$ joining the free ends of E_R and E_X. $O\ E$ is, however, the diagonal of the parallelogram whose sides are E_R and E : the line $O\ E$, Fig. 10, is, in fact, the same as $O\ E$ in Fig. 9. Hence the instantaneous value of the resultant of the two varying quantities is obtained from the diagonal of the parallelogram of which the quantities form the sides.

The result just obtained is of the greatest practical importance and shows that, when represented by vectors, two electromotive forces or two currents may be combined together in exactly the same way as may the forces or velocities illustrated in mechanics by straight lines, which are combined to form single resultant forces or velocities by the law of the parallelogram of forces.

A similar rule to this may therefore be given in the following terms. The resultant of two currents or electromotive forces in a circuit may be obtained by representing the two quantities in

magnitude and phase by two vectors drawn from a common point and both directed away from this point. The vector of the resultant quantity is the diagonal of a parallelogram of which the two vectors form the sides. Its direction will also be away from the common point. This form of the rule is illustrated by Fig. 9. The method shown in Fig. 10 for obtaining the resultant may be put into words as follows :—

Draw the vectors of the two quantities with the fixed end of one upon the rotating end of the other, *i.e.*, with the arrows of both directed towards the same end of the resulting bent line. The vector of the resultant is the line joining the free ends of the bent line thus obtained. Its direction will be such that its arrow points round the closed figure formed by the vectors in the opposite direction to the arrows of the component quantities.

This construction may be extended to any number of component quantities. The resultant of them all is always the line joining the free ends of the figure obtained by drawing all the quantities in regular sequence, the fixed end of one vector coinciding with the rotating end of the previous one. The angle which the resultant line makes with the components represents its phase relative to them.

The application of the construction just given may be illustrated by the following experiment :—

EXPERIMENT I.—DETERMINATION OF THE RESULTANT ELECTROMOTIVE FORCE OF TWO ALTERNATORS COUPLED TOGETHER.

DIAGRAM OF CONNECTIONS.

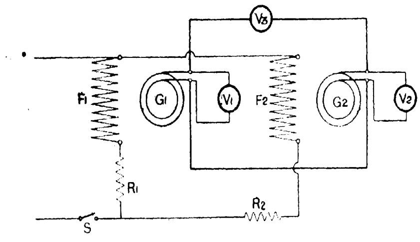


FIG. 11.

G_1, G_2 Two alternators coupled mechanically
F_1, F_2. Alternator field windings.
V_1; V_2 Voltmeters for reading voltage of each machine separately.
V_3. Voltmeter for reading resultant voltage.
R_1, R_2. Field regulating resistances.
S. Switch in field circuit.

Note.—In the case of this and a number of other experiments it is not necessary to use the full number of voltmeters shown in the diagram. A single voltmeter with throw-over switch may be used to take the readings of the two alternators. This is often preferable, not only in order to save instruments, but also in order to save errors due to uncertain constants of the instruments, &c.

INSTRUCTIONS.—Two alternators have their shafts mechanically coupled together by a coupling which can be adjusted to alter the relative angle of the rotating parts of the machines.

Excite the field windings from a supply of continuous current.

Insert in the field of each machine a field regulator, so that the fields can be independently regulated. Connect a voltmeter to the terminals of each alternator. Connect the two armatures together, putting a third voltmeter in this circuit, in order to read the resultant voltage of the two alternators.

Run the alternators at a constant speed, and take simultaneous readings on the three voltmeters, entering the readings in the proper columns of the table given below. Make the same observations for a series of relative positions of the machine coupling, which should be altered from the point where the machines are in series to the position when they are in parallel, or opposition.

From the readings of the voltmeters the angle of phase difference between the machines can be calculated, without reference to any graduation of the coupling.

RESULTANT VOLTAGE OF TWO ALTERNATORS.

Volts Machine A	Volts Machine B	Resultant Volts A and B	Phase Angle between Machines
72	72	136	36°
72	72	112	76°
72	72	52	136°

In order to determine the angle of phase difference between the machines, construct for each set of readings a triangle of which the sides represent to some convenient scale the three voltmeter readings. In doing this it will be convenient to draw a horizontal line OB (Fig. 12) to represent the resultant voltage. From one end of this line draw a circle whose radius is equal on the same scale to the voltage of one alternator. From the other end draw a circle with a radius equal to the voltage of the other alternator. Join the intersection A of these circles to the ends O, B of the horizontal line. The triangle OAB so obtained shows graphically the relations between the voltages of the two machines and the total voltage which they would supply to an external circuit. The

external angle between the sides representing the alternator voltages is the angle of phase difference between the machines. Fig. 12 shows the construction carried out for the first set of readings of the table given above. A curve should be plotted showing the resultant voltage at various angles of the coupling.

The resultant voltage will vary between a maximum when the voltage of both machines is added, and a minimum when the machines are in opposition, when the resultant voltage will be the difference between the two.

In carrying out the experiment just described, in order to determine the angle between the machines, it will be found convenient to excite the alternators to give *equal* voltages.

As an illustration of this experiment, the curve shown in Fig. 13 has been drawn. Two similar alternators with an adjustable coupling were excited to give 72 volts each From the graduation

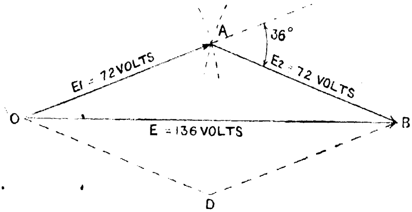


FIG. 12.—Construction for Determination of Angle between Two Alternators.

of the coupling the resultant value of the voltage was calculated for a series of relative positions of the machines. These values are shown on the curve, together with a set of observed values read on a voltmeter in series with the machines.*

The preceding experiment is not generally capable of any direct useful application, and is given here chiefly as illustrating the way in which two voltages may be applied to a circuit and the phase angle between them obtained by direct measurement of the resultant.

* The resultant value was calculated as follows :—

Let E = R.M.S. voltage of each alternator.

E_m = maximum voltage of each alternator

ϕ = phase angle between alternators.

Instantaneous values are

$$e_1 = E_m \sin \theta, \qquad e_2 = E_m \sin (\theta + \phi).$$

Instantaneous value of resultant voltage is

$$e_1 + e_2 = E_m \left[\sin \theta + \sin (\theta + \phi) \right] = 2 E_m \sin \left(\theta + \frac{\phi}{2} \right) \cos \frac{\phi}{2}$$

Voltmeter reading is R.M.S. value of this, or

$$2 \cos \frac{\phi}{2} \times \text{R.M.S. value of } E_m \sin \left(\theta + \frac{\phi}{2} \right) = 2 \cos \frac{\phi}{2} E.$$

The meaning of "R.M.S." value is explained later.

It should be noted, however, that two alternators which can be coupled at any angle form a valuable addition to the equipment of a test-house where meters, &c., have to be tested with currents of varying phase difference. In such cases current may be taken from one machine and voltage from the other, with any desired angle between them This experiment might in such cases be turned to practical account.

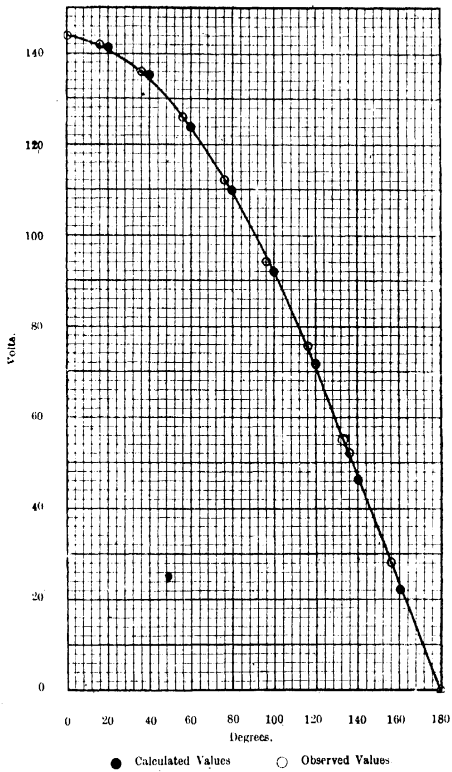


FIG. 13.—Curve of Resultant Voltage of Two Alternators in Series.

The geometrical construction made use of in the last experiment is employed in the following measurement, which has many important applications.

EXPERIMENT II.—DETERMINATION OF ANGLE OF LAG IN AN ALTERNATING CIRCUIT BY THREE VOLTMETERS.

DIAGRAM OF CONNECTIONS.

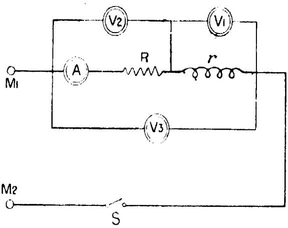


FIG. 14.

M_1, M_2. Source of alternating current.
r. Resistance partly inductive and partly non-inductive.*
R. Non-inductive resistance.
V_1 Voltmeter reading voltage of inductive resistance r
V_2 Voltmeter reading voltage of non-inductive resistance R.
V_3. Voltmeter reading total voltage of $R + r$.
A. Ammeter.

Instructions.—The portion of the circuit in which the angle of lag is to be determined will consist partly of inductive and partly of non-inductive resistances; this is represented by r in the diagram.

Insert in the circuit a non-inductive resistance, R, and an ammeter.

Connect three voltmeters † to points in the circuit so as to measure respectively (1) the voltage at terminals of non-inductive resistance (V_2), (2) voltage of r (V_1), (3) total voltage of circuit (V_3).

Take simultaneous readings on the ammeter and three voltmeters, and enter them in columns headed as given below:—

Current.	Voltage of Non-inductive Resistance V_2.	Voltage of Remainder of Circuit V_1.	Total Voltage of Circuit V_3.	Angle of Lag between V_2 and $V_3 = \phi$
9·0	84·0	44·0	105·5	23°
11·2	75·0	53·0	104·9	29°
12·14	71·3	56·0	103·5	31°
14·67	64·0	62·0	100·7	36·6°

* All so-called "inductive resistances" are partly non-inductive, since they necessarily have some ohmic resistance.

† A single voltmeter and multiple-way switch may be used instead of the three voltmeters.

The reading should, if possible, be repeated for several values of the current, obtained by altering the voltage of supply or by a resistance in series with R and r.

The angle of lag of the circuit must then be determined for each set of readings by construction similar to that described in the previous experiment.

Draw a horizontal line to represent to a convenient scale the total voltage of the circuit. Draw from opposite ends of this line circles of radius equal respectively to the two other voltages. By joining the intersection of the circles to each end of the horizontal line, complete the triangle. This will then be a triangle of electromotive forces for the circuit. (See Fig. 15).

The electromotive force measured at the terminals of the non-inductive resistance will be in phase with the current in the

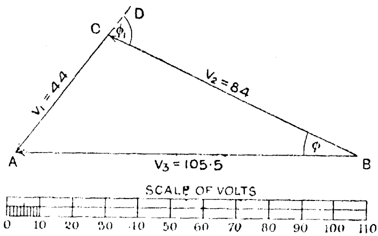


FIG. 15.—Triangle of E.M.F.

resistance, since there is no self-induction. The electromotive force V_2 is, therefore, always equal to $I \times R$, the product of the current and the constant resistance.

Consequently, the phase difference between this electromotive force and the electromotive force of the remaining part of the circuit will be the same as the phase difference between the current and the voltage of the remainder of the circuit. Thus in the triangle (Fig. 15) the angle ϕ_1 will be the angle of lag in the portion r of the circuit.

The angle ϕ between V_2 and V_3 will be the angle of phase difference between the current and total voltage of the circuit, *including* the added non-inductive resistance.

The readings entered on the table are those taken from an actual experiment, the corresponding vector diagram for the first set of readings being drawn in Fig. 15.

Example.—(1) An alternator giving a voltage of 200 supplies an alternating circuit in which is inserted a non-inductive resistance of 0·2 ohm. The current is found to be 32 amperes, and the voltage of the part of the circuit not including the ·2 ohm resistance is 198 volts. Draw a diagram of the various voltages in the circuit,

and ascertain therefrom the angle of lag between the current and voltage given by the alternator.

(*Note.*—In solving this problem remember that the value of the voltage in the non-inductive resistance is the product of current and resistance.)

(2) An arc lamp is connected to a 100-volt alternating supply in series with an inductive resistance. The arc is found to take 30 volts, and the inductive resistance 90 volts. Find by diagram the angle of lag of the circuit.

(*Note.*—The arc lamp is to be taken as behaving like a non-inductive resistance.)

Magnitude of Resultant Voltage.—We are now in a position to ascertain the relation between the resultant voltage in a circuit and the current, resistance, and self-induction.

Fig. 9 (page 31) shows the method of obtaining graphically the magnitude and phase of the resultant voltage when the component voltages are known. Since the two component voltages spent in overcoming respectively the resistance and inductance of the circuit are known to be always $\frac{1}{4}$ period, or 90°, apart in phase (one being in phase with the current and the other $\frac{1}{4}$ period in advance of it), the angle $E_R\,O\,E_X$ will always be a right angle. Consequently the length of the diagonal representing the resultant $= \sqrt{\text{sum of squares of 2 sides}}$, or

$$O\,E^2 = O\,E_R{}^2 + O\,E_X{}^2.$$

or (resultant voltage)^2 = (resistance voltage)^2 + (inductance voltage)^2.

We have seen that

resistance voltage $= I\,R$.
inductance voltage $= 2\,\pi\,f\,L\,I$. (See page 24.)

Hence if E = resultant voltage of the circuit

$$E^2 = I^2\,R^2 + (2\,\pi\,f\,L\,I)^2$$
$$\text{or } E = I\,\sqrt{R^2 + (2\,\pi\,f\,L)^2}$$

This result is of the greatest importance

If the circuit has no self-induction, *i.e.*, $L = O$, we have $E = I\,\sqrt{R^2} = I\,R$, as in the case of a direct current circuit.

A further important result follows from Fig. 9.

The voltage $O\,E_R$ is in phase with the current, while the relative phase of the resultant voltage of the circuit is represented by the line $O\,E$.

Hence the angle $E_R\,O\,E$ is the angle of phase difference between current and voltage in the circuit. This angle is usually denoted by the symbol ϕ, and will be frequently referred to hereafter as the "angle of lag" of the circuit.

CHAPTER II.

IMPEDANCE.

Impedance.—A most important consequence of the back electromotive force of self-induction in a circuit is that the current produced by an applied electromotive force is no longer numerically equal to the quotient of voltage divided by the resistance, and that Ohm's law apparently ceases to apply. Whenever an alternating voltage is applied to a circuit possessing self-induction, the current will be less than the value of the fraction

$$\frac{\text{voltage}}{\text{resistance}}.$$

The quotient $\frac{\text{voltage}}{\text{current}}$ is the *impedance* of the circuit, and is sometimes called the *apparent resistance*.

Thus we have the relation

$$\text{current} = \frac{\text{voltage}}{\text{impedance}}$$

$$\text{or impedance} = \frac{\text{voltage}}{\text{current}}$$

The impedance depends on two distinct properties of the circuit, viz., its resistance and its self-induction, the former opposing the flow of current, the latter opposing the change of current. The resistance of the circuit is independent of frequency of current, the shape of the conductor, or its magnetic surroundings. The effect of the self-induction varies with the frequency, and also with the form and surroundings of the circuit, if these affect the magnetic field set up by the current.

It has been shown on page 38 that the electromotive force to be applied to a circuit in order to maintain a current of I amperes in it is

$$E = I \sqrt{R^2 \times (2 \pi f L)^2}$$

Consequently the value of the impedance of a circuit is given by

$$\text{Impedance} = \frac{\text{voltage}}{\text{current}} = \frac{I \sqrt{R^2 + (2 \pi f L)^2}}{I} = \sqrt{R^2 + (2 \pi f L)^2}$$

In the triangle of electromotive force, Fig. 10, page 31, each side is proportional to the current multiplied by some function of the circuit.

The side $I\ R$ is $25 \times 1{\cdot}2 = 30$ units in length measured on the scale of volts, since $I = 25$ and $R = 1{\cdot}2$, as given on page 25.

Similarly, the side $2\ \pi\ f\ L\ I$ is $25 \times 1{\cdot}57$ units of length and

the third side $I\sqrt{R^2 + (2\pi f L)^2}$ is $25 \times 1{\cdot}98$ units. The quantity $2\pi f L$ is known as the *reactance* of the circuit; the quantity $\sqrt{R^2 + (2\pi f L)^2}$ has already been stated to be the impedance of the circuit. If the numerical value of each side of the triangle were divided by 25, we should have numbers proportional in magnitude to the length of the sides of the original triangle, but representing resistance, reactance, and impedance, instead of voltages. Suppose the triangle is redrawn to a scale such that the side $E_x E_R$ instead of representing $I R$ volts, represents R ohms. Then, if

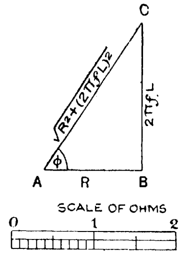


FIG. 16.—Triangle of Resistance and Impedance

the quantities representing the remaining sides are also drawn to this scale, after dividing their value by I, we shall get a triangle similar to the previous one, but drawn to a scale of ohms instead of volts. This has been done in Fig. 16, where each side of the triangle $O\, E_x\, E_R$, Fig. 10, has been divided by 25 (the current of the circuit), and the whole drawn to the scale of ohms shown below the triangle. For convenience the triangle has been drawn with the side representing the resistance horizontal. This direction is the one usually adopted where other circumstances do not afford any reason to the contrary.

It is usual to use the symbol Z to denote the impedance of a circuit and the symbol X to represent its reactance.

If a circuit has a resistance which is exceedingly small, the value of the impedance becomes practically $2\pi f L$, *i.e.*, equal to the *reactance*. If the self-induction is very small, the impedance approximates to the resistance of the circuit.

It is to be noted that the new triangle, Fig. 16, has no arrow heads, as the quantities represented by it are no longer variable, and the sides of the triangle do not now represent rotating vectors, but fixed quantities, or *scalars*.

The three sides represent respectively

$$A B = \text{resistance} = R.$$
$$B C = \text{reactance} = 2\pi f L = X$$
$$C A = \text{impedance} = \sqrt{R^2 + (2\pi f L)^2} = Z$$

the angle of lag of the circuit still being represented by the angle $C\ A\ B = \phi$, *i.e.*, the angle between the line of impedance and the line of resistance.

Hence from the triangle, $\cos\phi = \frac{A\ B}{A\ C} = \frac{R}{\sqrt{R^2 + (2\pi f L)^2}} = \frac{R}{Z}$

$$\sin\phi = \frac{B\ C}{A\ C} = \frac{2\pi f L}{\sqrt{R^2 + (2\pi f L)^2}} = \frac{X}{Z}$$

$$\tan\phi = \frac{C\ B}{B\ A} = \frac{2\pi f L}{R} = \frac{X}{R}$$

The impedance of a circuit is of very great importance, and the following experiments illustrate the various factors upon which it depends.

As a practical and convenient example, an ordinary arc-lamp choking coil may be taken for the purpose of the following measurements. The methods may, however, equally well be applied to any other form of inductive resistance, *e.g.*, an armature coil of an alternator, magnet winding, &c.

The three variable quantities in the expression $\sqrt{R^2 + (2\pi f L)^2}$ for the impedance are: (1) The resistance, R; (2) the periodicity, f; (3) the self-induction, L.

In order to determine the influence of each variable separately, two of them must be kept constant, while the third is varied.

Experiment III.—Determination of Dependence of Impedance upon Resistance.

If the coil on which the measurement is made contains iron, variation in current will produce variations in the permeability of the iron, and consequently variations in the self-induction of the circuit.

Therefore it is necessary in this experiment to maintain the current approximately constant, if the inductive portion contains iron.

Diagram of Connections.

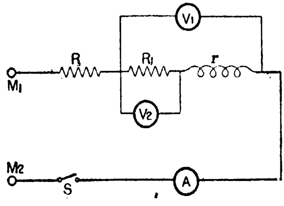


Fig. 17.

M_1, M_2. Source of alternating current.
A. Ammeter for reading current in circuit.
V_1. Voltmeter, reading voltage across impedance.
V_2. Voltmeter, reading voltage across non-inductive portion of impedance.
R. Variable resistance.
R_1. Non-inductive variable resistance forming part of impedance to be measured.
r. Inductive coil.
S. Switch.

Instructions. Connect in series two variable non-inductive resistances, the inductive coil, and an ammeter. One variable non-inductive resistance R_1 and the coil r together make the impedance to be measured. Connect one voltmeter to read the potential difference in R_1, and another voltmeter to measure the potential difference of the whole impedance, $R_1 + r$. In practice it will be found better to employ a single voltmeter which can be made to measure both potential differences in turn by means of a throw-over switch.

Supply the circuit thus formed with alternating current of constant periodicity.

By varying R and R_1 together, alter the value of R_1, while keeping the sum of R and R_1 about constant, so as to maintain the current at an approximately constant value, and take readings on the instruments for a series of values of R_1.

A single resistance may be used instead of the two R, R_1 shown in the diagram. This is then kept constant throughout the experiment, and the point of contact of the voltmeter is moved along it between each reading, thus changing the amount of the resistance R_1 included in the measured impedance, while leaving the total resistance in the circuit constant.

If the resistance of the coil r is not known, make a measurement of this by sending a measured direct current through it and noting the potential difference across its terminals.

Its resistance is then given by the relation

$$\text{resistance} = \frac{\text{potential difference}}{\text{current}}$$

For each series of readings taken with the alternating current, the value of the impedance should be calculated thus:

$$\text{Impedance} = Z = \frac{V_1}{I}$$

The value of the non-inductive part R_1 of the impedance should be calculated: $R_1 = \frac{V_2}{I}$, the resistance of the coil r (which, if not known, must be measured by direct current, as stated above) should be added to the value of R_1.

Readings should be tabulated as follows :—

DETERMINATION OF DEPENDENCE OF IMPEDANCE ON RESISTANCE.

Description of coil used........................

Resistance of coil r = ...ohm.

Current I.	Voltage across Impedance V_1.	Voltage across Non-inductive Resistance V_2.	Impedance $Z = \frac{V_1}{I}$.	Non-inductive Resistance $R_1 = \frac{V_2}{I}$.	Total Non-inductive Resistance $R_1 + r$.

The last column is the sum of the non-inductive resistance R_1 and the resistance of the coil r.

The results should then be shown graphically by drawing a curve with values of $R_1 + r$ plotted horizontally, and corresponding values of impedance plotted vertically.

A curve obtained in this way is shown in Fig. 18. The readings here plotted were taken on a 10 ampere arc-lamp choking coil

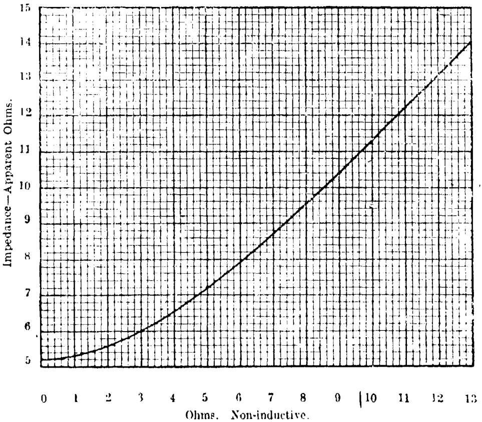


FIG. 18.—Curve Showing Dependence of Impedance upon Resistance.

having a laminated core and cast-iron base, completing the magnetic circuit. The resistance of the coil was 0·15 ohm, and the frequency of the current 45 cycles per second.

It is to be noticed that the curve is nearly horizontal at first, where the resistance is so small compared with the reactance as to exert scarcely any influence on the value of the impedance. The curve then bends, and ultimately approximates to a straight line inclined at 45° to the horizontal axis, showing that when the resistance is large, impedance and resistance increase at practically the same rate, the reactance having thus become unimportant.

From the curve it is seen that the value of the impedance when the resistance is zero is 5·2 ohms. This is consequently the value of the reactance of the coil $= 2\ \pi\ f\ L$. From this value L, the coefficient of self-induction of the coil, could be calculated from the known frequency.

The upper part of the curve approaches, and would ultimately become tangential to a straight line drawn through zero (which is not included in the figure) at an angle of 45°, *i.e.*, passing through the points having ordinates and abscissæ equal.

The next experiment shows that the impedance of a given circuit or coil does not depend only on the coil or resistance itself, but also on the frequency of the current sent through it.

EXPERIMENT IV.—DEPENDENCE OF IMPEDANCE UPON FREQUENCY.

DIAGRAM OF CONNECTIONS

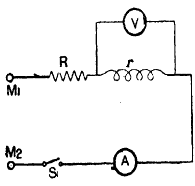


FIG. 19.

M_1 M_2. Source of alternating current.
r. Inductive resistance or coil of which impedance is to be measured.
R. Adjustable resistance.
A. Ammeter reading current in circuit.
V. Voltmeter reading voltage across inductive resistance.

Instructions.—Connect to the terminals of the alternator an inductive resistance or coil in series with an adjustable resistance and ammeter.

Connect a voltmeter to the terminals of the coil

Take readings on ammeter and voltmeter for a number of different speeds of the alternator. At each speed, before taking

readings, adjust the current approximately to a fixed value, which should be maintained throughout the experiment. Note the speed of the alternator in each case, and calculate the frequency thus :—

Frequency = revs. per minute × No of pairs of poles of alternator ÷ 60.

The readings should be tabulated thus :—

DETERMINATION OF DEPENDENCE OF IMPEDANCE ON FREQUENCY

Description of coil .....Cast-iron base, movable core.
Resistance ....0·15 ohms.
Current .... 3 amps.

Revolutions per Minute of Alternator.	Current in Circuit = I.	Voltage across Impedance = V.	Frequency = f.	Value of Impedance $Z = \frac{V}{I}$.
350	3·0	4·3	11·7	1·43
490	3·0	6·0	16·3	2·00
820	3·0	10·0	27·3	3·33
1500	3·0	18·3	50·0	6·1

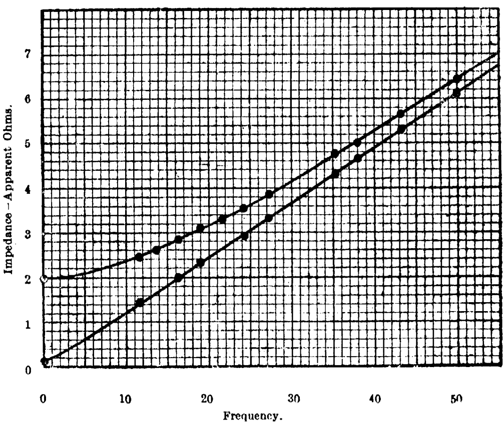


FIG. 20.—Curves showing Dependence of Impedance on Frequency. Lower Curve. Coil alone. Upper curve, Coil and Resistance.

A curve should be plotted with frequency measured horizontally and impedance vertically.

Fig. 20 shows two curves obtained in the manner just described. The lower curve gives the values obtained for the impedance of an arc lamp choking coil. The resistance of the coil was only 0·15 ohm, so that nearly all the impedance was due to its self-induction. That is, the impedance $\sqrt{R^2 + (2\pi f L)^2}$ was very nearly equal to the reactance $2\pi f L$, because the value of R was so small. Evidently the reactance increases in direct proportion to f, the frequency. This is shown to be the case by the straightness of the curve, which only bends very slightly at the bottom where the frequency is so small that the reactance is comparable with the resistance.

The upper curve shows readings of the impedance of the coil, and a resistance of 1·85 ohms in series. The resistance in this case is much larger compared with the reactance, and the curve is affected much more by the resistance near its lower end. At very high frequencies, the reactance would become so great that the resistance would again be negligible, and the two curves would then coincide, since the reactance factor of the impedance is the same for both curves.

The important point to be learnt from the results of this experiment is that the impedance of any circuit consists of two elements, resistance and reactance. The resistance is independent of frequency; the reactance depends directly on the frequency. The frequency of the current supplied determines the relative importance of the two elements.

It would be easy to calculate the value of the self-induction of the circuit from the readings shown on the curve.

If the resistance is known, this can be done directly from a single observation.

If the resistance is not known, two points on the curve will enable the calculation to be made.

Thus the upper curve shows

at frequency 10 impedance = 2·35

" 40 " = 5·3

In 1st case $R^2 + (2\pi f L)^2 = (2{\cdot}35)^2 = 5{\cdot}5$

" 2nd " $R^2 + (2\pi f_1 L)^2 = (5{\cdot}3)^2 = 28{\cdot}0$

By subtraction $(2\pi L)^2 (f_1{}^2 - f^2) = 22{\cdot}5$

$$L^2 = \frac{22{\cdot}5}{(f_1{}^2 - f^2)\, 4\pi^2} = \frac{22{\cdot}5}{1500 \times 4 \times 9{\cdot}87} = {\cdot}000379$$

$\therefore L = {\cdot}0195$ henries,

or 19·5 millihenries.

The determination of the effect of self-induction upon impedance is a problem which arises in many ways.

Choking coils for arc lamps are frequently made with a movable core, so that the self-induction can be varied, and the impedance thereby increased or decreased. In an alternator the armature coils have a variable amount of self-induction depending upon their position relative to the magnet poles.

The self-induction also depends on the degree of saturation of any iron forming part of the magnetic circuit, since this affects its permeability.

In such cases the self-induction of a circuit varies with the current in it.

The following experiment is consequently of very great practical importance, although the full calculations based upon it will not be given until later.

EXPERIMENT V.—DETERMINATION OF DEPENDENCE OF IMPEDANCE UPON SELF-INDUCTION.

(1) Due to alternation of magnetic circuit.

DIAGRAM OF CONNECTIONS.

As for Experiment IV. page 44.

It is assumed that the coil r in this case has a movable core, or is arranged so that its magnetic circuit can be varied by alteration of the air gap in it.

Instructions. — Make connections exactly as described for Experiment IV. The current must in this case also be kept approximately constant, if the effect of alteration of the magnetic circuit is to be considered alone. The current may, as before, be regulated by adjusting the resistance R. The frequency must remain constant.

For a series of positions of the movable part of the magnetic circuit take readings of the current and of the voltage across the terminals of the inductive winding. From these readings calculate in each case the impedance as in the previous experiment, and tabulate the results.

The alteration in the magnetic circuit should be carried out in such a way that an equal movement of the moving portion is made between each pair of readings; the observations may then be plotted as a curve with displacement measured horizontally and impedance plotted vertically.

A set of readings taken from a choking coil with a movable laminated core are reproduced in the form of a curve shown in Fig. 21.

DETERMINATION OF IMPEDANCE WITH VARYING AIR GAP.

Description of coil.........Resistance .... 0·15 ohms.
Current employed ....3 amps. at........................ 30 periods.

Length of Air Gap	Current $= I$	Volts across Coil V	Impedance of Coil $Z = \frac{V}{I}$
Inches			
9	3·0	1·2	0·4
6	3·0	1·9	0·63
3	3·0	5·7	1·9
0	3·0	17·0	5·7

The coil employed was the same as that referred to in the last two experiments. The air gap was varied by drawing out the core, leaving the measured gap between the end of the core and a projection on the base, which completed the magnetic circuit when the core was pushed home.

The value of the coefficient of self-induction for each position of the movable core may be calculated from the readings obtained in this experiment if the resistance of the coil and frequency of the current are noted. This is sometimes important in the case of rotating machinery, as the self-induction affects the wave form of the voltage generated. A more obvious use of the results is to show the range of application of a choking coil required to work on arc lamp circuits of various voltages.

The strength of the magnetic field produced in an iron core by a given current depends upon the permeability of the iron.

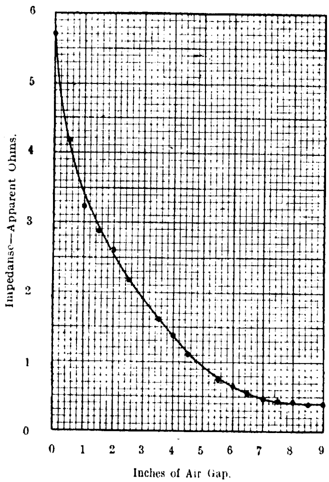


FIG. 21.—Dependence of Impedance on Air Gap.

Since the self-induction of a coil having a magnetic circuit partly composed of iron is proportional to the strength of field produced by each ampere of current flowing in the coil, any variation in the permeability of the iron will change the coefficient of self-induction. The permeability of iron changes as its magnetic saturation is altered. Consequently the permeability of the iron will depend upon the strength of the current producing the field, and the self-

induction of the coil will have a different value for each value of the current.

Obviously there will actually be a constant and rapid change in the permeability of the iron corresponding to the changing value of the alternating current. It is only the mean value for a given current with which we are at present concerned.

EXPERIMENT VI.—DETERMINATION OF DEPENDENCE OF IMPEDANCE UPON SELF-INDUCTION.

(2) Due to alteration in strength of current acting on an iron magnetic circuit.

DIAGRAM OF CONNECTIONS.

As for Experiment IV., page 44.

Instructions.—Connect in series with a supply of alternating current a variable resistance, the inductive coil to be experimented upon, and an ammeter. Connect a voltmeter to the terminals of the inductive coil.

By means of the resistance R vary the current in the circuit from a low value upwards. The frequency must be maintained constant. For each value of the current read ammeter and voltmeter, and tabulate the readings as follows:—

DETERMINATION OF IMPEDANCE AT VARIOUS CURRENTS.

Coil No. .... Frequency of Current.................37

Current I	Volts $=V$	Impedance $Z = \frac{V}{I}$
0·349	0·92	2·75
0·706	2·42	3·38
1·572	6·5	4·15
1·183	4·62	3·9

The readings should also be recorded in the form of a curve, as in Fig. 22, the current being plotted horizontally, and the impedance vertically.

The results shown in the upper curve Fig. 22 were obtained on the choking coil already experimented upon. The frequency was 37.

For low currents* the impedance is seen to be small. This is

* With these low currents a correction had to be made for the voltmeter current which was not negligible compared with the total current. The voltages were read upon an electro-magnetic voltmeter, as the electrostatic type would not read low enough. In making this correction it was found to be permissible to consider the voltmeter current to be in phase with the voltage at its terminals. Since this was not the case for the current in the coil, the current through the coil had to be determined by subtracting current in the coil, the current through the coil had to be determined by subtractin.. the voltmeter current from the component of the current in the coil which was in phase with the voltage, and determining the resultant of this energy voltage and the idle component of the voltage. The resultant current in the coil determined in this way is the value plotted in the curve. For the larger currents this correction was unnecessary. The lowest reading was obtained by calculation from the coefficient of self-induction (10.5 millihenries) measured on the secohmmeter.

A simple method of avoiding the correction for the voltmeter current would have been to employ a key in series with the voltmeter, which would be closed when the voltage is read and opened when the current is read. This would only be permissible when the voltmeter resistance is high compared with the resistance in the circuit.

owing to the well-known fact that the permeability of iron is low for very low inductions, but rises rapidly as the iron becomes more magnetised, until it reaches a point where the iron begins to be saturated. The permeability then falls again, but more gradually than it increased at first. These variations are very clearly to be traced in the curve. With a non-magnetic core the impedance would be constant at all currents.

The curve represents, in fact, the permeability curve for the core of the choking coil, and has the same properties as the permeability curves of iron, although the shape of the curve shown is

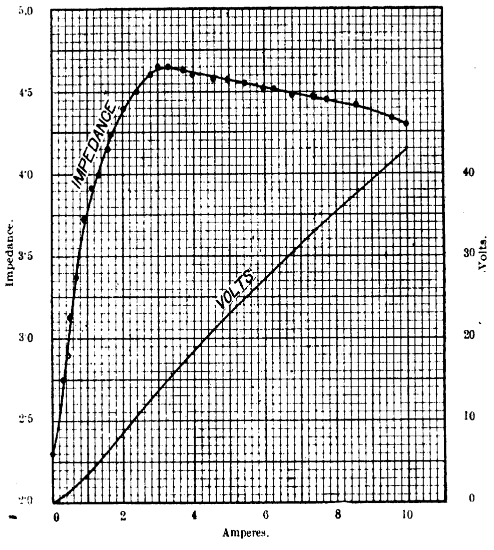


FIG. 22.—Variation of Impedance with Current.

influenced by the fact that the coil had a heavy cast-iron base, so that the variations in permeability were not uniform throughout the magnetic circuit.

The reason that the curve approximates so closley to the shape of a permeability curve is due to the fact that the resistance of the circuit was almost negligibly small. The value of R was 0·15 ohm, and was therefore small compared with the reactance $2 \pi f L$, the lowest value of which was 2·44, rapidly rising to a much higher value. Hence the impedance was nearly the same as $2 \pi f L$. Now the value

of L is directly proportional to the mean permeability of the magnetic path, and consequently the impedance varied in nearly direct proportion to the permeability.

The curve which has just been discussed must not be confused with the magnetisation curve, which shows the dependence of the strength of the magnetic field (not permeability) upon the magnetising current.

The lower curve in Fig. 22 is practically the magnetisation curve of the coil, and resembles the magnetisation curve discussed later in connection with transformers, &c.

The lower curve is obtained by plotting the voltage at the terminals of the coil instead of the impedance.

The voltage represented by this curve is almost entirely due to the back electromotive force of self-induction referred to on page 24, the drop in the resistance of the windings being only very slight. For simplicity, we may assume the curve to show the back electromotive force corresponding to each value of the current in the coil. From the results it would then be easy to calculate the number of magnetic lines formed in the core. The formula giving the back voltage in terms of the magnetic flux, which will be obtained later, is:—

$$4{\cdot}44\, F\, f\, T \times 10^{8} = V$$

where V = back electromotive force
F = number of lines
T = number of turns
f = frequency.

In the coil experimented upon the number of windings was 160. Hence, the number of lines corresponding to each volt = $\dfrac{10^{8}}{37 \times 160 \times 4{\cdot}44}$ = 3,800 nearly. The vertical scale might, consequently, be plotted in terms of lines of force, and similarly the horizontal scale might have been given in ampere turns. A curve of considerable theoretical interest would thus be obtained, although for practical purposes the scales actually employed are generally more useful.

As regards the form of the lower curve, it is practically straight after a small initial bend, but bends slightly to the right at the upper end. If the current were increased to much higher values, the curve would bend decidedly to the right, showing the well-known "knee" of a magnetisation curve. The magnetic densities for cores excited by alternating currents are chosen far below the knee, in order to avoid the heavy hysteresis and eddy current losses which would occur at higher saturation.

Graphic Calculation of Impedance.—The triangle of impedance shown in Fig. 16 forms the basis of a convenient method of obtaining the value of the impedance of a circuit from values of the self-induction and the reactance, requiring the use of *accurately ruled* squared paper only.

Thus, set out from the same point, the resistance of the circuit

horizontally and the reactance ($= 2 \pi f L$) vertically. The value of the impedance is then given by joining the extremities of the lines thus drawn. This is really the converse of the operation for determining the reactance and self-induction from observations of resistance and impedance given on page 40.

Example.—Plot a curve of impedance on a resistance base for a coil having a reactance of 10·4 ohms, in series with varying values of a non-inductive resistance. Points on the curve to be obtained graphically. This is equivalent to calculating the curve which would have been observed at double the frequency employed in the experiment illustrated in Fig. 18, page 43.

Inductive and Non-inductive Resistance.—Strictly speaking, no resistance is entirely non-inductive, and no conductor can have inductance without some resistance. It is desirable to form a

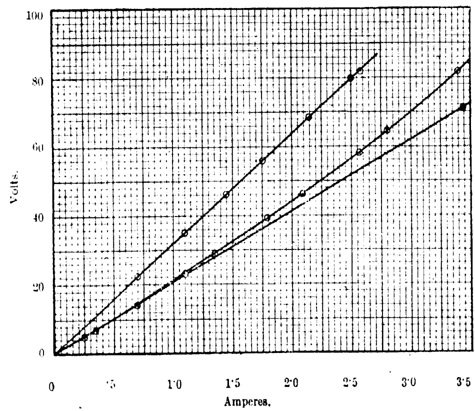


FIG. 23.—Impedance and Resistance of Wire Coils.

● Direct current. ○ Alternating current.

general idea as to what form of resistance may be considered non-inductive for practical working purposes. Convenient resistances in which the self-induction is entirely negligible at all ordinary frequencies may be made by grouping together incandescent lamps connected in parallel or series-parallel to suit the voltage required. Liquid resistances are also specially suitable for alternating-current work, since there is little eating away of the electrodes due to electrolysis. A solution of common washing soda or of aluminium

sulphate works satisfactorily, the strength of the solution being chosen according to the resistance required. The electrodes may be movable iron plates or tubes of any convenient form. Resistance elements consisting of wire woven into a fabric with asbestos are a very useful form of practically non-inductive resistance.

For general purposes the ordinary resistance frame made of spirals of resistance wire may be considered to be non-inductive, if any of the high-resistance alloys are used. Iron spirals, on the other hand, have considerable self-induction on account of the magnetic nature of the wire, and if employed for alternating currents must be looked upon as "partially-inductive" resistances. For specially high frequency currents any form of spiral may lead to errors if looked upon as non-inductive; the same caution should be applied in cases where the wave form of the current is specially "peaky."

Some tests made upon ordinary spiral wire resistances are recorded in the curves shown in Fig. 23.

The upper curve shows readings taken with alternating current at 42 cycles, and with continuous current upon a resistance consisting of spirals 1in. diam. of No. 22 S.W.G. resistance wire, the coils being spaced at an average of 5·5 turns per inch. The resistance consisted of 3 sets of 4 coils 27in. long, the three sets being in parallel, and the 4 coils of each set being in series. It will be seen that at this frequency no appreciable difference between the apparent resistance with alternating and direct current can be traced. This result can easily be verified by calculation.

By measurement with a secohmmeter and standard self-induction the induction of the coils was found to be 0·26 millihenries, while the resistance was 69 ohms.

The reactance of the coils was consequently $= 2\pi f L$

$$= 2\pi 42 \times \cdot 00026$$

$$= \cdot 069 \text{ ohm}$$

Hence ratio $\frac{\text{resistance}}{\text{reactance}} = \frac{1000}{1}$ and the reactance can safely be neglected.

The lower curve in Fig. 23 applies to a wire resistance formed of No. 14 S.W.G. galvanised-iron wire, coils 1·1in. diam., and wound 5 coils to the inch, 50 coils 26in. long being in series.

This curve has been drawn for a double purpose. Firstly, it serves to show to what extent a spiral wound with thick wire of fairly low resistance may be considered non-inductive. Secondly, it shows the influence of the magnetic properties of iron when employed as a resistance.

As may be seen from the cruve, the apparent resistance of the iron spirals for alternating and continuous currents is approximately the same with very small currents. The exact relation may be calculated from the following data.

Self-induction measured by secohmmeter = 2·3 millihenries.
Resistance = 20·66 ohms.
Reactance = $2\pi f L = 0{\cdot}604$ ohm, when $f = 42$.

$\therefore$ Impedance $= \sqrt{20{\cdot}66^2 + 0{\cdot}604^2} = \sqrt{426{\cdot}84 + 0{\cdot}365}$
$= 20{\cdot}664$ ohms.

Hence the error introduced in neglecting self-induction would only affect the impedance to the extent of about one-quarter per 1,000, when used for such small currents.

With a non-magnetic material the self-induction would be the same for all current densities, and the resistance might safely be considered non-inductive for any current of this frequency.

The important influence of the magnetic material of which the resistance in this case is composed is plainly shown by the wide and increasing divergence between direct and alternating current readings, as the current density increases, showing that iron resistances cannot be considered as non-inductive with larger currents, although a similar resistance of non-magnetic material may generally be considered to be so.

It may be stated generally that the larger the section of wire employed, the greater will be the divergence between its true resistance and its apparent resistance to an alternating current.

The figures given above form a useful rough basis for estimating the self-induction of any resistance spiral, as the self-induction varies approximately as the square of the number turns in the spiral, directly as the sectional area of the coil, and inversely as the length, so that it may be assumed that

$$\text{the self-induction} \propto \frac{T^2 A}{l}$$

Impedance of Armature of an Alternator.—A practical example of a circuit in which the impedance is affected by all the factors determined separately in the foregoing experiments is the armature winding of an alternator or synchronous motor.

The self-induction of the armature will depend on the exact position of the armature relative to the poles, since the magnetic circuit of the armature is completed through the poles or the pole faces. The extent to which the position affects the self-induction will depend on the nature of the winding and form of the magnetic system of the machine.

A distributed winding will show a small variation, since some of the conductors will always be opposite to part of the magnets while the maximum value of its self-induction will not be high.

A winding concentrated into single slots, on the other hand, will have a high self-induction when opposite to a pole, and a lower self-induction when situated between the poles, especially if the air gap between armature and poles is small.

The self-induction will also be affected by the amount of the armature current flowing, since the core will be more highly saturated with heavy currents, and its permeability will be lower. The magnetic saturation due to the main field will also affect the self-induction.

EXPERIMENT VII.—DETERMINATION OF THE IMPEDANCE OF THE ARMATURE OF AN ALTERNATOR WHILE STATIONARY.

DIAGRAM OF CONNECTIONS.

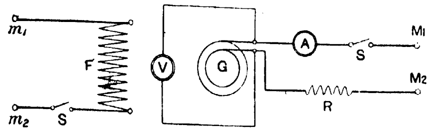


FIG. 24.

M_1 M_2. Source of alternating current.
m_1 m_2. Source of direct current.
G. Alternator armature.
F. Alternator field windings.
A. Ammeter.
V. Voltmeter.
R. Variable resistance.
S, *S*. Switches.

Instructions.—Connect the alternator armature to a source of alternating current which should give the same periodicity and similar wave form to that of the alternator whose armature is under test. Insert in the same circuit an ammeter, variable resistance, and switch. Connect a voltmeter to the armature terminals.

Close the switch, and give the resistance *R* its maximum value. Take readings on the ammeter and voltmeter for a series of positions of the armature, which should be turned through the same angle between each pair of readings, from a position in which the armature coils are midway between the poles, to the position bringing the coils symmetrically under the poles.

Repeat these readings with several values of the current, which should finally be increased to the full-load current of the machine.

Again take the same readings with the alternator fields excited to their normal extent. A field regulator and ammeter may be required in the exciting circuit for the purpose of adjusting the excitation. They are not shown on the diagram.

Finally, measure the resistance of the armature with direct current.

The readings should be entered in the manner indicated below, a separate table being taken for the experiments with the field unexcited and excited.

Note.—This experiment can only be carried out on alternators of small size in the manner described

DETERMINATION OF ARMATURE IMPEDANCE.

Alternator No........ Type........
Output.... amps.... volts...... frequency
Normal excitation......amps.

Armature Position.	Current.	Voltage.	Impedance.

For each set of readings corresponding to one particular current, the *mean* impedance for the various positions of the armature should be calculated and inserted in the table. Strictly, the "square root of mean squares" value should be taken as the effective impedance corresponding to any value of the current, but the error introduced by taking the *mean* value is very small, and the calculation is much simplified.

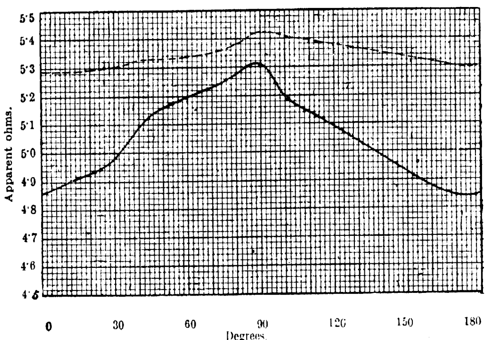


FIG. 25.—Curves of Impedance of Armature of Inductor Alternator.

Some difficulty may possibly be experienced in keeping the armature steady in the desired position when the magnets are excited, as it will tend to rotate into the position giving the maximum value of the impedance. With a little care it can usually be held steady with a wooden wedge under the pulley or a similar device.

The self-induction of the armature can be easily calculated from the results obtained above, since

$$\text{Impedance} = \sqrt{R^2 + (2\pi f L)^2}$$

When R = resistance of armature.
L = self-induction coefficient of armature.
f = periodicity of current.

The self-induction should be calculated for readings obtained with the magnets excited, and the armature current at its maximum and minimum values, respectively.

Fig. 25 gives two curves obtained in the manner just described, the experiment being carried out on the armature of a small Pyke and Harris inductor alternator. The lower curve shows the results with the field fully excited with 3·8 amps., and the armature carrying its normal current of 5 amperes. The upper dotted curve shows the results obtained with the field unexcited, and the same armature current.

In this type of machine the armature is wound in two sets of windings between which the soft-iron inductors rotate; the variation of self-induction is therefore specially large in this case. The armature resistance was high, being 3·65 ohms. The effect of the field excitation is to partially saturate the iron cores of the armature and of the inductors. This reduces the permeability of the magnetic path, and consequently also the self-induction of the armature. Hence the impedance of the armature when the fields are unexcited is considerably higher. The difference between the two curves would be still greater if the armature resistance had not been so high. The impedance was so largely composed of resistance that variation in self-induction makes comparatively little difference to the total value of the impedance.

In order to make sure that the difference between the two curves was not due to induced currents in the field winding, the voltage at the terminals of the winding was measured when open. No induced voltage was found. In most types of alternators this transformer effect may be noticed, and in large alternators the voltage induced in the field coils would be so high as to make this method of measuring the armature impedance inadmissible.

Another effect which has an important influence on the self-induction and impedance of the armature must be mentioned. This is the currents which are induced in the pole-faces, poles, and field-windings by the alternating flux set up by the current in the armature. This is referred to at greater length in the chapter on alternators.

CHAPTER III.

POWER AND POWER-FACTOR.

Power in an Alternating Circuit.—The nature of power is that of a force exerted at a certain speed; for instance, the resistance to turning of a shaft overcome at the rate of some number of revolutions per minute.

In a direct-current electric motor the torque exerted by the armature is proportional to the current in the armature, since the turning effort is produced by the action of the armature current upon the field. The speed of the motor is independent of current, but is proportional to the voltage applied to the motor. Hence the power, which is proportional to the product of turning effort and speed, is proportional to the product of current and volts supplied to the motor armature. The actual power supplied to the motor is measured in watts, and is numerically equal to the product of current and voltage.

It would, however, be possible to apply current and voltage to the armature alternately, instead of simultaneously.

Thus, if the armature were held stationary, a large current could be sent through it at a very low voltage. If the motor, on the other hand, is allowed to run without load, a high voltage may be applied, causing the motor to run very rapidly, while taking only a very small current. In either of these cases the power exerted by the motor would be zero or very small.

A somewhat analogous condition may arise in an alternating circuit and with corresponding results. The voltage and current may be considerable, but unless they have a high value *simultaneously* the power which they represent may be small.

Power of an Alternating Current.—The power due to an alternating current in a circuit at any instant is numerically equal to the product of the current multiplied by the voltage of the circuit. Since both current and voltage change their direction twice in every cycle, this product may be either positive or negative, and will usually be positive during part of each period, and negative during the remainder.

If the current and electromotive force act in opposing directions in the circuit, their product must be considered as negative, *i.e.*, the power given to the circuit by the current is negative. This is illustrated in Fig. 26, which shows a curve of current C and a curve of voltage V which differ in phase by 30°. A third curve of *watts* W has been obtained by calculating the product of current and volts, and represents the fluctuation of the power in the circuit.

When a point on the curve of watts is obtained by multiplying together a current and a voltage which are represented by ordinates on opposite sides of the base line, this product is negative and the point must be plotted below the base line to represent *negative watts.*

We must consider that when the power given to the circuit is negative, the circuit is not receiving power from the generator to be spent in heating conductors, driving motors, &c., but is at such times giving back power to the generator (or to some other part of the circuit) in virtue of the self-induction (analogous to inertia) possessed by the circuit. When the watts are negative, the power is therefore actually to be considered as given back by the circuit to the generator to assist in driving it. The energy delivered to the circuit is the mean value of the power of the circuit (obtained after subtracting the negative power from the positive power) multiplied by the time of its duration.

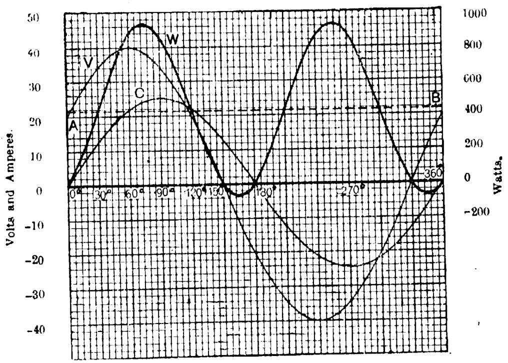


FIG. 26.—Curves of Current, Voltage, and Watts.
Angle of lag = 30°

The **Average Power** developed in the circuit is the average value of the product (amperes × volts) in the circuit. This value is shown in Fig. 26 by a horizontal dotted line A B, obtained by adding together the ordinates of the curve of watts corresponding to each vertical line of the squared paper, and dividing by the number of these ordinates. Ordinates below the base line must be subtracted from the sum of the ordinates above the line. The value of the average power is seen in this case to be 430 watts.

The average value of the power in a circuit can be measured directly by a wattmeter.

The wattmeter has two coils, one carrying the current of the circuit, and one connected, in the same manner as a voltmeter, to carry a small current proportional to the voltage of the circuit to which it is connected.

The deflection of the instrument is produced by the mutual action of these two coils, so that it is proportional to the product (amperes × volts). Since the inertia of the moving parts of the instrument is far too great to allow them to follow the rapid variations occurring during every period, the deflection is steady, and is proportional to the mean value of the power in the circuit.

It is to be noted that the *average* power measured in this way is what we shall call later the *true power* of the circuit.

Power Factor.—It was pointed out that in Fig. 26 the curves of current and voltage are not in phase, since they do not pass

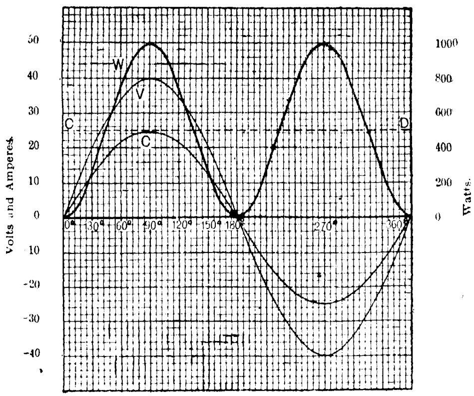


FIG. 27.—Curves of Current, Voltage, and Watts
Current and voltage in phase.

through their zero values simultaneously. In consequence of this a portion of the power represented by the product (current × volts) is negative during each period.

If the same current and voltage existed in a circuit without self-induction, there would be no difference in phase between them, and the two curves would always pass through zero at the same time, and at every instant would both be on the same side of the horizontal axis. Hence in this case their product would always be positive, and the average useful power of the circuit would be correspondingly greater than is the case for the condition illustrated in Fig. 26, where the power is partly positive and partly negative.

In order to illustrate this, the same curves are redrawn in Fig. 27 with the volts and amperes coincident in phase. W is the curve showing the power in the circuit in this case. The dotted line C D gives the average height of this curve, and shows the average power given to the non-inductive circuit. The value of this average power is 500 watts.

From a comparison of the curves in Figs. 26 and 27, it is evident that with given values of current and voltage, the power developed will not be the simple product of these quantities, but will depend upon the difference in phase between them. When the current and voltage are in phase, there is no negative power, and the power developed in the circuit has its greatest value, which is equal to the product of the current and voltage as read upon an ammeter and voltmeter.*

The product of current and voltage is often called the ***apparent watts*** of the circuit, or sometimes the ***volt-amperes.***

The reading of an ammeter or voltmeter is independent of the phase difference between current and voltage. Thus, the product of ammeter and voltmeter readings will only give the true value of the power in the circuit when current and voltage are in phase with one another. Under these conditions the ***apparent*** power, *i.e.*, the product of amperes and volts, is also the real power.

Under any other circumstances the product of ammeter and voltmeter readings will give a value of the "apparent watts," which is greater than the true power.

The greater the difference of phase between current and voltage, the greater is the negative power, and the less the resultant output. If the phase difference is exactly 90°, or a quarter of a period, the positive and negative powers are equal, and the average power developed is zero. This should be verified by the student, who should draw the curves shown in Figs. 26 and 27 with 90° difference of phase between them, and calculate the average power under these conditions.

The Power-factor is the ratio of the true watts to the apparent watts or volt-amperes.

$$\text{Power-factor} = \frac{\text{actual power in watts}}{\text{volts} \times \text{amperes.}}$$

Power of circuit = volts × amperes × power-factor.

As explained in the next chapter, an ammeter in a circuit carrying a current of the form shown in Figs. 26 and 27, with a value of 25 amps. would read 17·68 amps., and similarly the voltmeter would give a reading of 28·3 volts with the maximum voltage of 40.

The product of these readings, 17·68 × 28·3 = 500, is, therefore, the apparent power in both cases shown in Figs. 26 and 27.

In both cases the power-factor of the circuit is the ratio of the average power to this quantity.

* The relation between the reading of an ammeter or voltmeter and the maximum value of the variable current and voltage is given in the next chapter.

Hence in Fig. 26

$$\text{Power-factor} = \frac{430}{500} = 0{\cdot}86.$$

While in Fig. 27

$$\text{Power-factor} = \frac{500}{500} = 1{\cdot}0.$$

The comparison of the true power in a circuit with the "apparent watts" or volt-amperes forms one of the most direct methods of determining the power-factor of a circuit.

The following experiment illustrates the measurement of the power-factor in both inductive and non-inductive portions of a circuit.

EXPERIMENT VIII.—DETERMINATION OF THE POWER-FACTOR OF A CIRCUIT.

DIAGRAM OF CONNECTIONS.

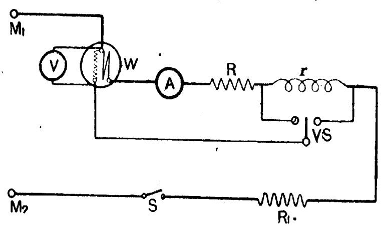


FIG. 28.

M_1 M_2 Source of alternating current.
r Inductive portion of circuit in which power-factor is to be determined.
R Non-inductive portion.
R_1 Resistance for varying current.
W Wattmeter for measuring power of circuit.
V Voltmeter for measuring voltage of circuit.
A Ammeter for measuring current of circuit.
V S 2-way voltmeter switch.

Instructions.—Connect in series an inductive resistance, a non-inductive resistance, and variable resistance.

Put an ammeter in the circuit, and connect a voltmeter across the portion in which the power-factor is to be determined.

Connect a wattmeter to read the same voltage and current as the voltmeter and ammeter. In order to be able to measure the power-factor in the non-inductive portion of the circuit alone, connect the common lead from the voltmeter and wattmeter shunt to a 2-way switch, in such a way as to be able to put the instruments

across the non-inductive part of the circuit or across both resistances as desired.

Take simultaneous readings of voltmeter, ammeter, and wattmeter for each position of the voltmeter switch. Repeat the readings with several different values of the current obtained by altering the resistance R_1 outside the portion of the circuit on which the measurement is being made. In cases where the power-factor of a motor, coil, or other piece of apparatus is required, the current should always be varied to give the range of readings likely to occur subsequently, since the power-factor will probably vary with the current on account of iron saturation.

For the purpose of forming a suitable experiment, it is convenient to keep r constant and to determine the power-factor of the circuit for various values of R.

Enter the readings under headings as follows :—

DETERMINATION OF POWER-FACTOR IN AN INDUCTIVE CIRCUIT.

Current $= I$.	Inductive Circuit.				Non-inductive Circuit			
	Volts $= V_1$.	Watts $= W_1$	Volt-amps. $I \times V_1$	Power-factor $= \frac{W_1}{I \times V_1}$	Volts $= V_2$	Watts $= W_2$	Volt-amps. $= I \times V_2$	Power-factor $= \frac{W_2}{I \times V_2}$
2·1	108·5	226	228	·991	105	220	220·7	1·0
5·4	109·0	570	589	·967	97	524	524	1·0
14·67	100·7	1185	1478	·802	64	939	939	1·0

The power-factor in the non-inductive portion of the circuit should, of course, be constant and equal to unity. The readings taken on the non-inductive circuit form a valuable check on the correctness of the readings of the instruments, and the extra connections and time involved are well repaid by the added certainty of the results whenever a non-inductive resistance forms a part of the circuit. It is therefore strongly recommended that the watts and volt-amperes in a non-inductive portion of the circuit should be taken *with the same instruments* employed for ascertaining the power-factor of an inductive circuit whenever possible.

The results of the test should be plotted in a curve, with current measured horizontally and power-factor vertically. This is illustrated by the curve in Fig. 29, which was taken on a circuit composed of a choking coil and non-inductive resistance in series. In this case the non-inductive portion of the circuit was varied for each reading.

A resistance of about 100 ohms was put in series with the coil, giving a power-factor for the whole circuit of nearly unity. This resistance was then gradually cut out, enabling an increased current to pass, and at the same time reducing the power-factor of the circuit.

The curve shows the power-factor for the whole circuit, including both inductive and non-inductive portions. Three of the readings actually taken are shown in the table above as an example.

Energy and Idle Voltage.—It has been shown already that the electromotive force of the circuit consists of two components which differ by a quarter of a period in phase (see page 25). One of these components (equal to $I\ R$ when there are no iron losses in the circuit) is in phase with the current, the other (equal to $I\ X$) is a quarter of a period out of phase with it.

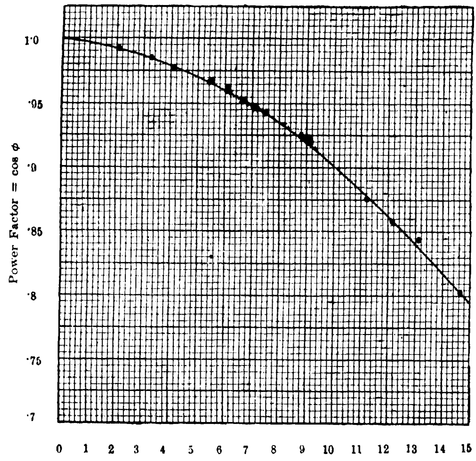


FIG. 29.—Curve Showing Variation of Power Factor with Current.

It has also been stated that the average product of the instantaneous values of the current and volts is zero when the variations of current and voltage have a quarter-period phase difference, whereas the value of this product is numerically equal to the product (current × voltage) when the current and voltage are in phase.

Hence the two components of the voltage correspond respectively to the portion of the voltage which does not affect the power of the circuit, and the portion which when multiplied by the value of the current represents the total power of the circuit.

These two components may be suitably called the "Idle" and the "Useful," or "Energy" components of the electromotive force.

Hence, power due to current in circuit = useful component of electromotive force × current = energy voltage × I

Referring to Fig. 30, representing the three electromotive forces of the circuit, $\cos F E O = \frac{F E}{O E}$

$$\text{or } F E = O E \cos F E O ;$$

but the angle $F E O$ is the angle of phase difference between the current and voltage of the circuit $= \phi$

Hence, $F E = O E \cos \phi$

$\therefore$ Useful component or energy voltage = total voltage $\times \cos \phi$

Consequently if V = total voltage of circuit, the power given out $= I \times$ energy voltage $= I \times V \cos \phi$

But the power of the circuit $= I \times V \times$ power-factor, and consequently we have power-factor $= \cos \phi$

The **power-factor** of a circuit is the cosine of the angle of phase difference between current and voltage.*

Returning now to the experiment just described, each set of readings would enable us to construct a voltage diagram for the circuit.

The energy voltage E_e is obtained by dividing the watts by the current, since

$$\text{watts} = \text{energy voltage} \times \text{current}.$$

The following construction gives the idle voltage overcoming the self-induction of the circuit.

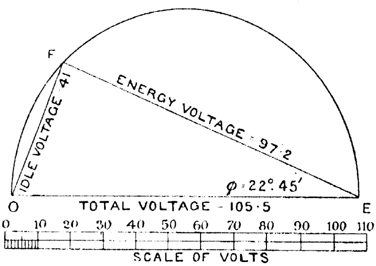


FIG. 30.—Diagram of Electromotive Force.

Draw a horizontal line $O E$ (see Fig. 30) to represent the total voltage E. On $O E$ describe a semi-circle, and from E, with radius equal to the energy voltage, describe a circle cutting the semi-circle in F. Then $O F$ represents the magnitude and phase of the self-induction electromotive force, and the sides $F E$, $O E$ represent in

* We are throughout this chapter assuming a sinusoidal wave-form.

phase and magnitude the energy voltage and total voltage respectively.

The angle of lag is the angle $F E O$ between $F E$ and $O E$, since $O E$ is the voltage of the circuit and $F E$ is in phase with the current.

Fig. 30 represents such a diagram for the point on the curve in Fig. 29, for which total voltage of circuit was 105·5, the current 9 amperes, and the true watts 875. The energy voltage was consequently $\frac{875}{9} = 97{\cdot}2$. The curve in Fig. 29 shows the value of the power-factor or $\cos \phi$ to be ·921, whence $\phi = 22° 45'$. This value for ϕ is seen to correspond with that of the angle in Fig. 30.

The construction adopted is based on the fact that the angle contained by a semi-circle is always a right angle. The useful and idle voltage vectors are always mutually perpendicular, and have as a resultant the constant voltage represented by the diameter of the semi-circle.

Choking Coil.—A coil of copper wire wound on an iron core will have a high self-induction and consequently a high **apparent resistance,** although its true, or ohmic, resistance is small. It may consequently be used in series with an arc lamp in order to reduce the current to the value required, and answers then the purpose of the "ballast" resistance which is necessary to make the lamp burn steadily.

Since the voltage absorbed by the self-induction does not represent loss of power (*i.e.*, is not energy voltage), a coil possessing low ohmic resistance and high self-induction may be used more economically than an ordinary resistance to reduce the current in an alternating circuit.

The following experiment illustrates the use of a choking coil and also the saving which may be effected by it.

Experiment IX.—Determination of Power Lost in a Choking Coil, and Power Saved by its Use.

Diagram of Connections.

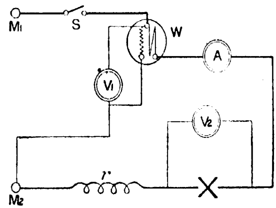


Fig. 31

M_1 M_2 Source of alternating current.
X Arc.
r Choking coil.
V_1 Voltmeter reading voltage of circuit.
V_2 Voltmeter reading voltage of arc.
A Ammeter reading current in circuit.
W Wattmeter reading power of circuit.
S Switch.

Instructions. Connect an alternating arc lamp and choking coil in series. Join them to a source of alternating current in series with an ammeter and the series coil of a wattmeter. Connect the shunt coil of the wattmeter and a voltmeter across the terminals of the circuit. Connect also a voltmeter across the arc.

Adjust the choking coil* so that the lamp burns with its correct current and voltage. Take several readings at short intervals of the current and watts, and of the voltages of the whole circuit and of the arc.

The readings should be tabulated as in the following table. The watts taken by the arc may be taken as numerically equal to the product of the current and the voltage across the arc.†

The watts lost in the choking coil and leads are the difference between the total watts of the circuit and the watts taken by the arc.

The power saved by using the choking coil, instead of an equivalent resistance, is the difference between the total volt-amperes of the circuit, *i.e.*, the "apparent watts," and the watts registered by the wattmeter, since with a non-inductive resistance in place of the coil, the total power taken would be the product of current and voltage of the circuit.

DETERMINATION OF POWER ABSORBED IN CIRCUIT CONTAINING ARC LAMP AND CHOKING COIL.

Voltage of Circuit = V_1.	Current I.	Voltage across Arc V_2.	Watts in Circuit = W	Watts of Arc = IV_2.	Watts Lost in Circuit = $W - IV_2$.	Watts Saved $IV_1 - W$.
100	10	40	568	400	168	432

The power factor of the circuit is the value of the fraction $\frac{W}{I\,V_1}$ and the angle of lag is the angle whose cosine is $\frac{W}{I\,V_1}$ or $\cos^{-1}\frac{W}{I\,V_1}$.

* Or if this cannot be done, adjust a resistance in the main circuit, so as to alter the voltage of the circuit shown in the diagram.

† The shunt coil and the series coil forming part of the mechanism of the lamp will be inductive, and may modify this relation slightly.

The watts spent in heating the coil are chiefly spent in heating the windings ($= I^2 R$ watts). Power will also be spent in hysteresis of the core, and in producing eddy currents in the metal of the core, screws, &c. The copper and iron losses can be separated by making a careful measurement of the resistance of the winding of the coil at the working temperature and calculating the power lost in this resistance.

The total voltage of the circuit is less than the sum of the voltages of the coil and arc. This is due to the fact that the voltage in the coil is not in phase with the voltage of the arc, because it is the resultant of an energy voltage and voltage of self-induction. The energy voltage is in phase with the voltage of the arc, since the arc lamp may usually be assumed to have a negligibly small self-induction.

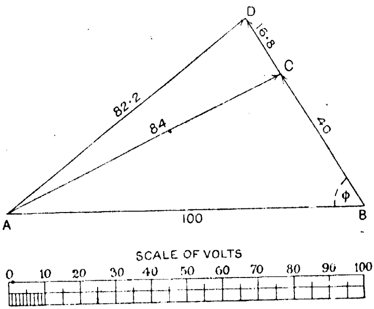


FIG. 32.—Diagram of E.M.F for Arc Lamp and Choking Coil.

The three voltages may be drawn as the three sides of a triangle, of which the total voltage forms the hypotenuse (see Fig. 32). The construction for this has been given in Experiment II., page 37. The values chosen represent those of the circuit in which the readings in the table (page 67) were taken.

Let $B\ A$ represent the total voltage.
$B\ C$ represent voltage of arc.
$A\ C$ represent voltage of coil.

Produce $B\ C$ and from A draw $A\ D$ perpendicular to $B\ C$, cutting it in D. Then the $\triangle\ A\ D\ C$ will be the $\triangle$ of voltages for the coil, $C\ D$ being the energy voltage in phase with the current, and $A\ D$ being the self-induction voltage. The power lost in the coil will be the product of the voltage represented by $C\ D \times$ current

in the circuit. The voltage represented by $A\ D$ is the product of the reactance of the coil $\times$ current $= X \times I$. Consequently the coil would have to be designed so as to have the self-induction calculated in this way. This may be put in other words by saying that the back voltage of the coil must be the voltage represented by $A\ D$.

The relation between the voltages in a circuit composed of an arc lamp and choking coil is so instructive that we shall illustrate it by a numerical example.

Calculation.—As an example to illustrate the preceding statements, the following example of a choking coil and arc lamp are given. The figures are those for which Fig. 32 is drawn.

An arc lamp requires 40 volts and 10 amps., and a choking coil is to be put in series with it to reduce the current to the required value. The voltage of supply is 100 volts. It is found by calculation, the details of which are explained later, that the power spent in heating the windings and overcoming hysteresis and eddy current losses is 168 watts.

The energy voltage represented by this lost power $= \dfrac{\text{watts}}{\text{current}}$ $= \dfrac{168}{10} = 16{\cdot}8$. The watts taken by the arc, which acts like a non-inductive resistance, $=$ (current $\times$ voltage) $= 10 \times 40 = 400$ watts; hence, total power given to circuit $= 400 + 168 = 568$ watts.

The total energy voltage $= 16{\cdot}8 + 40 = 56{\cdot}8$.

The total voltage applied to the circuit $= 100$, hence the idle voltage, or voltage spent in overcoming the self-induction of the coil, will be the third side of a right-angled triangle, having its hypotenuse 100 and one side $56{\cdot}8$. This is shown in Fig. 32.

This reactance voltage may be got by measurement or calculation. Its value $= E_0 = \sqrt{100^2 - (56{\cdot}8)^2} = 82{\cdot}2$ volts. The total voltage across the coil is the resultant of this voltage and of the energy voltage of the coil $16{\cdot}8$.

Total volts across coil $= E_1 = \sqrt{(82{\cdot}2)^2 + (16{\cdot}8)^2} = 84$ volts.

This could also have been obtained by direct measurement from the figure as shown. The angle of lag for the whole circuit is the angle between the energy voltage $B\ D$ and the total voltage $B\ A$. Referring to the figure, $\cos\phi = \dfrac{56{\cdot}8}{100} = {\cdot}568$, which is the power-factor of the circuit. The angle of lag for the coil alone is the angle $D\ C\ A$. For the coil, $\cos\phi = \dfrac{16{\cdot}8}{84} = {\cdot}2$.

The coil has consequently to be designed so that the back

electromotive force produced in it by self-induction is 82·2 volts. From the formula given previously, and explained on page 116,

$$E_0 = 4{\cdot}44\, F\, T\, f\, 10^{-8}.$$

Where E_0 = back voltage due to self-induction.
F = number of magnetic lines in core.
T = number of windings,
f = frequency of current.

Hence the product of $F\,T$ can be calculated. Usually the value of F will be determined by the size of core it is desired to use, and the magnetic density found desirable to avoid excessive iron losses.

Suppose in the present instance periodicity = 50.

$$\text{Then } F\,T = \frac{82{\cdot}2 \times 10^8}{4{\cdot}44 \times 50} = 37{,}000{,}000.$$

If it is decided to use a type of coil having a core with an iron section of 3 sq. in., and the maximum induction to be employed is 30,000 lines per square inch, the value of F = 3 × 30,000 = 90,000.

$$\text{Hence the number of turns of wire must be } \frac{37{,}000{,}000}{90{,}000} = 410.$$

Resultant of Two Currents Not in Phase.—An alternating current may be looked upon as the sum of two component currents, in the same way as we have already regarded an E.M.F. as capable of resolution into separate voltages. In this case the instantaneous value of the total current is at any moment the sum of the instantaneous values of the two component currents.

Also, if represented on a vector diagram, the line representing the total current is the diagonal of the parallelogram having the component currents as sides drawn to scale and in correct phase relation.

A case in which currents differing in phase and magnitude are combined together occurs when a circuit is formed by the junction of two branches of unequal resistance and inductiveness. Thus in Fig. 33 two circuits of this kind lie between the points X and Y. Between X and Y is an alternating voltage. One branch circuit is non-inductive, and consequently the current in this branch is in phase with the voltage maintained between X and Y. The other branch is, however, partly inductive, and the current lags behind the voltage in phase.

Thus in the two branch circuits there are two currents differing in phase and in magnitude, but the resultant current flowing from X to Y must be the same as flows in the single conductor forming the remainder of the circuit. On account of the difference in phase between the two currents, the sum of the currents measured in each circuit separately will not be equal to the total current. They must be combined in exactly the same manner as two electromotive forces which are not in phase.

The composition of currents in this way is illustrated by the following experiment, in which the currents first flow in two branch circuits, where they are measured separately, and are then made to flow together through a single conductor where the resultant current is measured.

EXPERIMENT X.—DETERMINATION OF RELATION BETWEEN CURRENTS IN BRANCH AND MAIN CIRCUITS.

DIAGRAM OF CONNECTIONS.

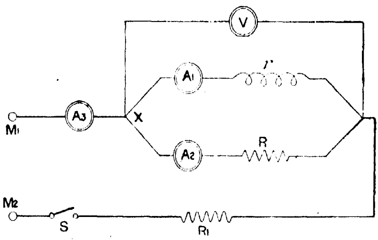


FIG. 33.

M_1, M_2	Source of alternating current.
R	Non-inductive resistance.
r	Inductive resistance.
R_1	Variable resistance.
A_1	Ammeter reading current in r.
A_2	Ammeter reading current in R.
A_3	Ammeter reading total current.
V	Voltmeter reading voltage of branch circuits
S	Switch for breaking circuit

Instructions.—Connect in parallel, to one terminal of the source of supply an inductive and a non-inductive resistance with an ammeter in series with each.

Join them both to the other terminal of the supply in series with a variable resistance, ammeter, and switch.

Connect a voltmeter to the points of junction of the two branch circuits.

Take simultaneous readings on the three ammeters and the voltmeter for several values of the current, which may be varied either by alteration of the voltage of the alternator supplying the circuit, or by varying the non-inductive resistance in the common branch of the circuit.

For each set of readings construct a triangle to represent the magnitude and phase relations of the currents. From this triangle obtain by measurement the value of the angle of lag between current and voltage of the complete circuit ($= \phi$): this will be the angle between the lines representing I_2 (in the non-inductive resistance) and I_3 the total current.

This is because I_2 is in phase with the voltage of the circuit and would therefore be parallel to it in the diagram of current and

voltage. Similarly the angle of lag in the inductive branch circuit r is the angle between I_2 and I_1 produced. Calculate also the impedance of each branch circuit, and the joint impedance of the two circuits by dividing the voltmeter reading by the current in each circuit. These values may be drawn as a triangle of impedance for the part of the circuit between X and Y

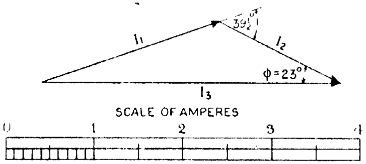


FIG. 34.—Diagram of Currents.

As an example of the readings taken and of the method of entering up, one out of a series of actual values, together with the corresponding diagram, is given below and on Fig. 34.

Voltage = V.	Current in $R = I_2$.	Current in $r = I_1$.	Current in $R + r = I_3$.	Angle of Lag.		Impedance.		
				In Joint Circuit = ϕ.	In $r = \phi_1$.	Of $R = \frac{V}{I_2}$.	Of $r = \frac{V}{I_1}$.	Of Joint Circuits = $\frac{V}{I_3}$.
20·6	1·48	2·12	3·41	23°	$39\frac{1}{2}°$	13·9	9·72	6·04

Idle Current.—In the discussion previously given of the conditions in a circuit in which a difference of phase exists between current and voltage, it was shown that the total voltage of the circuit might be considered as being composed of two components, viz.: the energy component ($= E \cos \phi$) in phase with the current, and the idle component ($= E \sin \phi$) $\frac{1}{4}$-period out of phase with the current.

For some purposes it is preferable to treat the total *current* as if formed of two components respectively in phase and $\frac{1}{4}$-period out of phase with the *voltage*. It is immaterial from a mathematical standpoint whether the current or voltage is looked upon as consisting of two components, since there are actually one current and one voltage in the circuit, and it is only for convenience that we consider either of them to be of a composite character

Following out this idea, Fig. 35 has been drawn to represent the same current, voltage, and angle of lag as shown in Fig. 7, page 26.

In this case, however, the current is resolved into two components whose values are $I \cos \phi$ in phase with the voltage, and $I \sin \phi$ $\frac{1}{4}$-period behind the voltage in phase. These components are usually termed the *energy current* and the *wattless* or *idle current* respectively, and are shown by the dotted lines marked C_e and C_i. It is evident from the curves that at any instant the sum of the values of energy and wattless currents is equal to the total current.

The currents are shown in Fig. 35 both in the form of curves and in the form of vectors. Thus $O N$, $N C$ are the two components of $O C$ which represents the total current.

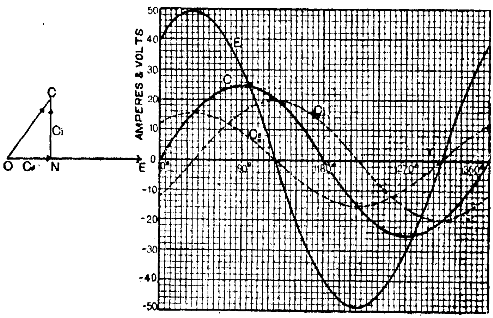


FIG. 35.—Curves of Voltage and Component and Resultant Currents

As in the previous discussion, it is only the current and voltage which are coincident in phase which represent power given to the circuit. Hence in this case

$$\text{power of circuit} = \text{energy current} \times \text{total voltage}$$
$$= I \cos \phi \times V = I V \cos \phi \text{ as before.}$$

Actually the two points of view illustrated by Figs. 7 and 35 lead to the same result, the difference lying in this, that in one case we speak of an *idle component of the voltage* overcoming the effects of self-induction, whereas in the other case we speak of an *idle current*, which may also be termed the magnetising current, and which is the portion of the total current spent in producing the magnetic field which is the manifestation of the self induction.

The following experiment is given as an example of this method of regarding the quantities in the circuit, and should be carefully compared with Experiment VIII. The measurements are a repetition of those made in Experiment VIII, but the conclusion drawn from them are obtained by a different course of reasoning.

EXPERIMENT XI.—DETERMINATION OF IDLE AND ENERGY CURRENT IN A CIRCUIT.

DIAGRAM OF CONNECTIONS.

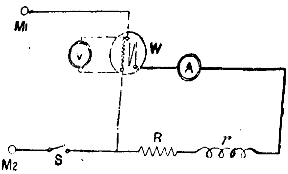


FIG. 36.

M_1 M_2. Source of alternating current.
W. Wattmeter.
R. Non-inductive resistance.
r. Inductive resistance.
A. Ammeter measuring current of circuit.
V. Voltmeter measuring voltage of circuit.
S. Switch for breaking circuit.

Instructions.—Connect in series an inductive and a non-inductive resistance, ammeter, the series coil of a wattmeter and switch. Connect a voltmeter and the shunt coil of the wattmeter across the two resistances, so as to read the total voltage of the circuit.

Take readings on the ammeter, voltmeter, and wattmeter for several values of the resistances.

For each set of readings determine graphically the idle and energy current and angle of lag as described below, and tabulate the results as shown in the following table :—

DETERMINATION OF IDLE AND ENERGY CURRENTS.

Voltage $=V$	Watts W	Total Current $=I$.	Energy Current $\frac{W}{V}$	Idle Current.	Angle of Lag.
100·7	1185	14·67	11·7	8·89	$36\frac{1}{2}°$

To separate the idle and energy currents, proceed as follows :—

Draw a horizontal line O C (see Fig. 37) to represent the total current of the circuit to a scale of amperes. Describe a semi-circle on O C as diameter. Determine the value of the energy current by dividing the measured watts by the voltage, and describe a circle

with centre C and radius equal to the energy current on the scale of amperes, cutting the semi-circle in D ; then the line O.D represents the idle current in phase and magnitude.

The figures inserted in the table above give a sample reading, and Fig. 37 is the diagram corresponding to these figures.

The value of the angle of lag may be obtained by measurement from the triangle, since the voltage of the circuit is in phase with the energy current, and consequently the angle D C O is the angle of lag. ϕ may also be got from the value of the power-factor ; thus

$$\frac{\text{watts}}{\text{amperes} \times \text{volts}} = \text{power factor} = \cos \phi.$$

As a check on the accuracy of the figures inserted in the table it should be remembered that

$$(\text{energy current})^2 + (\text{idle current})^2 = (\text{total current})^2 ;$$

indeed, it is by the use of this equation that the idle current would generally be calculated from the total and energy currents.

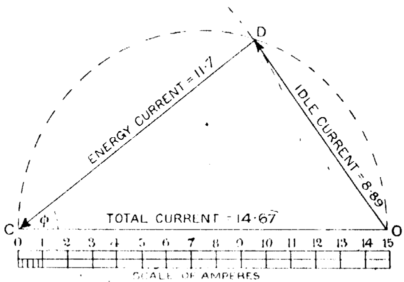


FIG. 37.—Diagram of Currents.

3-Voltmeter Measurement of Power.—A direct useful application may be made of the measurements made by 3 voltmeters in Experiment II., page 36. It was shown how the 3 voltmeter measurements could be represented graphically as the sides of a triangle of voltages. Since the voltage in the non-inductive portion of the circuit is in phase with the current, the angle of lag in the whole circuit is the angle between the lines representing the voltage in the non-inductive part and the total voltage respectively.

Similarly, the angle of lag for the inductive portion is the angle between the lines representing non-inductive voltage and voltage of the inductive portion.

Thus, referring to Fig. 37, page 25, the voltage represented by B C is in phase with the current in the circuit, and consequently the angle C B A is the angle of phase-difference between the current

and total voltage, *i.e.*, the angle of lag for the whole circuit. Similarly the angle D C B is the angle of lag for the inductive portion of the circuit.

Since the power in any circuit or portion of a circuit is given by $I \times E \times \cos \phi$, where I and E are the current and voltage of the part considered, we have in the diagram of the three electromotive forces all the information necessary to determine the power in the whole circuit and each portion of it, when we know also the value of the current.

Consequently in the diagram Fig. 15 we have

Energy in whole circuit $= I \times A\ B \times \cos C\ B\ A$.
Energy in inductive part of circuit .... $= I \times C\ A \times \cos D\ C\ B$.
Energy in non-inductive part .... $= I \times B\ C$.

where in each case the lines are taken as the voltages which they represent.

If, therefore, a non-inductive resistance forms part of a circuit, or if a non-inductive resistance can be inserted in the circuit for the purposes of the measurement, three readings of a voltmeter and a reading of the current enable us to measure the power of the circuit. This is frequently of great advantage where an accurate wattmeter is not available.

The diagram of connections necessary for such a measurement is Fig. 14. In the following experiment a wattmeter is introduced into the circuit so that a comparison may be made between the readings obtained with the wattmeter and by the 3-voltmeter method.

The ammeter is unnecessary if the value of the non-inductive resistance is known, as will be seen from the calculation given below.

EXPERIMENT XII.—MEASUREMENT OF POWER BY THE 3-VOLTMETER METHOD.

DIAGRAM OF CONNECTIONS.

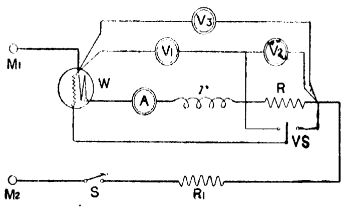


FIG. 38.

M_1 M_2 Source of alternating current.
r Inductive resistance.
R Non-inductive resistance.
R_1 Resistance for varying current in circuit.
V_1 Voltmeter measuring voltage in inductive portion of circuit.
V_2 Voltmeter measuring voltage in non-inductive portion of circuit.
V_3 Voltmeter measuring voltage of total circuit.
A Ammeter reading current in circuit.
W Wattmeter reading either total power or power in inductive portion.
V S Voltmeter switch for changing connections of wattmeter shunt coil.

Instructions.—Connect in series with an ammeter and the series coil of a wattmeter an inductive and a non-inductive resistance. Connect these to a source of alternating current through a switch and a variable resistance, in order to enable the current in the circuit to be varied.

Connect either three voltmeters, or a single voltmeter with 3-way switch, to read the voltages of the whole circuit, of the inductive and the non-inductive portions. Connect the shunt coil of the wattmeter to a 2-way switch so as to read alternately the power in the total circuit and in the inductive portion

Take readings on all the instruments for several values of the current, noting both total watts and watts in inductive resistance. Tabulate the results as shown below, where a few readings actually determined in the manner described are given as examples :—

MEASUREMENT OF POWER BY THREE VOLTMETERS.

Current.	Volts.			Watts observed.		Watts calculated	
	Total.	Inductive.	Non-inductive.	Total.	Inductive.	Total.	Inductive.
2·1	108·5	11	105	226	6	226·2	7·5
4·09	110	22·2	103	440	20	440	19·6
10·25	105	49	79	965	155	964·5	154·5
13·12	103	58	69	1140	236	1141	236·1

If the value of cos ϕ used in calculating the power from the readings of the ammeter and voltmeter is obtained by actual measurement, a high degree of accuracy is not possible. The power can be obtained entirely by calculation from the ammeter and voltmeter readings, as is shown by the following investigation :—

By the ordinary trigonometrical relations in a triangle we have (see Fig. 15, page 37) :— $V_1^2 = V_2^2 + V_3^2 - 2\, V_2\, V_3 \cos\phi$, but $V_2 = I\, R$ where R is the resistance of the non-inductive portion of the circuit.

$\therefore\; V_1^2 = V_2^2 + V_3^2 - 2\, R\, I\, V_3 \cos\phi.$

Since the total power of the circuit $= I\, V_3 \cos\phi$, we have

$$\text{Power in circuit} = \frac{V_2^2 + V_3^2 - V_1^2}{2\, R}$$

The power in the inductive portion r is obtained by subtracting the watts spent in the non-inductive portion from this expression.

The watts in the non-inductive resistance are $I^2\, R = \dfrac{V_2^2}{R}$

Hence watts in inductive part of circuit

$$= \frac{V_3^2 - V_2^2 - V_1^2}{2\, R}$$

In making a measurement of the power of the circuit it is not necessary to measure the current if the value of the non-inductive resistance is known. It will often be the easiest method of determining this resistance to put an ammeter in the circuit, as shown in the diagrams given.

For greatest sensitiveness of measurement the readings of the voltmeters connected to the inductive and non-inductive parts of the circuit should be as nearly equal as possible.

3-Ammeter Measurement of Power.—This method is analogous to the preceding, but employs three ammeters instead of three voltmeters for making the measurement. From the readings, a triangle of currents, instead of a triangle of voltages, gives the construction for determining the angle of lag.

In this case a non-inductive resistance must form a parallel circuit to the inductive part of the circuit, and must be added if not already part of the connections. The connections for making the measurement are practically those given in Diagram 33, page 71.

As in the previous case, the following experiment is arranged for a comparison between the measurement made by this method, and the measurement with a wattmeter :—

EXPERIMENT XIII.—MEASUREMENT OF POWER BY THE 3-AMMETER METHOD.

DIAGRAM OF CONNECTIONS.

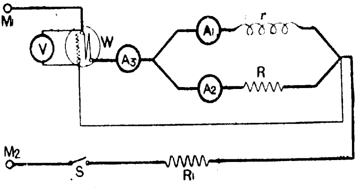


FIG 39.

M_1 M_2 Source of alternating current.
r Inductive resistance.
R Non-inductive resistance.
R_1 Resistance for varying current.
A_1 Ammeter measuring current in inductive circuit.
A_2 Ammeter measuring current in non-inductive circuit.
A_3 Ammeter measuring total current.
V Voltmeter measuring voltage of circuit.
W Wattmeter measuring total power of circuit.

Note.—The wattmeter may be connected so as to read the power in the inductive circuit only, by placing the current coil in the branch circuit instead of the main circuit.

Instructions.—Connect in parallel an inductive and a non-inductive resistance, each in series with an ammeter. Connect the joint resistance thus formed to a source of alternating current in series with an ammeter and a variable resistance for changing the current. Connect a voltmeter to measure the voltage applied to the branch circuits, and insert a wattmeter to read the power in them.

For several values of the current read all the instruments, and tabulate the results as below. The figures in the table are taken from actual readings, and given as an example.

The watts may be calculated after drawing a triangle of currents for each set of readings from the formula : Power $= I\ V \cos\phi$. The value of ϕ is the angle between the lines representing the current in the non-inductive branch and in the circuit for which the power is to be calculated.

MEASUREMENT OF POWER BY THREE AMMETERS.

Voltage.	Current.			Watts Observed.	Watts Calculated.	
	Total.	Inductive.	Non-Inductive.	Total.	Total.	Inductive.
20·6	3·41	2·12	1·48	63·5	64·2	33·7
27·7	4·70	2·92	2·12	117·4	117·7	59·3
32·0	6·13	3·80	2·56	185·0	185·9	103·9
45·8	7·98	4·80	3·76	333·3	334·3	162·0

As in the last experiment, the results should be checked by a calculation. In this case the calculation depends upon the formula obtained as follows :—

In the triangle of currents (see Fig. 34, page 72) we have by trigonometry

$$I_1^2 = I_2^2 + I_3^2 - 2\ I_2 I_3 \cos\phi.$$

Also $I_2 = \frac{E}{R}$ where E is the voltage at the terminals of the branch circuits, and R is the value of the non-inductive resistance.

$$\therefore\quad I_1^2 = I_2^2 + I_3^2 - \frac{2}{R} I_3 E \cos\phi.$$

But $I_3 E \cos\phi$ is the total power given to the branch circuits Hence,

$$\text{Total power in branch circuits} = \frac{R}{2}\left[I_2^2 + I_3^2 - I_1^2\right]$$

$$\text{The power in the non-inductive branch} = I_2^2 R.$$

$$\therefore \text{ Power in inductive branch} = \frac{R}{2}\left[I_3^2 - I_2^2 - I_1^2\right]$$

For maximum sensitiveness, the current in the two branches should be approximately equal.

If the value of the resistance in the non-inductive branch is known, it is not necessary to employ the voltmeter. Usually the addition of a voltmeter forms the simplest method of determining the resistance.

In the case of both 3-voltmeter and 3-ammeter measurements, the power is usually calculated directly by the use of the formulæ given, and not by construction of a triangle, or by the calculation of $\cos\phi$.

It will be seen that in order to measure the power in an inductive circuit by the 3-voltmeter method it is necessary to insert a series non-inductive resistance in the circuit, and thus to absorb a considerable proportion of the power during the measurement.

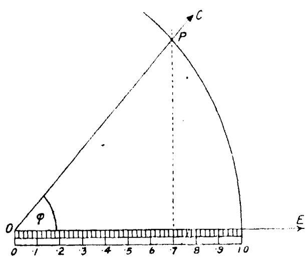


FIG. 40.—Graphic Determination of φ from Given Power Factor.

In making a measurement with three ammeters it is necessary to have a parallel branch circuit which takes a large part of the total power.

For these reasons it is not usual to employ these methods, except in such cases as the measurement of the load of a machine under test, where the manner in which the power of the circuit is absorbed is not of importance. For continuous reading of the power in a circuit employed in doing useful work, the method is not admissible.

In carrying out the experiments just given, if fairly small

currents are used, the losses in the voltmeters, ammeters, and wattmeter shunt coils may be found to affect seriously the accuracy of the measurement. A knowledge of the resistance of each instrument is necessary to determine the best arrangement for minimising these errors, and the experiments, if carefully carried out, form a valuable exercise in judgment as to their use.

Graphic Construction of Angle between Current and Volts.—We may here introduce the simple construction, which is of constant use in constructing vector diagrams, for drawing the current and voltage of a circuit in their correct phase relation when the power-factor of the circuit is known, and where it is not desired to ascertain the value of Θ from a table of cosines.

From O as centre, draw the arc of a circle having a radius of 10 units of length (say, 10 cm.). (See Fig. 40.)

Along the line of voltage mark off a number of units equal to 10 times the power-factor.

From the point thus found draw a line perpendicular to the voltage vector, to cut the circle in P. A line drawn from O through P will show the phase of the current.

In Fig. 40 the construction is shown for a power-factor of 0·7, while the divided scale shows how any other power-factor may be similarly treated.

The same construction may, of course, be carried out by starting with the current vector and obtaining the phase of the voltage by construction.

CHAPTER IV.

VIRTUAL VALUE OF AN ALTERNATING CURRENT.

Curve-tracing by Contact Maker.—The easiest method of determining experimentally the wave form of an alternating current or electromotive force is to fix a rotating contact maker to the shaft of the generator. This contact maker acts as a switch, which is only closed for an instant once during each revolution of the shaft. If the terminals of the alternator are connected to an electrostatic voltmeter through this contact switch, the voltmeter will indicate the voltage of the machine at the instant when the circuit is closed.

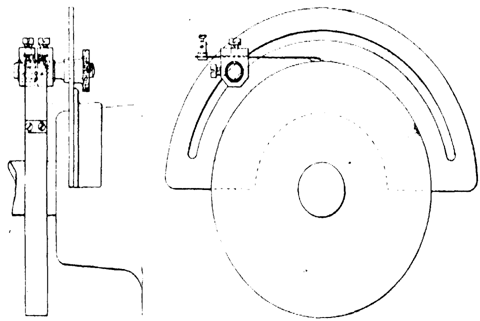


FIG. 41.—Rotating Contact Maker.

By varying the position of the contact maker, the voltage of the alternator can be observed for a series of positions of its rotating part, and the variation of voltage during a complete cycle may be traced.

One of the simplest forms of such a contact maker is a disc of ebonite, with a metal strip let into the edge at one point. Two insulated brushes, each formed of a strip of spring steel, are mounted side by side so as to press on the edge of the disc. Once in each revolution they are connected together by the rotating strip. By mounting the brushes on a movable arm which can be clamped to a divided sector, the point of contact can be varied by any desired angle. A convenient form of this arrangement is shown in Fig. 41.

Another device which is very easy to apply is to provide a rotating pin on the shaft or coupling of the alternator. The pin

comes into contact with a light spring once during each revolution, and so completes the circuit. The spring in this case also must be insulated, and provided with means for moving it round a graduated dial.

Contact is usually made to the rotating pin through the frame and shaft of the machine. In some cases this would be objectionable, and the pin must then be insulated, and connected to a slip ring.

The readings are generally taken upon an electrostatic voltmeter, since the readings of such an instrument are independent of the duration of the time of application of the voltage. It is usually necessary to put a condenser in parallel with the voltmeter, in order to insure steady readings, since the leakage which takes place is often sufficient to partially discharge the voltmeter between successive instants of contact, especially if the speed be low. By adding a condenser, a practically uniform voltage is maintained at the terminals of the voltmeter, in spite of a small leakage. In Fig. 41 it will be noticed that the spring brushes are put one in advance of the other, so that the contact is made only momentarily, and not during the whole time necessary for the contact to pass under the brushes.

Another method of taking the readings is to replace the electrostatic voltmeter by a condenser, which is then discharged by a switch through a ballistic galvanometer, when a reading is to be made. By this method a momentary throw, instead of a steady reading, is obtained.

EXPERIMENT XIV.—DETERMINATION OF WAVE FORM OF AN ALTERNATOR.

DIAGRAM OF CONNECTIONS.

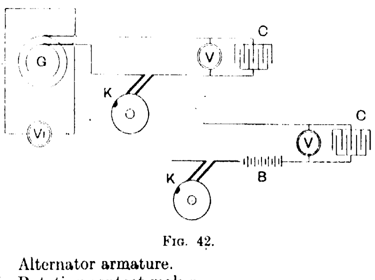


FIG. 42.

G Alternator armature.
K Rotating contact maker.
V Electrostatic voltmeter for reading instantaneous value of voltage.
C Condenser.
V_1 Voltmeter for reading virtual value of voltage of alternator.

Instructions.—Connect the alternator terminals to an electrostatic voltmeter in series with the rotating contact maker, so that

the voltmeter is only momentarily connected to the alternator by the contact maker once during each revolution. It will usually be found necessary to connect a condenser in parallel with the electrostatic voltmeter in order to get reliable readings. Connect a second voltmeter, which need not be electrostatic, to the terminals of the alternator. Insert a regulating resistance in the alternator field circuit.

Throughout the experiment keep the alternator voltage, as read on voltmeter V_1, constant, by regulation of the field when necessary.

For a series of positions of the contact maker take readings of the voltmeter. V.

Plot the results as a curve, measuring displacement of contact horizontally and voltage vertically.

It is probable that difficulty will be experienced in reading the lower voltages on account of the uneven scale of the electrostatic voltmeter. Thus a voltmeter reading up to 80 volts cannot be used for voltages much below 20, and a voltmeter reading to 160 will probably have a scale not extending below 80. For taking the lower readings, a small battery may be inserted in the position indicated by B in the lower part of Fig. 42. The voltage of this battery is then in series with the voltage to be measured, and should be chosen of a suitable value to bring even the lowest reading on to the scale. The true value of the voltage to be measured is obtained by subtracting the battery voltage from the voltage recorded on the voltmeter. This auxiliary battery has not to supply any appreciable current, and may consequently be formed of very small cells. These should give a fairly constant voltage, and must be well insulated. A battery of small secondary cells answers the purpose well, or a set of small dry cells, which are inexpensive. The continuous current supply mains may be also employed for this purpose, if precautions are taken to ensure that leakage cannot affect the readings.

An additional advantage gained by the addition of a continuous voltage is that it enables a distinction to be made between positive and negative values of the voltage. Thus the battery will be in series with the voltage when acting in one direction and in opposition to it when it changes sign and the volt curve crosses the zero line. Without such a device it is often difficult to judge the exact point of reversal, since the voltmeter readings are always in the same direction, irrespective of reversal of the applied voltage. A key should be arranged for throwing the battery directly on to V, so that its voltage may be read at short intervals, and subtracted from the total readings.

The same method of tracing curves may be applied to read the instantaneous values of the current in an alternating circuit, and so to obtain the current wave form. In this case the terminals of a non-inductive resistance in the circuit are connected periodically to the voltmeter by means of the contact maker. In a non-inductive resistance the voltage is always proportional to the current,

and consequently the voltmeter readings when divided by the constant resistance give the values of the current.

By means of a throw-over switch the simultaneous values of current and voltage in a circuit may be obtained by connecting the contact maker and electrostatic voltmeter alternately to the terminals of the alternator, so as to read the voltage of the circuit, and to the terminals of a non-inductive resistance, so as to read the voltages proportional to the current.

This is the principle of the following experiment :—

EXPERIMENT XV.—DETERMINATION OF SIMULTANEOUS CURVES OF CURRENT AND VOLTAGE IN A CIRCUIT.

DIAGRAM OF CONNECTIONS.

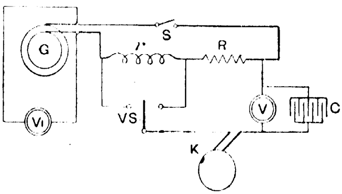


FIG. 43.

G Alternator armature.
R Non-inductive resistance.
r Resistance partly inductive.
K Rotating contact maker.
V Electrostatic voltmeter for reading instantaneous voltage.
V_1 Voltmeter for reading virtual voltage of alternator
VS Throw-over switch.
C Condenser in parallel with electrostatic voltmeter.
S Switch.

Instructions.—Connect in series to the terminals of the generator a non-inductive resistance and the inductive resistances which are to form the circuit. Connect to the centre terminal of a throw-over switch an electrostatic voltmeter in series with the rotating contact maker.

Connect the other pole of the voltmeter to the terminal of the non-inductive resistance which is supplied direct from the alternator.

To one side of the 2-way switch connect one terminal of the generator ; to the other side connect the terminal of the non-inductive resistance nearest to the same generator terminal. Connect a voltmeter to the terminals of the generator. If found necessary, connect a condenser in parallel with the electrostatic voltmeter. A battery may be used in series with the electrostatic voltmeter as described in the previous experiment, if necessary to read the low voltages.

For each position of the contact maker read the instantaneous voltage of the alternator, and the instantaneous voltage of the non-inductive resistance, by putting the switch first to one side and then to the other. Throughout the experiment maintain the voltage of the alternator constant by regulating the field if necessary. Results should be entered in tabular form, and curves of current and voltage should be plotted on the same sheet of squared paper. An example of a curve taken in this way is shown in Chapter XIII.

A little care will have to be used in deciding which points near the zero line are positive and which are negative, since the voltmeter will always give readings in the same direction If the contact maker is moved rapidly at this point, the sign of the voltage may change before the voltmeter needle has time to go to zero and rise again as the zero point is passed. By moving the contact maker slowly, and carefully watching the voltmeter, the approximate point of reversal can be obtained with certainty ; the exact point can only be seen after plotting the curve. As already mentioned, the use of a battery is a great assistance at this part of the curve.*

Zero Method.—An alternative method, which can be made very sensitive and which does not involve the use of an electrostatic voltmeter, may be shortly indicated.

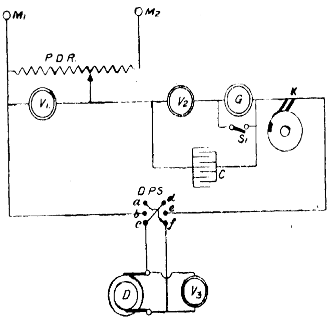


FIG. 44.

The connections will be followed from Fig. 44, together with the following list :—

D machine whose wave-form is required.

M_1 M_2 source of continuous voltage, either supply mains or

* A slight modification of the arrangement shown in Fig. 43 enables the low values of current and voltage to be measured with increased accuracy By connecting the voltmeter to read alternately the voltage across r, R and $R + r$ (instead of R and $r + R$ only), the separate readings will check each other. Thus, since the quantities read are *instantaneous* values, the sum of the voltages across r and R must be equal to the voltage of $R + r$, and thus the two readings across the separate resistances should always be together equal to the third reading. Since the circuit is inductive, it will seldom happen that more than one of the quantities is small at the same time. Thus the value of a low reading can generally be calculated as the difference of two readings of more convenient magnitude

battery, having voltage at least equal to maximum value of voltage wave to be measured.

G sensitive dead-beat galvanometer to be used for indicating zero current.

$P\ D\ R$ potential dividing resistance, which should be finely divided.

V_1 moving-coil voltmeter, upon which are read the successive values of the voltage wave.

V_2 moving-coil voltmeter put in series with the galvanometer to act as series resistance, and also to indicate approximate point of balance before the galvanometer is allowed to read (by opening S_1)

V_3 alternating-current voltmeter indicating virtual voltage of the source which is being tested.

S_1 short-circuiting switch, which is kept closed after each fresh adjustment of the contact maker until approximate balance is indicated by V_2.

C condenser which enables much steadier readings to be obtained

K revolving contact maker.

$D\ P\ S$ double-pole reversing switch.

The method of procedure is to adjust the movable contact of the potential dividing resistance until V_2 indicates zero. Switch S_1 is then opened and further adjustment made, if necessary, for obtaining as nearly zero reading as possible on G. For zero deflection of G, the reading of voltmeter V_1 is that for the point on the wave being measured. Usually exact balance is not to be obtained, and the position of balance must be estimated from deflections produced on G in opposite directions with two positions of the potential divider.

The current through G will be a pulsating uni-directional one, but sufficiently constant to give a definite deflection.

For a description of the Oscillograph and Ondograph, see Chapter XIII.

Comparison of Values of Alternating and Direct Current.—The simplest standard of comparison between alternating and direct currents is that of their heating effects when passed through a resistance, since this is independent of direction and dependent only on the strength of the current.

The rate at which heat is developed in a conductor whose resistance is R ohms is I^2R joules per second, where I = current in amperes. Thus a direct current of I amperes would develop I^2R joules in one second. An alternating current of *equivalent value* will also develop I^2R joules in one second. Hence the alternating current of I amperes, which is equivalent to a direct current of I amperes, must be such that the average value of I^2 (alternating) = average value of I^2 (direct), *i.e.*, $\sqrt{\text{average value of } I^2 \text{ alternating}} = \sqrt{\text{average value of } I^2 \text{ direct}}$.

Now $\sqrt{\text{the average value of } I^2 \text{ direct}}$ is I direct, but $\sqrt{\text{the average value of } I^2 \text{ alternating}}$ is not the same as the average value of I alternating.

If n represents a series of different terms, the $\sqrt{\text{average value of } n^2}$ is not equal to the average value of n, as can be shown at once by taking a numerical example.

For instance, the average value of the numbers 1, 2, 3, 4 is

$$\frac{1+2+3+4}{4} = \frac{10}{4} = 2{\cdot}5$$

The average value of their squares is

$$\frac{1+4+9+16}{4} = \frac{30}{4}$$

The square root of this $= \dfrac{\sqrt{30}}{2} = \dfrac{5{\cdot}47}{2} = 2{\cdot}73.$

The student should convince himself by actual trial of the difference between the average value and the root of the mean of values squared. For this purpose he should draw a curve such as that shown in Fig. 4, and measure the length of a number of equidistant ordinates. By adding these lengths together and dividing by the number of ordinates taken, he will obtain the *average* value of the current represented by the curve. By squaring each ordinate, adding the ordinates together, dividing by the number of ordinates, and then extracting the square root, he will obtain the value of the alternating current which is the equivalent, as regards heating, of a direct current of that number of amperes.*

It can be proved mathematically that the average ordinate of a sine curve is $\dfrac{2}{\pi} = {\cdot}635$ of the maximum ordinate, whereas $\sqrt{\text{the average value of the squares of the ordinates}} = \dfrac{1}{\sqrt{2}}$ or ${\cdot}707$ of the maximum ordinate.

It is important always to remember that the *average* value of an alternating current is never employed, since it would not be equivalent to a direct current of the same number of amperes.

The value of the alternating current which is equivalent to a direct current as regards heating effect is called the *virtual*, *effective*,

* If the value of a sinusoidal voltage be written in the form $E \sin \theta$, where E is the maximum value of the voltage, we obtain the *average* value as follows :—

$$e_{av} = \frac{1}{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} E^2 \sin\theta \, d\theta = \frac{2E}{\pi}\Big[-\cos\theta\Big]_0^{\frac{\pi}{2}} = \frac{2}{\pi}E$$

The virtual or R.M.S. value of the same function is :—

$$e_{virt} = \left\{\frac{1}{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} E^2 \sin^2\theta \, d\theta\right\}^{\frac{1}{2}} = \left\{\frac{2E^2}{\pi}\int_0^{\frac{\pi}{2}} \frac{1-\cos 2\theta}{2} \, d\theta\right\}^{\frac{1}{2}}$$

$$= E\left\{\frac{2}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{2}}\right\}^{\frac{1}{2}} = E\left\{\frac{2}{\pi}\cdot\frac{\pi}{4}\right\}^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{2}}E.$$

or *R.M.S.* (root of mean squares) value, and it is this which is recorded on an ammeter and is generally understood when an alternating current of a given number of amperes is spoken of.

The virtual value of an alternating current of regular wave form is therefore $\frac{1}{\sqrt{2}}$ or ·707 of its maximum value.

Exactly the same statement applies to an alternating electromotive force, the virtual voltage being $\frac{1}{\sqrt{2}}$ of the maximum value.

The ratio of the virtual value of a wave to its average value is sometimes called the *form factor* of the wave For sinusoidal waves it is evident that the form factor has a value $\frac{0{\cdot}707}{0{\cdot}635} = 1{\cdot}11$.

Scales of Instruments.—Both ammeters and voltmeters give readings proportional to the mean values of the square of current or voltage, and would consequently give, in most cases, a nearly even scale for (amperes)^2 or (volts)^2. The reason for this is as follows: The reading of all alternating-current measuring instruments is produced either by the expansion of a heated wire or by the mutual action of a fixed and a movable part. In hot-wire instruments the average heating effect on the wire must obviously be due to the average value of $I^2 R$ as explained above, and the expansion of the wire is approximately proportional to the rate at which heat is given to it. In all other instruments the deflection depends upon the force with which the fixed and movable parts act upon one another. But both fixed and moving portions exert forces proportional to the current or voltage being measured. The resultant force depends, consequently, on the product of the two forces exerted by the fixed and moving parts, *i.e.*, on the product of two quantities, each separately proportional to the quantity to be measured. It thus results that the average force producing the deflection is the average of a force due to (current $\times$ current) or (voltage $\times$ voltage), *i.e.*, to the average of the square of the quantity to be measured.

As stated above, the scale of an alternating-current instrument would usually be evenly divided if graduated in (amperes)^2.

An excellent example of this is the Siemens dynamometer, in which the scale is uniformly divided, and the readings give the average value of (current)^2. The virtual current is obtained by extracting the square root of the reading.

For convenience in reading, the scale of most instruments is marked in volts or amperes, instead of (volts)^2 or (amperes)^2.

In consequence, direct-reading alternating-current ammeters and voltmeters have an uneven scale, and the distance between successive sub-divisions increases approximately in the ratio of the square of the value of the quantity being measured. In many instruments arrangements are adopted by which a more even graduation is obtained by making the deflection corresponding to a given force smaller at higher points of the scale, or by arranging

that the action between fixed and moving parts becomes less as the deflection increases.

In cases where we wish to distinguish between the *maximum* value of alternating currents, voltages, &c., and their *effective* or virtual values; it is convenient to adopt the general convention of indicating the maximum value by a capital letter and the virtual value by the corresponding small letter.

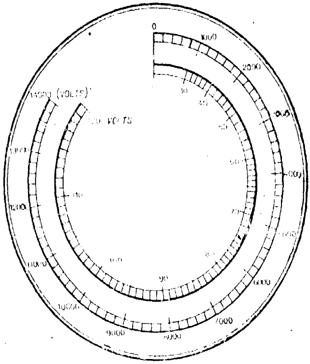


FIG. 45.—Scales of Volts and (Volts)^2.

Thus I, V represent maximum values ;
i, v are virtual, or *R.M.S.* values.

Consequently for simple harmonic wave form we have

$$I = \sqrt{2}\, i. \qquad V = \sqrt{2}\, v., \&c.$$

In order to show that the reading of an alternating-current instrument is almost proportional to the square of the current taken by it, the scale of a Cardew hot-wire voltmeter is shown in Fig. 45, where the inner scale represents the scale of volts, and the outer evenly-divided scale shows the graduations, which are proportional to the expansion of the wire producing the deflections. It is thus seen on the figure that the deflection corresponding to 70 volts is four times the deflection for 35 volts, and that a practically uniform scale would be obtained if the graduations were measured in (volts)^2 instead of volts.

A simple way of showing the relationship between the *R.M.S.* value of a current and its maximum value is to employ a current with a frequency of about 1 cycle per second and to send this through a hot-wire ammeter and a central-zero moving-coil ammeter connected in series. The hot wire instrument will give a practically steady reading, while the moving-coil instrument will follow the variations of the current.*

* See J. H. Buchanan, *Electrician*, July 18th, 1919.

Average Power in a Circuit.—Let us examine the curve of power (marked W) in a non-inductive circuit given in Fig. 27, page 60. From the symmetrical form of the curve it is evident that the average distance of the curve above the zero line is half the height of its highest point. In fact, the dotted line $C\ D$ which is drawn at this average height forms an axis about which the watt curve is symmetrical. This relation is universally true, and the *average* value of the power developed in non-inductive circuits is half the *maximum* value of the power. In the case of measurement of power, it is the *average* value which is required for all purposes of comparison, since the total work done by the current in the circuit is the sum of all products of the form (power at any moment $\times$ time during which it is developed) ; this is equivalent to (*average power* $\times$ *time considered*). The work done in a circuit in five minutes, for instance, is (average value of watts $\times$ 5 $\times$ 60 secs.) joules.

The maximum value of the watts in a non-inductive circuit = maximum amperes $\times$ maximum volts (see Fig. 27), hence average watts $= \frac{1}{2}$ (maximum amperes $\times$ maximum volts) $= \frac{1}{\sqrt{2}}$ maximum amperes $\times \frac{1}{\sqrt{2}}$ maximum volts

= virtual amperes $\times$ virtual volts.

Hence we have the important relation for a non-inductive circuit :—

Average power in circuit = product of virtual amperes and virtual volts.

The relation just obtained may be extended to an inductive circuit if we substitute the *energy* voltage for the total voltage. This is permissible, since the energy voltage is in phase with the current, and is the only part of the voltage which influences the power of the circuit.

The energy voltage $= e \cos \phi$, and consequently in an inductive circuit.

average watts = virtual amperes $\times$ virtual volts $\times \cos \phi$.

The above relations may be summarised in symbols as follows, capital letters representing *maximum* values, and small letters virtual values.

For non-inductive circuits

$$w = \frac{1}{2} W = \frac{1}{2} I E$$

$$\therefore w = i e.$$

For inductive circuits

$$w = \frac{1}{2} W = \frac{1}{2} I E \cos \phi$$

$$\therefore w = i e \cos \phi.$$

It is important to remember that it is the *average* and not the R.M.S. value of the watts which gives the true power in a circuit. A wattmeter deflection is proportional to the average value of the watts in a circuit.

CHAPTER V.

EFFECT OF CAPACITY.

Capacity in an Alternating Circuit.—A condenser usually consists of two metal plates very near together, but separated by a thin layer of insulating material called the dielectric.

When the two metal plates are connected to points which have a difference of potential between them, the condenser becomes chargd by positive electricity flowing into one plate and negative into the other. This flow of electricity will continue until the potential difference of the condenser plates is the same as that of the points to which they are connected. The quantity of electricity which flows into the condenser in order to establish this condition of equilibrium depends on the *capacity* of the condenser and on the difference of potential produced between the plates.

The practical unit of capacity is the *Farad.*

The unit of quantity of electricity is the *Coulomb,* and is the quantity which is transmitted by a current of 1 ampere in 1 second.

Thus the *charge* of a condenser = number of coulombs which have flowed into it = number of amperes × duration of flow in seconds.

$$\text{Also capacity} = \frac{\text{amount of charge}}{\text{potential difference due to charge}}$$

$$= \frac{\text{amperes} \times \text{seconds}}{\text{volts}}$$

The Farad being too large to correspond to the dimensions of capacities occurring in practice, the *microfarad, i.e.,* one millionth of a Farad, is usually employed as a unit of capacity.

Almost all conductors may be considered to form one plate of a condenser, the other being the earth, a metal sheathing, or another conductor. If such a condenser has its electrodes connected to the terminals of a generator, the generator will set up a difference of potential between the electrodes, equal to that which exists between its own terminals. In order to produce this result, the generator will supply a certain *quantity* of electricity, so as to charge the condenser. If the generator supplies alternating current, the charges given to the condenser plates will be alternately positive and negative, and there will thus be an alternating flow of electricity (*i.e.,* a current) between the machine and both poles of the condenser.

The following general explanation is given for ths sake of the student who is not already familiar with the nature of electrical capacity.

Effect of Capacity in the Circuit: General Explanation.—If a constant electromotive force be applied to a circuit possessing neither self-induction nor capacity, a current is instantly produced, having the value

$$\text{Current} = \frac{\text{electromotive force}}{\text{resistance}}$$

If, however, the circuit possesses electrostatic capacity as well as resistance, a charging current will have to flow into the circuit before the voltage acting upon the resistance rises to the full value of the applied electromotive force. Consequently, at the first moment of application of the voltage, a current will flow into the circuit, and the strength of this current will depend mainly upon the capacity of the circuit, and only partly upon its resistance. It is only after this initial current has charged the circuit, that the normal value of the current will be established, and the current strength will be that given by Ohm's law. Thus in a circuit containing a condenser, or having electrostatic capacity, there will be an initial rush of current which may be considered as flowing in advance of the normal current of the circuit.

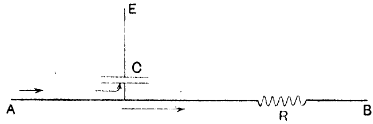


FIG. 46.—Circuit Possessing Capacity and Resistance

This may be illustrated by the diagram Fig. 46. Suppose that C represents a condenser with one pole connected to the circuit, and the other pole earthed. A is a point in the circuit which is suddenly connected to a source of high potential. This will cause a flow of current from A to B. Before the high potential applied at A can produce a current through the resistance R, the potential at the end of R nearest to A must be raised by the flow of electricity from A. It is only after the charging current has raised the potential of the condenser plates to the potential of A that the full potential of A can act upon R so as to send current through it.

In the case of an alternating-current circuit possessing capacity, there will be a charging current in the circuit at each reversal of the electromotive force, and the total current flowing will be the sum of the capacity-charging current and the normal current obeying Ohm's law.

When a condenser is first connected to the positive and negative terminals of a source of electromotive force, equal and opposite charging currents will flow into the condenser from the two sides. If we consider a positive charge to flow in at the positive terminal of the condenser, a negative charge will simultaneously flow in at the negative terminal. But a displacement of a positive charge of

electricity in one direction is equivalent to a displacement of a negative charge in the opposite direction. Consequently current may be considered as actually flowing in the same direction in the circuit on both sides of the condenser, and is often said to flow " in " at one terminal and " out " at the other, or to flow *through* the condenser. As soon as the condenser is charged, the flow must necessarily cease until the difference of potential of the source changes.

Relation between Voltage and Current.

The following symbols will be employed :—

Q Quantity of electricity in coulombs.
C Capacity of condenser in farads.
E Difference of potential between terminals of condenser.
I Current in amperes flowing into condenser.

When connected to a constant source of electromotive force, no current flows through the condenser. In this case

$$\text{Charge} = Q = C\,E.$$

During the process of charging,
Current flowing to condenser = rate of change of charge.

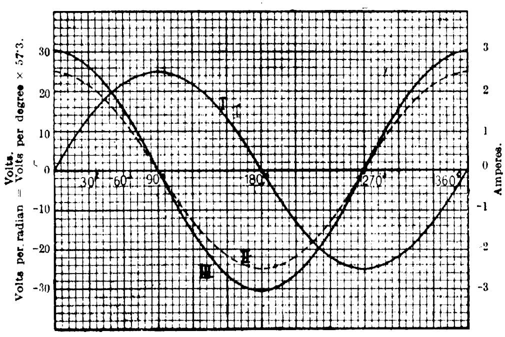


FIG. 47.—Curves of Electromotive Force, Change of Electromotive Force and Charging Current. Capacity = 200 Microfarads.

Curve I.—Voltage. Curve II.—Rate of Change of Volts.
Curve III.—Charging Current.

When connected to a source of alternating electromotive force, the difference of potential between the terminals will change at the same rate as that of the alternating source. Consequently $Q = C\,E$ will change also. Since current = rate at which quantity flows, the rate of change of $C\,E$ (measured in coulombs per second) will be numerically equal to the current flowing to the condenser.

Hence charging current of condenser = capacity × rate of change of voltage at terminals.

It is now possible to represent graphically the relation between the voltage applied to a condenser and the charging current.

This is done in Fig. 47, where Curve I. is a curve of voltage whose maximum value is 25 volts. Curve II. is a curve of rate of change of voltage (measured in volts per radian*), and may be obtained by a similar construction to that given on page 22 for finding the "rate of change" of a current. Since in this case the rate of change is measured per *radian*, instead of per cycle, the maximum height of the curve obtained is equal to the maximum height of the curve of voltage, because the maximum rate of change per radian of a sine function is equal to the maximum value of the function. The maximum rate of change of volts per *cycle* would be 20 times as great. Curve III. is the charging current plotted to a scale of amperes, each value being obtained from Curve II. by multiplying by the factor ·125, which is equal to the product of the capacity of the condenser (= 200 microfarads) and $20\ \pi f$ the number of radians per second at a frequency of 100. The maximum current is thus found to be 3·14 amperes.

If E = maximum value of voltage

then instantaneous value of voltage $= E \sin \theta$

and instantaneous value of rate of change of voltage

$$= E \sin (\theta + 90^\circ) = E \cos \theta.$$

Hence value of current at any instant $= 2\ \pi\ f\ C\ E \cos \theta$.

It will be seen, both from the figure and from the expressions just given, that the variations of the charging current will be similar to those of the voltage in character, but will always occur *one-quarter period earlier in phase*. Hence the charging current will be similar to the *idle current* discussed on page 72, except that its phase is ¼-period *before* that of the voltage of the circuit, instead of being ¼-period later. By the same reasoning as that given previously, it is evident that the charging current is a wattless or idle current.

In a circuit possessing both capacity and resistance, two currents will arise when an electromotive force acts on the circuit, viz., (1) the energy current in phase with the voltage, (2) a charging current leading the voltage in phase by ¼-period. The total current flowing at any instant will be the algebraic sum of the instantaneous values of these two. The resultant current may therefore be represented by a rotating line obtained in magnitude and relative phase as the diagonal of a rectangle having the maximum values of the two component currents as the lengths of its sides. The reason in this case follows exactly as in the case of the resultant of two electromotive forces given on page 30.

This resultant current will differ in phase from the voltage, but will lead by an angle ϕ depending on the relative magnitude of the resistance and capacity.

* The radian is the unit angle in circular measure, and is equal to about 57·3°. There are $2\ \pi$ radians in a complete circle, or 360°.

The charging current will be the component which is one-quarter period in advance of the voltage, while the energy current will be the component in phase with the voltage. As in the case of an inductive circuit, the power given to the circuit is the product of energy current, and voltage or the product of total current × total voltage × cos ϕ, when ϕ is the angle of lead of current and cos ϕ is the power factor of the circuit.

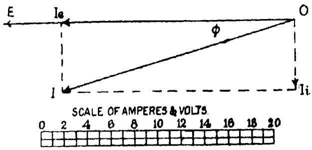


FIG. 48.—Diagram of Voltage and Currents in Circuit, with Capacity.

The relation between these quantities is shown in diagram form in Fig. 48, where $O\,E$ is the total voltage and $O\,I$ the resultant current in the circuit. The charging current is $O\,I_i$ and the energy current $O\,I_e$.

The angle of lead is $I_e\,O\,I$, *i.e.*, the angle between total voltage and total current.

The values of the quantities represented are

Total voltage $= E = 25$ volts.
Energy current $= O\,I_e = 20$ amps.
Idle current $= O\,I_i = 2\,\pi\,f\,C\,E = 5$ amps.
Total current $= O\,I = \sqrt{20^2 + 5^2} = 20{\cdot}6$ amps.

Impedance of Circuit having Capacity.—In the case of a circuit with capacity as with self-induction, the impedance or apparent resistance is the value of the fraction

$$\frac{\text{voltage of circuit}}{\text{current in circuit}}$$

and in this case, also, the impedance, or apparent resistance, is always greater than the true, or ohmic, resistance.

Its value may be obtained by drawing a triangle of impedance similar to that obtained on page 40 for an inductive circuit. The components of the impedance at right angles to each other will be, respectively, R, the resistance of the circuit, and $\frac{1}{2\,\pi\,f\,C}$ which is the reactance due to the capacity,

since $$\frac{\text{total voltage}}{\text{charging current}} = \frac{E}{2\,\pi\,f\,C\,E} = \frac{1}{2\,\pi\,f\,C}$$

The resultant of these components representing the total impedance of the circuit $= \sqrt{R^2 + \left(\frac{1}{2\,\pi\,f\,C}\right)^2}$.

These results are illustrated in the following experiments :—

Experiment XVI.—Determination of the Effect of a Condenser upon the Phase Relations in a Circuit.

Diagram of Connections.

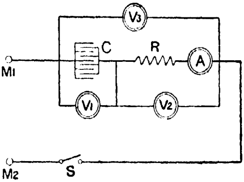


Fig. 49.

M_1 M_2 Source of alternating current.
R Non-inductive resistance.
C Condenser.
A Ammeter.
V_1 Voltmeter for reading voltage across condenser.
V_2 Voltmeter for reading voltage of non-inductive resistance.
V_3 Voltmeter for reading total voltage of circuit.

Instructions.—Connect in series to the source of supply a condenser or other form of capacity, a non-inductive resistance, ammeter, and switch.

Connect voltmeters*, or a single voltmeter with 3-way switch, to read respectively the voltage across the capacity, resistance, and total circuit.

For several values of the current obtained by variation of the resistance R take readings on the voltmeters and ammeter.

For each set of readings construct a triangle of electromotive forces and deduce the angle of lead of the current by measurement of the angle from the diagram. Calculate in each case the power factor by dividing the energy voltage by the total voltage. From the value of the power factor ($= \cos\phi$) find the value of ϕ from a table of cosines.

Readings should be tabulated as follows :—

Determination of Phase Relations in Circuit with Capacity.

Voltage of Circuit $= V_3$	Voltage of Non-inductive Resist. $= V_2$	Voltage across Condenser $= V_1$	Current $= I$	Angle of Lag from Diagram.	Power Factor $\frac{V_2}{V_3}$	Angle of Lead by Calculation $\cos^{-1}\frac{V_2}{V_3}$	Impedance $= \frac{V_3}{I}$
114	26	111	·76	76·5	228	76·75	150
112·5	53·4	99	·68	61·5	·475	61·75	165·5
110	85·7	69	·47	38·5	·779	38·75	234

* It is best to use electrostatic voltmeters for the experiments on condensers in order to avoid errors due to voltmeter currents.

Periodicity of circuit = 54.

From results of this experiment the value of the capacity can be calculated, as well as the value of the resistance of the circuit.

Thus the idle voltage $V_1 = \frac{I}{2\pi f C}$ from which the value of C can be calculated.*

Further, the energy voltage = $V_2 = I R$, from which the value of R can be determined.

The figures given in the table above are three sets of readings taken from a series by way of illustration. The diagram of voltage corresponding to these results is shown in Fig. 50. In the case of

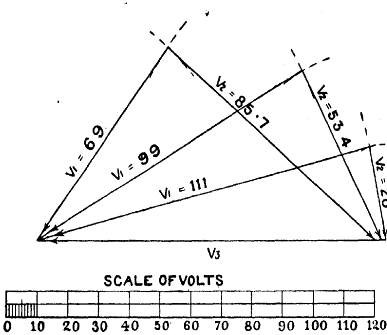


FIG 50.—Diagram of Volts for Circuit with Capacity.

the third set of readings, the value of the capacity is given by the formula thus :—

$$K = \frac{I}{2\pi f V_1}$$
$$= \frac{\cdot 47}{2\pi \times 54 \times 69} = \cdot 00002 \text{ farad.}$$
$$= 20 \text{ microfarads.}$$

Also the resistance of the circuit is given by

$$R = \frac{V_2}{I} = \frac{85\cdot 7}{\cdot 47} = 182 \text{ ohms.}$$

This resistance was actually a single 16 c.p. 100-volt carbon filament incandescent lamp.

The readings also enable us to calculate the power of the circuit, since we have determined the total current and voltage, and also

* This calculation can only be employed when the wave form of the current is practically a sine curve.

the power factor of the circuit. The total power = amperes × volts × power factor.

The calculation might also be carried out as in the Experiment XII. by the 3-voltmeter method.

The power was calculated from all the readings of the series from which the examples in the tables above were taken. At the same time the power was measured on a wattmeter, and, except in the case of the first reading, where the power factor is very low, the results of the calculation agreed with the readings of the wattmeter within a fraction of 1 per cent.

The idle and energy currents in the circuit might be calculated from the readings taken in the last experiment, since idle current $= I \sin \phi$, and energy current $= I \cos \phi$.

Instances of circuits containing sufficiently large capacities to seriously affect the nature of the current chiefly arise where long feeders have to transmit current to a considerable distance, or in the high-tension windings of transformers, &c. Instances of a current which leads the voltage in phase from other causes are, however, not at all infrequent, and will be referred to later in connection with the running of synchronous motors.

The following experiments illustrate the effect of various factors upon the impedance of a circuit having capacity.

Experiment XVII.—Determination of Dependence of Current in a Circuit upon Capacity.

Diagram of Connections.

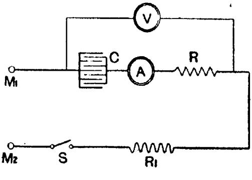


Fig. 51.

M_1 M_2 Source of alternating current.
R Non-inductive resistance of known value.
R_1 Variable non-inductive resistance for maintaining the voltage of V constant.
C Condenser capable of giving several values of capacity.
A Ammeter for measuring current in circuit.
V Voltmeter for measuring voltage of portion of circuit consisting of C and R.
S Switch for breaking circuit.

Instructions.—Connect in series a known non-inductive resistance R, condenser capable of giving several values of capacity, an ammeter and variable non-inductive resistance. Connect a voltmeter across the condenser and resistance R, so as to read the drop in both together.

Supply the circuit with alternating current, and maintain constant the periodicity and the voltage as read on the voltmeter.

First, short-circuit the resistance R, so as to read the current in the circuit having capacity and a negligibly small resistance. Take a series of readings, each corresponding to a different value of the capacity.

Then, for one or two values of the resistance R take similar sets of readings, keeping the resistance constant during each set.

Unlike the case of an inductive resistance with an iron core, the capacity of the condenser, and consequently the impedance of the circuit, will be the same for all values of the current.

Determination of Dependence of Current upon Capacity.

Resistance in Series with Condenser	Capacity Microf.	Current	Voltage	Impedance
0	2	·059	100	1695·0
0	10	·295	100	339·0
0	20	·590	100	169·5

Periodicity = 32 cycles per second.

The voltage should be kept constant by means of an additional variable resistance R_1 in the circuit or by the regulation of the generator voltage. For slight variations, the current readings may be corrected by increasing them in the same ratio as any decrease in the voltage observed, and vice versâ.

In this experiment, since the total current is small, care must be taken not to introduce errors due to the voltmeter current. If the voltmeter is not of the electrostatic type, it should be connected so that its current does not pass through the ammeter.

Enter the readings in tabular form as shown, and plot a curve for each set of readings, plotting capacity horizontally and current vertically.

Plot also a curve comparing impedance and capacity with the same horizontal scale as before.

The curves given in Fig. 52 show the results obtained with a subdivided condenser of 20 microfarads capacity, three of the readings being shown on the table above. The curves drawn as a

continuous line show the readings with the condenser alone in circuit. The dotted curves show the results with a resistance of 4 ohms in series with the condenser. Throughout the experiments the voltage was maintained at 100, the frequency being 32.

With low values of the capacity, the current is very small, and the resistance is practically without effect; it becomes relatively more important as the capacity and current in the circuit increase. Evidently when the capacity of the condenser is zero no current will

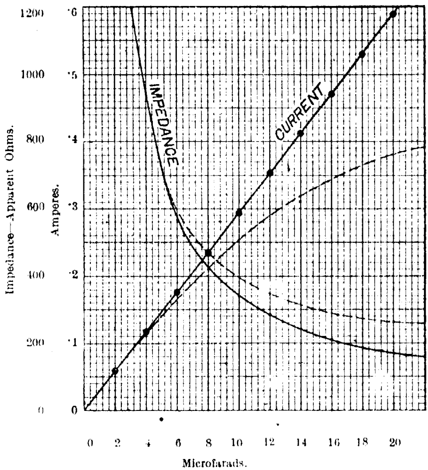


FIG. 52.—Variation of Charging Current with Capacity.

flow, since the condenser will then be simply a break in the continuity of the circuit, making all current impossible.

The flow into and out of a condenser depends not only on the capacity of the condenser, which determines the amount of the charge at each reversal, but also on the frequency of charge and discharge, *i.e.*, on the periodicity of the current. As already expressed in the formulæ given on page 95, the current varies directly with the frequency. This follows immediately from the fact that a current is a rate of flow, or displacement of electricity, and consequently varies directly as the number of charges per second.

The next experiment illustrates these statements.

Experiment XVIII.—Determination of Dependence of Charging Current upon Frequency.

Diagram of Connections.

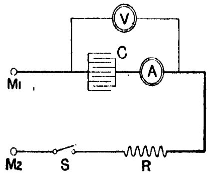


Fig. 53.

$M_1 M_2$ Source of alternating current.
C Condenser.
V Electrostatic voltmeter.
A Ammeter.
R Adjustable resistance for keeping voltage across condenser constant, if this cannot be done by other means.
S Switch for breaking circuit.

Instructions.—Connect in series a condenser, ammeter, and switch. Connect these to a supply of alternating current of variable frequency. Unless this supply can be maintained at a constant voltage, a regulating resistance must be put in the circuit, so that the voltage across the condenser may be maintained at a constant value. Connect a voltmeter across the condenser. This voltmeter should be of the electrostatic type, to avoid errors due to voltmeter current.

For a range of different frequencies of current, adjust the condenser voltage to a constant value, and observe the frequency and current. The frequency will usually have to be determined by a speed counter applied to the alternator supplying the circuit, the frequency being given by—

$$\text{Frequency} = f = \frac{n \times p}{60} \qquad \begin{aligned} n &= \text{revs. per min.} \\ p &= \text{No. of pairs of poles.} \end{aligned}$$

The readings should be entered in tabular form, and a curve plotted with frequency, or revolutions per minute, horizontal and charging current vertical.

Fig. 54 shows the results of an experiment carried out in this way on a 20-microfarad condenser, the current being read on a hot-wire ammeter. The alternator was a 4-pole machine, and the voltage was adjusted by variation of its excitation and not by a resistance as indicated in the diagram of connections above. The

voltage was maintained at 100. The curve given by the readings is seen to be a straight line, showing that the current increases in direction proportion to the frequency.

The curve goes through zero, since with a frequency of O the current would become a continuous current, which can only charge the condenser once, and then cease to flow.

The results may be used to check the accuracy of the formula given previously, viz. :—$I = 2 \pi f C E$.

For the last reading at a speed of 1,620, we should have as the value of the current

$$I = 2 \pi \frac{2 \times 1620}{60} \times \cdot 0002 \times 100 = \cdot 678 \text{ amp.}$$

The actual value observed was ·68 amp.*

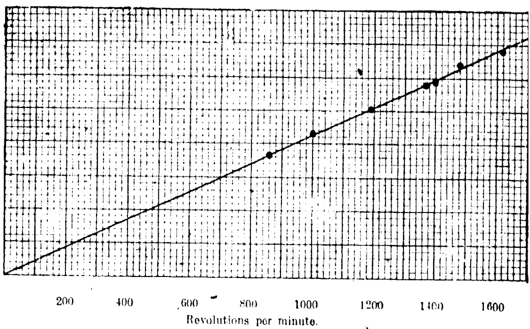


FIG. 54.—Variation of Charging Current with Frequency.

The preceding experiments have shown how the capacity and frequency of charge and discharge of a condenser affect the amount of current in the circuit.

The quantity of each charge depends not only on the capacity of the condenser, but also on the voltage to which the condenser is charged each time, *i.e.*, on the effective terminal voltage. This is the point to be illustrated in the next experiment.

EXPERIMENT XIX.—DETERMINATION OF EFFECT OF TERMINAL VOLTAGE UPON CHARGING CURRENT OF A CONDENSER.

DIAGRAM OF CONNECTIONS.

Same as for Experiment XVIII., Fig. 53, page 102.

Instructions.—Connect a condenser in series with an ammeter to the source of the alternating current. Arrange matters so that the

* Again it must be mentioned that such results are only obtainable with an alternator giving sine wave form. With any other wave form the formula would only be true of each of the harmonics of which the wave is formed.

terminal voltage of the condenser can be varied, either by means of a variable resistance in the circuit, as shown in the diagram, or by means of a field-regulating resistance in the generator field circuit. Connect a voltmeter to the condenser terminals if the voltmeter is electrostatic, or so as to measure the voltage across both ammeter and condenser, if the voltmeter takes current which might affect the ammeter readings.

Throughout the experiment keep the frequency constant, and vary the voltage at the condenser terminals. For each value of the voltage read the value of the current.

Enter readings in tabular form. A curve should be plotted with voltage horizontal and charging current measured vertically.

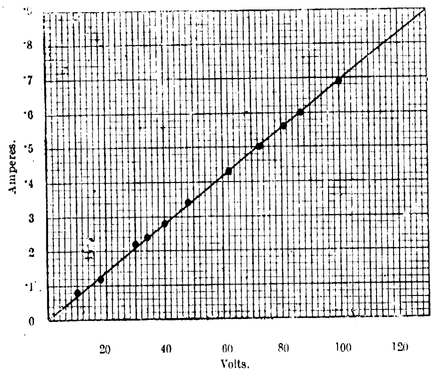


FIG. 55.—Variation of Charging Current with Voltage.

Fig. 55 shows the curve plotted from a set of readings obtained with a 20-microfarad condenser at 54 periods per second. Again the results give a straight line, showing that the charging current varies in direct proportion to the voltage applied to the condenser. As these readings were taken with an alternator giving a sine wave form, a calculation might be made to confirm the results, as in the case of the previous experiment.

Measurement of Power in a Circuit possessing Capacity.—From what has already been said, it will be apparent that the power in a circuit possessing capacity cannot be determined by a simple measurement of the current and voltage. The power can, however, be obtained by the use of a wattmeter, or by either of the methods given in Experiments XII. and XIII. for inductive circuits. The diagrams of connections there given apply to the present case also,

with the substitution of the capacity for the inductance. A special form of experiment is consequently not necessary.

The following record of an actual experiment will serve as a guide as to the observations to be taken. The readings practically form a companion experiment to the 3-voltmeter method given as Experiment XII.

As in that case, the total power in the circuit is given by the formula

$$W = \frac{V_3^2 + V_2^2 - V_1^2}{2R},$$

while the power in the portion exclusive of the non-inductive resistance is

$$W^1 = \frac{V_3^2 - V_2^2 - V_1}{2R}$$

Where V_3 = total voltage of circuit.
V_2 = voltage of non-inductive portion.
V_1 = voltage across portion including capacity.
R = value of non-inductive resistance.

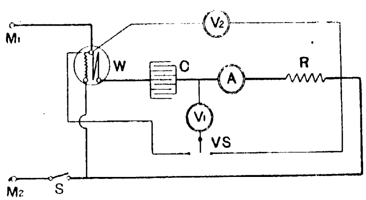


FIG. 56.—Diagram of Connections for Determination of Power in Circuit with Capacity.

Example of Determination of Power in Circuit containing Capacity.—The actual connections made are shown in the diagram Fig. 56, which is identical in principle with that shown in Fig. 38, page 76, but shows the modification, already frequently alluded to, of employing a single voltmeter with a two-way switch. Since the voltmeter V_2 is only used to indicate the voltage of the whole circuit, it may be of the hot-wire type. The voltmeter V_1 must be electrostatic, so as to take no current.

The non-inductive resistance R was a bank of lamps; the condenser had a capacity of 20 microfarads. The periodicity was 54 per second.

DETERMINATION OF POWER IN CIRCUIT WITH CAPACITY

Watt-meter Reading	Total Volts V_3	Non-inductive Resistance Volts V_2	Condenser Volts V_1	Current I	Calculated Power of Circuit $\frac{V_3^2 + V_2^2 - V_1^2}{2R}$
0	115·0	0	115	79	0
20	114·0	26·0	111	76	19·4
30	113·0	41·7	105	·72	30·0
37	112·5	53·4	99	·68	37·0
40	111·5	65·8	90	·62	40·0
40	110·0	85·7	69	·47	40·0

Capacity and Self-induction in Series. It has been shown that the effect of self-induction in a circuit is to cause the current to lag in phase behind the voltage, and that a capacity causes the current to lead in phase. When both capacity and self-induction are present, the effect is to produce a difference of phase between current and voltage, which is less than the angle of lag or lead, which would be due to either acting separately.

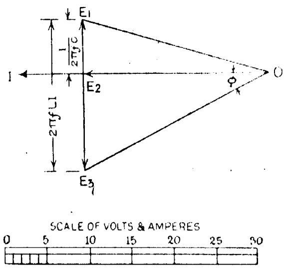


FIG. 57.—Diagram of Current and Voltage in Circuit with Capacity and Self-induction.

This result is best shown in diagram form by drawing a line OI to represent the current horizontally (see Fig. 57). The energy voltage spent in overcoming the non-inductive resistance of the circuit ($= I \times R$) is then marked off as the length $O\,E_2$ to the scale of volts. At E_2 the line $E_2\,E_1$ is drawn vertically upwards to represent the volts spent in overcoming the capacity of the circuit

$\left(= \frac{I}{2 \pi f C}\right)$. From E_1 the length $E_1 E_3$ is measured downwards to represent the voltage overcoming self-induction ($= 2 \pi f L I$). The triangle $O E_2 E_3$ then represents the following quantities, viz. The total voltage $O E_3$, the resultant idle voltage $E_2 E_3$, and the energy voltage $O E_2$. The angle ϕ, which may be either an angle of lead or of lag, is then the angle $E_2 O E_3$.

Impedance of Circuit containing both Inductance and Capacity. Since the energy voltage is $I R$, and the idle voltage of the circuit is $\left(2 \pi f L I - \frac{I}{2 \pi f C}\right)$, the expression for the total voltage may be obtained from the relation

$$E^2 = I^2 \left[R^2 + \left(2 \pi f L - \frac{1}{2 \pi f C} \right)^2 \right]$$

Hence, Impedance $= Z = \frac{E}{I} = \sqrt{R^2 + \left(2 \pi f L - \frac{1}{2 \pi f C}\right)^2}$

If ϕ = the angle of lag of the circuit, from the Fig. 57,

$$\tan \phi = \frac{\left(2 \pi f L - \frac{1}{2 \pi f C}\right)}{I R}$$

$$= \frac{2 \pi f L}{R} - \frac{1}{2 \pi f C R}$$

and the angle of lag $= \tan^{-1} \left(\frac{2 \pi f L}{R} - \frac{1}{2 \pi f C R}\right)$

These are the general expressions for the impedance and angle of lag in an alternating circuit. If the circuit is non-inductive, L becomes zero.

In a circuit with self-induction and capacity

Reactance $= X = 2 \pi f L - \frac{1}{2 \pi f C}$

or with capacity only

$$X = \frac{-1}{2 \pi f C}$$

where the negative sign indicates that the reactance voltage is opposite in phase to that which would be required to overcome an inductive reactance.

The following experiment serves to illustrate what has just been said.

EXPERIMENT XX.—DETERMINATION OF POWER AND PHASE RELATIONS IN A CIRCUIT CONTAINING RESISTANCE, SELF-INDUCTION, AND CAPACITY.

DIAGRAM OF CONNECTIONS

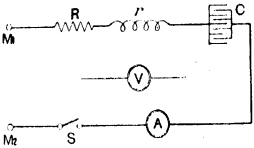


FIG 58

M_1 M_2 Source of alternating current.
R Non-inductive resistance
r Inductive resistance
C Condenser
A Ammeter for measuring current in circuit.
V* Voltmeter for measuring respectively voltage of inductive and non-inductive resistance, condenser, and total circuit.
S Switch for breaking circuit.

Instructions.—Connect in series to the source of supply a non-inductive resistance, inductive resistance, condenser, and ammeter. Connect a voltmeter to read in turn the total voltage of the circuit, and the voltage across each resistance and the condenser and across the inductive and non-inductive resistances together. It is an advantage to insert a wattmeter in the circuit to serve as a check on the accuracy of the other readings.

For several values of the non-inductive resistance and current, take readings of the current and voltage of inductive and non-inductive resistances, and also the resultant voltage across both resistances. Read also the voltage at the terminals of the condenser and the total voltage of the circuit. For each set of readings construct a diagram showing the relation between the voltages of the circuit, proceeding as in the example given below. Hence calculate the total power of the circuit and the power in the inductive and non-inductive portions.

The method of drawing the diagram and determining the phase relations between the voltages of the various portions of the circuit are best discussed in connection with an actual example.

The readings tabulated in the annexed table show the readings obtained in an experiment in which a 20 microfarad condenser formed the capacity, incandescent lamps were used as the non-inductive resistance, and two sections of a small transformer were

* Only a single voltmeter is actually necessary, and should be connected successively to the different terminals indicated in the diagram.

employed as inductive resistance. In this case a wattmeter was also used to check the accuracy of the results. The figures given in brackets are calculated values, the other figures are those actually observed

Taking the current vector horizontal, we shall have idle voltages due to self-induction (which precede the current in phase) drawn vertically downwards, and idle voltages due to capacity (and which lag behind the current) drawn vertically upwards.

POWER AND VOLTAGES IN CIRCUIT WITH CAPACITY, RESISTANCE, AND SELF-INDUCTION.

Volts Across							Watts in.		
Total Circuit V_0	Capacity V_3	Inductive Resistance V_1	Non-inductive Resistance V_2	Capacity + Inductance V_5	Inductive and Non-inductive Resistance V_4	Amps.	Total Circuit.	Non-inductive Resistance.	Inductive Resistance
97·0	125	92·0	45 (45)	61 (60·9)	119	·78	67 (67·2)	34·8 (35·7)	32·2 (32·5)
90·0	110	84·0	40·8 (40·7)	55·5 (55·24)	110	·69	57 (56·77)	28·3 (28·2)	28·7 (28·57)
80·0	91	74·8	35 (34·5)	49·5 (49·6)	98	·57	43 (42·23)	19·56 (19·67)	23·5 (23·26)
72·2	76	69·5	31 (30·2)	43 (43·2)	90·5	·475	33 (32·92)	14·3 (14·35)	18·7 (18·53)
60·0	62	52·6	27 (26·9)	35 (35·4)	71·5	·38	21·74 (21·62)	10·43 (10·22)	11·31 (11·40)
70·0	47·5	42·0	42 (42·4)	30 (29·6)	76	·297	20·43 (20·315)	12·6 (12·6)	7·83 (7·72)
50·2	32	22·0	39 (38·9)	(16)	52·5	·20	9·78 (9 72)	7·83 (7·78)	1·95 (1·94)

The voltage V_2 in the non-inductive resistance is drawn horizontally as $O\ A$ (see Fig. 59), since this is an energy voltage, and in phase with the current. The voltage V_1 taken across the inductive resistance r will consist partly of energy and partly of idle volts, and is drawn downwards from A, making an obtuse angle with $O\ A$. In order to determine the correct position for V_1 on the diagram, describe a circle with radius equal to V_1 from A as centre, and a second circle about O of radius V_4, the resultant voltage across both inductive and non-inductive resistances. The intersection of these circles will give the point B, and $A\ B$ represents in phase and magnitude the voltage V_1.

The triangle $O\ A\ B$ resembles the triangle obtained in Experiment XII., in a circuit with inductance and resistance only. The total energy voltage of the circuit is obtained by drawing a vertical line $B\ C$ to meet $O\ A$ produced in C. Then $O\ C$ is the energy component of the total voltage of the circuit. The line $C\ B$ represents the idle voltage due to self-induction.

The condenser voltage will be in opposition of phase with respect to $C B$, *i.e.*, it will be 180° out of phase with $C B$.

Hence the condenser voltage V_3 is represented by the vertical line $B D_1$ measured from B upwards. Finally, the total voltage of the circuit is represented in phase and magnitude by the line $O D_1$, which, in Fig. 59, is seen to be later than the current in phase, so that

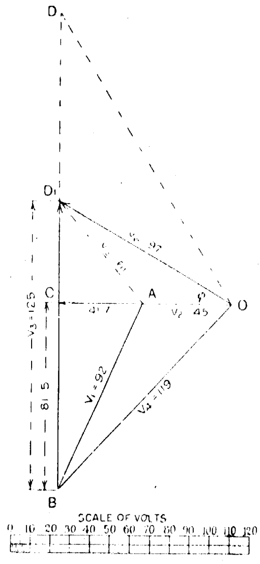


FIG 59.—Diagram of Voltages in Circuit with Capacity and Self-induction.

there is a leading current in the circuit. The line $B C$ represents the portion of the condenser voltage which is neutralised by the idle voltage of self-induction of r. Thus, if the circuit had been non-inductive, the resultant voltage of the circuit would have been the dotted line $O D$, where $C D$ is equal to the condenser voltage V_3 or $B D_1$.

It is thus seen that the voltage required to produce a given current may be reduced by the presence of both inductance and capacity. If the inductance had been greater, so that $C B = B D_1$, the self-induction and capacity would exactly have neutralised each

other, and the voltage of the circuit would only have been of the magnitude necessary to send the given current through a non-inductive resistance.

The diagram shown in Fig. 59 is drawn for the first set of readings given on the table above. It would form an excellent exercise for the student to draw diagrams for the remaining readings, if he has not the opportunity of carrying out a set of readings for himself.

Determination of Power from the Diagram. Having drawn this diagram, it is an easy matter to calculate the power in each part of the circuit from it.

In each case the power = current × energy component of voltage. Thus the power in the inductive portion r is the product of the current and the energy component of $A\ B$, *i.e.*, $A\ C$.

$$\text{Power} = W_{,} = I \times A\ C.$$
$$= \cdot 78 \times 41\cdot 7 = 32\cdot 5 \text{ watts.}$$

Similarly, in the total circuit, the energy component is represented by the line $O\ C$, and the total power

$$= W_{t} = I \times O\ C.$$
$$= \cdot 78 \times 86\cdot 7 = 67\cdot 6 \text{ watts}$$

In the non inductive resistance R the power

$$= W_{n} = I \times O\ A$$
$$= \cdot 78 \times 45 = 35\cdot 1 \text{ watts}$$

In the condenser the power is zero, since the voltage is entirely idle voltage ¼ period behind the current.*

Evidently the power in the circuit could be calculated directly from the voltmeter readings, in the same way as in Experiment XII. The triangle $O\ A\ D_{1}$ would in this case be the triangle whose sides would represent the voltage to be employed in the calculation. Another and more direct way would be to calculate the power from the triangle $O\ A\ B$ obtained from the inductive and non-inductive portions of the circuit, and omit the consideration of the voltage across the condenser. Obviously this might not be possible if the capacity were not confined to one portion of the circuit.

Resonance.—The conditions just alluded to may give rise to what is called "Electric Resonance." It has been shown in the preceding discussion that when special relations exist between the capacity C, the self-induction L, and the frequency in a circuit, the voltage at the terminals of the self-induction and capacity connected in series may be less than the voltage at the terminals of one of them. Thus, if an alternator be connected to a circuit, containing a condenser and choking coil, it is possible that the voltage at the terminals of the condenser or choking coil might be greater than the voltage supplied by the alternator. An example will serve to illustrate this.

* The small losses due to resistance and dielectric hysteresis in the condenser are neglected as being too small to affect the results.

The total voltage of the circuit

$$= \sqrt{(\text{energy voltage})^2 + (\text{idle voltage})^2}.$$

Suppose the energy voltage to be due to 10 amperes flowing through ·5 ohm, its value will be

$$10 \times \cdot 5 = 5 \text{ volts.}$$

Let the inductance idle voltage $= 2\pi f L I = 200$ volts, and the capacity idle voltage $= \dfrac{-I}{2\pi f C} = -160$ volts, the negative sign indicating that the phase of the capacity idle voltage is opposite to that of the inductance idle voltage.

The total idle voltage of the circuit is then $= 200 - 160 = 40$ volts.

The total voltage of the circuit $= \sqrt{5^2 + 40^2} = \sqrt{25 + 1600} = \sqrt{1625} = 40 \cdot 3$ volts, whereas the voltage in the inductance is 200 and across the condenser is 160.

Cases of this kind occur in practice in connection with long concentric mains which have considerable capacity. The self-induction of the generator armature, of the cable itself, or of machines connected to the cable, may supply the inductance in the circuit.

If $2\pi f L I = \dfrac{I}{2\pi f C}$ *i.e.*, if $f = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{L C}}$ the idle voltages due to capacity and to inductance will always be equal and opposite, and may each have very high values, although the circuit is supplied from a source of relatively low voltage. The particular value of the frequency just given is that at which a maximum resonance will occur. It thus appears that for any circuit resonance will be possible at some frequency. Usually this critical frequency is far higher than that of the voltage supplied to it. The frequency given by the relation for maximum resonance is sometimes called the "natural frequency" of the circuit.

Similarly, $t = \dfrac{1}{f} = 2\pi\sqrt{L C}$ is called the "natural period" of the circuit. These values are of great importance in connection with the calculation of pressure rise in cables.

Capacity and Inductance in Parallel.—In the previous paragraph it was found that the effect of a capacity and self-induction connected in series was that the idle electromotive force produced by each was partly or entirely neutralised by the other.

When the capacity and self-induction are in parallel circuits, the electromotive force at their common terminals depends only upon the voltage of the supply, and is clearly independent of the nature of the two branch circuits. It is, however, possible for the inductance and capacity to have a combined effect on the current in the external circuit, for there may be a lagging current produced in the branch containing self-induction, which may partially or entirely neutralise the leading current flowing in the branch containing capacity as regards the external circuit.

Suppose, for instance, that an inductance r of L henries and negligible resistance, and a condenser C with capacity C farads, are connected in parallel, as shown in Fig. 60, with a voltage V between T_1 and T_2. The current through r will be $\frac{V}{2 \pi f L}$ and through C it will be $2 \pi f C V$. These currents will differ in phase by 180°, because the first will be 90° behind the phase of V, and the second 90° in front.

Consequently, when current flows from T_1 to T_2 through r, the current through C will be directed from T_2 to T_1. We may

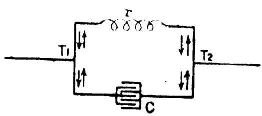


FIG 60.—Condenser and Inductance in Parallel.

thus have large currents in both parallel circuits which may neutralise one another, so that only a very small current flows in the alternator armature and the external circuit joining T_1 to T_2. This effect is called current resonance.

Example.—As an example of what has just been said, may be quoted an experiment of Mr. W. N. Mordey.* Mr. Mordey connected an alternator giving 2,050 volts at 100 cycles to a 5½-mile length of 37/15 concentric rubber-covered cable, one pole of the alternator being connected to the inner core and the other pole to the outer core, which was earthed, as was also the steel sheathing.

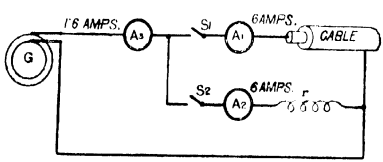


FIG. 61.—Inductive Resistance Neutralising Idle Current Due to Capacity of Cable.

The capacity of the cable between conductors was ·86 microfarad per mile. Thus the charging current taken from the alternator was calculated to be

$$I = 2 \pi f E C = \frac{2 \pi \times 100 \times 2{,}050 \times \cdot 86 \times 5{\cdot}5}{1{,}000{,}000}$$
$$= 6{\cdot}092 \text{ amps.}$$

* Proc. Inst. Elec. Eng., vol. xxx. p. 378.

The current actually observed was 6 amp. A choking coil was then put in circuit connected to the alternator in parallel with the cable. The connections were then as shown in Fig. 61. The coil was so designed that with switch S_1 open and S_2 closed, so that the alternator supplied current to the coil only, the current taken by the coil was 6 amps.

On closing both switches, so that the alternator supplied the two circuits in parallel, the current given out by the alternator was found to fall to 1·6 amps. Thus ammeters A_1 and A_2 both indicated 6 amps., while A_3 only registered 1·6 amps.

The explanation was that the current in the inductive circuit lagged behind the voltage of the alternator, while the current in the capacity circuit was in advance of the voltage, so that the two branch currents were nearly opposite in phase, and neutralised each other to a large extent.

It is hardly necessary to point out that the principles dealt with in this chapter are of the greatest importance to students of wireless telephony.

CHAPTER VI.

THE TRANSFORMER.

The Transformer.—One of the greatest advantages of alternating currents is the ease with which power transmitted by them may be converted from a low current at high pressure to a high current at low pressure, and *vice versa*, without the employment of rotating machinery.

The transformer is the means by which this transformation is accomplished. The principles underlying its construction and behaviour must now be considered.

Fig. 62 represents a rectangular frame consisting of a number of soft-iron stampings placed side by side, so that the section of the frame is approximately square. This forms a magnetic circuit of very high permeability without any air gap. Two separate windings of insulated wire are indicated as being wound upon this magnetic circuit, one winding being connected to an alternator. A current sent through either winding will produce a magnetic flux in

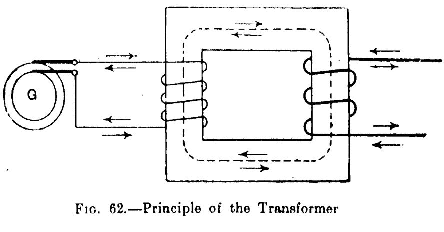


FIG. 62.—Principle of the Transformer

the core, upon which both windings are mounted. The general direction of this flux is indicated in the diagram by a dotted line. An alternating current flowing in the coil connected to the alternator will produce an alternating field in the magnetic circuit as indicated in the diagram, where the upper arrows show the direction of current and flux during one-half period, and the lower arrows indicate their direction after reversal of the current.

Voltage Relations of Transformer without Resistance.

Let F = maximum number of lines of force produced in the core by the current,

f = periodicity of current,

T_1 = number of turns composing the winding I. connected to the alternator,

T_2 = number of turns composing the other winding II.,

E_1, E_2 = maximum value of the voltage at the ends of the windings I. and II.

The number of the magnetic lines will change with the current producing them.

With each complete cycle of current the magnetic lines in the transformer core will undergo the following changes :—

(1) The lines are formed in one direction as the current rises to its maximum value. F lines are formed.

(2) The lines disappear as the current falls to zero. F lines are withdrawn.

(3) The lines are formed in the reverse direction as the current rises to its opposite maximum. F lines are formed in the reverse direction.

(4) The lines disappear as the current falls to zero. F lines are withdrawn.

It follows that the total change of the lines passing through the transformer windings is $4\ F$ per cycle.

This is equivalent to an average rate of change of $4\ F\ f$ lines per second.

The electromotive force generated in one turn of winding = rate of change of magnetic lines $\div\ 10^8$.

Hence the *average* voltage induced in a single turn

$$= E = \frac{4\ F\ f}{10^8}$$

and the *average* voltage induced in winding II. having T_2 turns

$$= E_{2\,av} = \frac{4\ F f T_2}{10^8}$$

The *virtual* voltage will be equal to the average voltage multiplied by the "form factor" (see page 89) for the wave form of the voltage.

With sine waves the form factor has the value 1·11, and in this case the virtual voltage of the winding is

$$e_2 = 4{\cdot}44\ F\ f\ T_2\ 10^{-8}$$

which is a fundamental formula for the voltage induced in the winding of a transformer.*

But the magnetic flux passes through the windings of the coil I. connected to the alternator, as well as through winding II. Consequently, there will be induced a back voltage in coil I., which we have previously called the electromotive force of self-induction, opposing the voltage of the alternator. The value of this back voltage is obviously

$$e_1 = 4{\cdot}44\ f\ F\ T_1\ 10^{-8}$$

since it may be calculated by the same reasoning as that just given, and it is induced in a winding having T_1 turns.

* The general expression for the voltage, applicable to any wave-form of flux variation is $e = 4\ b\ F\ f\ T\ 10^{-8}$ where b is the form factor for the actual wave-form employed.

On account of the complete magnetic circuit of soft-iron on which the coil I. is wound, its self-induction is exceedingly high, and when the transformer is not supplying current, the primary applied electromotive force is nearly all spent in overcoming the self-induction and will only be able to send a very small current through the winding. Consequently only a small part of the applied voltage will be spent in overcoming the resistance of the winding.

Thus in the case of a transformer with unloaded secondary winding, the back voltage very nearly equals the applied voltage, and the current in the winding is exceedingly small. The current which flows is, in fact, only the magnetising current necessary to produce the flux of F lines in the iron circuit.

In any transformer the winding which has a voltage applied to it from an external source and which carries the magnetising current is called the **Primary winding.**

The **Secondary Winding** embraces the same magnetic circuit, and has a voltage induced in it due to the magnetic changes set up by the change of current in the primary.

Thus with no load on the secondary, the ratio of voltages will be very nearly

$$\frac{\text{Primary voltage}}{\text{Secondary voltage}} = \frac{4{\cdot}44\, f\, F\, T_1}{4{\cdot}44\, f\, F\, T_2} = \frac{T_1}{T_2}$$

$$= \frac{\text{Number of primary turns}}{\text{Number of secondary turns}}$$

Effect of Current in Secondary Circuit. The preceding statements were based on the assumption that the magnetising current in the primary winding was the only current acting upon the magnetic circuit of the transformer. The magnetising current will lag practically 90° behind the applied voltage in phase, since it flows in a highly inductive circuit.

It is important to remember that the applied voltage determines the amount of magnetic flux set up in the core, since, by the formula on page 116, $e_1 = F \times$ constant, so that with a constant applied voltage, the flux in the core of a transformer must always remain constant. The current taken by the primary winding will be of the magnitude necessary for the production of this flux and will be very small, because a small current will be sufficient to produce a large flux in a magnetic circuit without air-gap.

Let us now imagine the secondary winding of the transformer to be connected to a load circuit so that the induced electromotive force of the winding will produce a current. This current, in flowing through the secondary winding, will tend to set up a flux in the transformer core which previously did not exist. But since the applied voltage at the primary terminals is unchanged, the flux in the core must remain unchanged, in order that the applied voltage may still be balanced by the induced electromotive force which is due to this flux. This result can only be brought about by an additional current flowing in the primary winding producing a magnetising effect exactly equal and opposite to that of the current

in the secondary winding. We thus have the important relation between primary and secondary currents of a tranformer.

Increase in primary ampere turns = secondary ampere turns.

We may look upon the secondary current as at first tending to lessen the flux in the core of the transformer. This makes the back electromotive force induced in the primary winding fall below the applied electromotive force, so that an increased current flows from the supply mains. This current will continue to increase until the flux is restored to its original value. When this has been done, the increased primary ampere turns will exactly balance the increased secondary ampere turns, which is the condition stated above.

If we disregard the small no-load current taken by the primary winding (which continues to flow at a practically constant value at all loads), we may state the ratio of primary to secondary currents as being given by $T_1 I_1 = T_2 I_2$, from which

$$\frac{I_1}{I_2} = \frac{\text{Primary current}}{\text{Secondary current}} = \frac{\text{Secondary turns}}{\text{Primary turns}} = \frac{T_2}{T_1}$$

Experimental verification of the statements made regarding the voltage and current ratios will be found in all the experiments which follow, and a special experiment for this purpose is therefore not given.

Effect of Power Factor of Secondary Circuit.—The current in the secondary circuit is always balanced by a corresponding current in the primary, producing at any instant an equal and opposite number of ampere turns. If the secondary circuit is non-inductive, the secondary current will be in phase with the secondary induced electromotive force, and the primary current which corresponds to this will be in phase with the primary applied voltage overcoming the primary induced back electromotive force.

It also follows that if the load circuit has a power-factor cos ϕ both secondary and primary currents will be later in phase by ϕ, as the primary and secondary ampere-turns would not otherwise be equal at *every instant*. Thus, any alteration in the power-factor of the secondary circuit is exactly reproduced in the primary circuit; in fact, the power-factor under load will be practically identical at primary and secondary terminals. The small no load current taken by the primary circuit and tending to increase the lag of the current in that circuit is hardly appreciable when the transformer is fully loaded.

Effect of Resistance of Windings.—The current flowing in both primary and secondary windings requires a certain amount of voltage to be spent in overcoming the resistance of these windings. Let

i_1 = current in primary winding.
i_2 = current in secondary winding.
r_1 = resistance of primary winding.
r_2 = resistance of secondary winding
T_1 = number of primary turns.
T_2 = number of secondary turns.

The voltage required to overcome the resistance of the primary windings $= i_1 r_1$ volts

The effective voltage spent in producing the alternating flux in the core is reduced by this amount. Consequently, if $e_1 =$ primary applied voltage, the effective voltage which produces the flux affecting the secondary winding $= e_1 - i_1 r_1$.

Here the symbol $-$ indicates the *vector difference* between the two quantities, and not the simple algebraic difference, as the applied voltage will not be in phase with the energy voltage $i_1 r_1$. Thus the voltage induced in the secondary winding is

$$(e_1 - i_1 r_1) \times \frac{T_2}{T_1}$$

The voltage at the terminals of the secondary will only have this value when the secondary current is zero. When a current flows in the secondary, there will be a loss of voltage due to the resistance of these windings, and consequently the secondary terminal voltage

$$= e_2 = e_1 \frac{T_2}{T_1} - \left(i_1 r_1 \frac{T_2}{T_1} + i_2 r_2 \right)$$

$$\text{But } \frac{i_1}{i_2} = \frac{T_2}{T_1} \text{ or } i_1 = i_2 \frac{T_2}{T_1}.$$

$$\therefore e_2 = e_1 \frac{T_2}{T_1} - \left(i_2 \frac{T_2}{T_1} r_1 \frac{T_2}{T_1} + i_2 r_2 \right)$$

$$= e_1 \frac{T_2}{T_1} - i_2 \left[r_1 \left(\frac{T_2}{T_1} \right)^2 + r_2 \right]$$

or putting $\frac{T_1}{T_2} = k$, the ratio of the turns,

$$e_2 = \frac{e_1}{k} - i_2 \left[\frac{r_1}{k^2} + r_2 \right]$$

If there were no loss of voltage, e_2 would be $\frac{e_1}{k}$, consequently the effect of the resistance of both windings is to diminish the voltage at the secondary terminals by the amount $i_2 \left(\frac{r_1}{k^2} + r_2 \right)$, which must be subtracted vectorially from the no-load voltage. The transformer may consequently be treated as if it had no resistance in the primary, and a resistance of $\left(\frac{r_1}{k^2} + r_2 \right)$ ohms in the secondary. For this reason the quantity $\left(\frac{r_1}{k^2} + r_2 \right)$ is called the *equivalent resistance* of the transformer referred to the secondary circuit. This we shall indicate by the symbol R_2.

It would not be difficult to carry through the calculation just given so as to arrive at the result

$$e_1 = \frac{T_1}{T_2} e_2 + i_1 (r_1 + k^2 r_2),$$

which shows that the resistance of both windings may also be treated as equivalent to a single resistance $r_1 + k^2 r_2$, in the primary circuit.

The quantity $r_1 + k^2 r_2$ is called the equivalent resistance of the transformer referred to the primary circuit, and may be denoted by R_1.

Effect of Magnetic Leakage.—The alternating field produced by the current in the primary winding will only produce the voltage given by the formulæ on page 116, if the whole of the magnetic lines follow the magnetic path formed by the core, so as to pass through both primary and secondary windings. Any lines which are formed in the primary which do not pass through the secondary will produce a back electromotive force in the primary in the same way as the remainder of the field, but will not produce any secondary voltage. Similarly, the current in the secondary may produce some magnetic lines which do not pass through the primary. These lines will act in opposition to the main primary field in the secondary winding, lessening the flux and the consequent voltage produced, but will not affect the flux and back voltage of the primary.

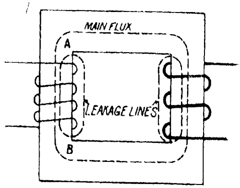


FIG. 63.—Diagram showing Leakage in Transformer

The lines which penetrate either winding without affecting the other constitute what is called the leakage field. There may be thus both primary and secondary leakage fields.

The effect of the primary leakage field is like that of an inductive resistance put in series with the winding.

The effect of the secondary leakage field (which, of course, only occurs when there is a secondary current) is similar to that of an inductive coil put in series with the secondary winding.

In either case, the result of magnetic leakage is to reduce the useful effect of the voltage of the winding in which it occurs, and to produce the result of a self-induction in the coil.

It is an important fact that the magnetic leakage increases with the load on the transformer. In order to explain the reason of this Fig. 63 is drawn to represent the primary and secondary coils of a transformer, wound in a way to produce considerable leakage. At no load the primary receives magnetising current and produces

an alternating field, most of which will follow the magnetic circuit of the iron core, because this path has the least reluctance or magnetic resistance. If the reluctance of the iron path is $\frac{1}{99}$th of the reluctance of the leakage path A B through the air, then $\frac{1}{100}$th of the lines will pass through the air from A to B, the remainder following the iron path. If the secondary coil carries current, its effect is to oppose the flux produced by the primary, and the current in the primary increases in order to force the same number of lines round the circuit. The primary ampere-turns acting on the circuit are thus increased, while the difficulty of sending lines from A to B through the *iron* path increases on account of the secondary reverse ampere-turns in the same proportion. The reluctance of the *air* path from A to B remains, however, as before, so that the increased magnetising force applied will send a correspondingly increased leakage field through this path. Similar reasoning may be applied to trace the effect of load on the secondary leakage field. In either case the amount of leakage in either winding increases in practically direct proportion to the current in that winding.

Since both primary and secondary leakage are practically equivalent to the addition of an inductance in these circuits, we may treat these windings as having each a definite coefficient of self-induction *due to leakage inductance alone.* It is usual to speak of the self-induction of transformer windings signifying thereby the self-induction due to leakage flux, and not that due to the main flux. The reactance of either winding is accordingly $2 \pi f L$, where L is the coefficient of self-induction of the winding due to leakage inductance only.

Reactance and Impedance of Windings.—The value of the coefficient of self-induction of a winding due to the stray field will be given by the formula of page 24 :—

$$L = \frac{\text{stray lines} \times \text{turns linking with them*}}{i \times 10^8}$$

L being the coefficient measured in henries ;
i being the current in the winding in which the stray flux is set up.

The reactance due to the magnetic leakage lines will be

$$X = 2 \pi f L \text{ ohms.}$$

Evidently this reactance will absorb voltage. This voltage will have a value

$$e_1 = i X \text{ or } 2 \pi f L i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the total voltage necessary to overcome the reactance and resistance of the transformer windings will be

$$i \times Z = i \sqrt{R^2 + X^2} = i \sqrt{R^2 + (2 \pi f L)^2} \quad . \quad . \quad (2)$$

where Z is the impedance of the winding.

Both primary and secondary windings of the transformer will have reactance, and, as in the case of resistance, the voltage spent in both windings will reduce the secondary terminal voltage. By

* It is to be noted that the *virtual* value of the stray flux must be used here, not the *maximum* flux, because i gives the *virtual* current value.

a similar process of reasoning to that carried out in the case of the resistance (see p. 119), we can show that the effect of the reactance of both windings upon the output voltage of the transformer will be the same as that of a single "equivalent reactance," $X_2 = x_2 + \frac{x_1}{k^2}$ in the secondary circuit, or of a single reactance $X_1 = x_1 + k^2 x_2$ in the primary circuit. These expressions are termed the *equivalent reactance* of the transformer referred to the secondary and primary circuits, respectively, when x_1 and x_2 represent the reactance of primary and secondary windings, due to leakage.

Similarly, if z_1 and z_2 are the impedance of the primary and secondary windings, the equivalent impedance of the transformer referred to the primary circuit is

$$Z_1 = z_1 + k^2 z_2$$

and referred to the secondary,

$$Z_2 = z_2 + \frac{z_1}{k^2}$$

The voltage overcoming the impedance of the transformer windings is $i_2 Z_2$ volts, measured in terms of the voltage of the load circuit, or $i_1 Z_1$ volts if measured in terms of the voltage applied to the primary terminals.

The voltage absorbed in the production of the leakage field may also be calculated directly from the value of the leakage lines.

Writing

F_1 for the leakage lines (maximum value), and

t for the turns linking with them,

we may express the induced voltage due to this part of the alternating flux in the same way as we have already done for the voltage arising from the main flux (see p. 116).

Voltage due to leakage flux $= e_e = 4{\cdot}44\ f\ F\ T\ 10^{-8}$ . . (3)

This equation is the equivalent of equation (1) previously found, but is expressed in terms of the flux, instead of the coefficient of self-induction.

It is desirable to remember in connection with all kinds of alternating-current problems, that the idle voltage of a circuit may be expressed in these two ways, which are really identical :—

$$e_e = 4{\cdot}44\,f\,F\,T\,10^{-8} = 2\,\pi\,f\,L\,i.$$

EXPERIMENT XXI.—DETERMINATION OF REGULATION OF A TRANSFORMER.

DIAGRAM OF CONNECTIONS.

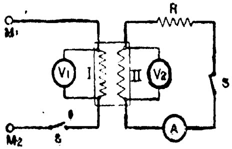


FIG. 64.

M_1, M_2 Source of alternating current.
I, II Primary and secondary windings of transformer.
R Variable non-inductive resistance of varying transformer load.
V_1, V_2 Voltmeters for reading primary and secondary voltage.
A Ammeter for reading output of secondary.
S, S Switches for breaking primary and secondary circuits.

Instructions.—Connect the primary of the transformer to a source of alternating current, and the secondary to a variable non inductive resistance in series with an ammeter and a switch.

Connect voltmeters to the terminals of the primary and secondary windings.

Beginning with open secondary circuit, read primary and secondary voltage. Then close the switch, and repeat the readings for increasing values of secondary current. The primary voltage and frequency should be maintained constant throughout the experiment. Measure the resistance of both windings of the transformer, employing continuous current. It is most important to remember that it is no use to determine the drop in the transformer windings until they have attained their normal working temperature. It will therefore be necessary to run a large transformer for several hours before taking readings in order to make them correspond with working conditions. Small transformers will arrive at steady temperatures much sooner. It is usually convenient to make the regulation test during, or after, the trial run in the case of a new transformer.

If the primary voltage cannot be kept constant, a simple proportional correction should be made in the secondary volts to bring them to terms of a constant applied voltage.

Tabulate the results under headings, as follows:—

Regulation Test of a Transformer

Transformer No.......... Type....................
Output................. kw. .... cycles per second
Ratio of transformation..........to..................volts.

Current Output = I_2	Primary Voltage = V_1	Secondary Voltage = V_2	Ratio of Transformation = $\frac{V_1}{V_2}$	Drop in Transformer* = $\frac{V_1}{k} - V_2$ or $V_{02} - V_2$

* $k = \frac{\text{Number of primary windings}}{\text{Number of secondary windings}} = \frac{\text{No-load primary voltage}}{\text{No-load secondary voltage}}$

V_{02} = No-load secondary voltage.

At no load the ratio of primary to secondary volts will be practically equal to the ratio of the number of the turns.

If no loss occurred in the transformer, this would, therefore, be the ratio of transformation at all loads. The difference between this theoretical value of the secondary voltage and the actual value is termed the "drop" of the transformer. The "drop" is obtained by subtracting the value of the secondary terminal voltage from the no-load secondary voltage. It is the *arithmetic* difference, not the *vector* difference.

Plot a curve of regulation from the results, plotting load current horizontally and voltage vertically. Draw through the no-load voltage a horizontal line. The ordinate between this horizontal line and the curve of voltage gives the value of the drop corresponding to any load.

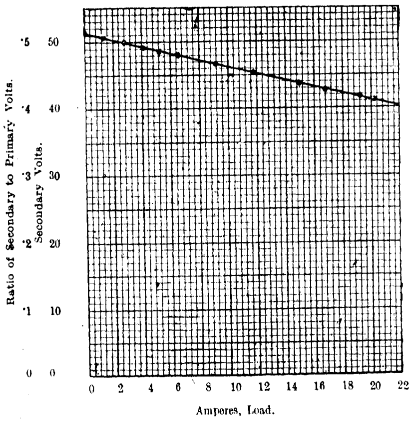


FIG. 65.—Curve of Secondary Voltage and Ratio of Transformation of Transformer.

From the experimental results and the measured resistance of the windings, calculate the equivalent reactance of the transformer.

The curve shown in Fig. 65 gives the results of a test made in this way on a small 1 kw. transformer with a nominal voltage ratio of 100 : 50. The primary voltage was kept constant at 100. In this case a single curve represents both secondary voltage and ratio of secondary to primary voltage on the two scales given at the side of the Fig. 65.

A careful measurement of the resistance of the windings of this

transformer made while they were still warm gave the following results :—

$$r_1 = \cdot 685 \text{ ohm.}$$
$$r_2 = \cdot 183 \text{ ohm.}$$

The ratio of the number of turns was $\frac{248}{128} = 1{\cdot}935$.

Calculation of Transformer Reactance from Regulation Curve.—We know that the drop in voltage under load is due partly to resistance and partly to reactance in the windings. As we can calculate the voltage spent in overcoming resistance, we may calculate the equivalent reactance of the transformer from the value of the drop, as just found in the experiment, and from the known resistances.

Since the transformer is supplying a non-inductive circuit, its load current is in phase with its secondary terminal voltage. The voltage $I_2 R_2$ spent in overcoming the resistance of the windings is also in phase with the current, whereas the voltage overcoming

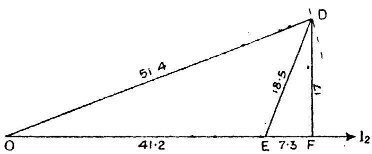


FIG. 66.—Voltages in Transformer.

reactance is at right angles to the current in phase. The terminal voltage of the transformer will be equal to the total voltage induced in the secondary at no load, diminished by the voltage lost in the windings. In order to show the voltages in their proper phase relations, we may draw a diagram (see Fig. 66) in which the secondary terminal voltage $O\ E$ is shown to be equal to the total no-load voltage $O\ D$, diminished by the resistance voltage $E\ F$, and the reactance voltage $F\ D$. $E\ D$ represents the voltage overcoming transformer impedance, and we have the condition that

No-load voltage = Terminal voltage + Impedance voltage.

The sign + indicating *vectorial* addition.

In order to construct the diagram we must first calculate the value of the resistance voltage $E\ F$.

The equivalent resistance of the transformer referred to the secondary circuit is (see page 119)

$$R_2 = \frac{r_1}{k^2} + r_2$$
$$= \frac{\cdot 685}{1{\cdot}935^2} + \cdot 183 = \cdot 183 + \cdot 183 = \cdot 366 \text{ ohms.}$$

At full load the current was 20 amps., and the voltage overcoming resistance was, therefore,

$$20 \times \cdot 366 = 7 \cdot 3 \text{ volts.}$$

We can now construct the diagram (Fig. 66) by drawing the horizontal lines, terminal volts $= O\ E = 41 \cdot 2$ volts (taken from the curve).

$$\text{resistance volts} = E\ F = 7 \cdot 3 \text{ volts.}$$

At F a perpendicular is erected, and from O a circle is described with radius 51·4 volts, the no-load voltage, to cut the perpendicular line at D.

$F\ D$ is now the reactance voltage. Its value is given by

$$F\ D^2 = (51 \cdot 4)^2 - (41 \cdot 2 + 7 \cdot 3)^2 = 2642 - 2352 = 290$$

$$\therefore F\ D = 17 \text{ volts.}$$

The equivalent reactance referred to the secondary circuit is thus $X_2 = \frac{17}{20} = 0 \cdot 85$ ohms.

It is to be noticed that the voltage overcoming the impedance of the transformer windings ($= E\ D$ on the diagram) is $\sqrt{17^2 + 7 \cdot 3^2} = 18 \cdot 5$ volts, while the "drop" in volts due to the load, being the *arithmetical* difference between volts on open circuit and under load, is

$$\text{drop} = 51 \cdot 4 - 41 \cdot 2 = 10 \cdot 2 \text{ volts.}$$

It is important to remember the difference between these two quantities.

The equivalent impedance of the transformer windings referred to the secondary circuit is

$$\frac{18 \cdot 5}{20} = 0 \cdot 925 \text{ ohm.}$$

Determination of Regulation by Method of Opposition.—The method just given for determining the drop in a transformer is difficult to carry out with great accuracy, especially if the voltages to be dealt with are fairly high.

For instance, an error of 1 per cent. in the readings of the voltmeter may produce an error of 10 to 20 per cent. or more in the value of the drop.

Thus if the secondary voltage is 200, it will probably be difficult to read reliably the difference between 199 and 200 volts on the voltmeter. Since the total drop even at high loads would possibly be only six to eight volts, it is evident that the percentage accuracy in readings of the drop will not be high. Also, if the primary voltage is high, say 1,000 volts, it is impossible to regulate this so as to maintain it exactly at a constant value.

The following method has the advantage that the drop of voltage is measured directly, instead of being only a small fraction of the total volts registered. In this case two similar transformers, or two transformers giving the same ratio of transformation, are required, and are connected in parallel to the supply mains—primary and primary or secondary and secondary, whichever is the more convenient.

When two similar transformers have their primary windings connected to the same supply, their secondaries will give the same voltage, and if joined together in opposition would exactly neutralise each other. Consequently a voltmeter connected in series with the two windings under these conditions would read zero.

If, now, one transformer be loaded, it will give a slightly lower secondary voltage on account of the drop in its windings. The voltmeter would then give a reading which is the vector difference between the secondary voltage of the transformer without drop and that of the loaded transformer. That is to say, the voltmeter will read the drop in the loaded transformer directly, if the two secondary voltages are approximately in phase.

It is obvious that the second transformer is prevented from contributing current to the load circuit by the voltmeter in series with it. (See Fig. 67.)

EXPERIMENT XXII.—DETERMINATION OF DROP IN TRANSFORMER. (DIRECT READING METHOD.)

DIAGRAM OF CONNECTIONS.

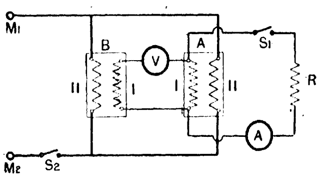


FIG 67.

M_1, M_2 Source of alternating current.
I, II Primary and secondary windings of transformers *A* and *B*.
R Variable non-inductive resistance for altering load of transformer *A*.
A Ammeter for reading output of transformer *A*.
V Voltmeter for reading drop of transformer *A*
S_1 Switch for breaking load circuit.
S_2 Switch for connecting transformers to supply

Instructions.—Connect* both transformers to the source of supply through a switch, the two primary or the two secondary windings being chosen according to the voltage available.

Connect the free windings of the two transformers together in opposition, inserting a low-reading voltmeter in the circuit.

* The advantage of making the connection of both transformers to the supply by a single switch is that the voltmeter does not receive a high voltage, as would be the case if one transformer were connected to the supply before the other.

Connect one of these windings also to a load circuit comprising a variable resistance, ammeter, and switch.

Before putting the low-reading voltmeter in the circuit, it is necessary to make sure that the windings connected to it are in opposition and not in series. If they are in series the voltmeter will get double the voltage of a single transformer. First, therefore, put in place of the voltmeter a set of lamps in series or a high-reading voltmeter, and note if there is considerable voltage on closing S_2. If so, interchange the connections of one transformer.

After connecting to the supply, note the reading of the voltmeter with the switch S_1 open. If the transformers are exactly similar, this will be zero.

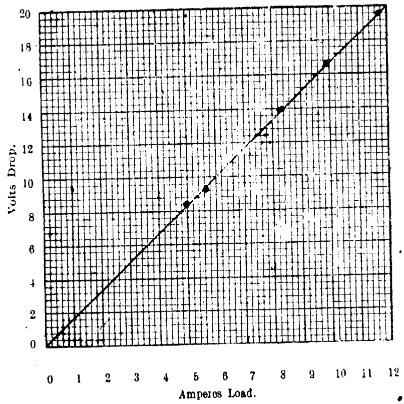


FIG. 68.—Voltage Drop in Transformer.

Close the switch, and gradually increase the load on the transformer A, until the maximum load contemplated is reached. For each value of the load read the voltmeter. The voltage applied to the transformers should be maintained fairly constant, although a small variation will not affect the results, since the voltages of both transformers will vary together.

The readings should be tabulated and plotted in the form of a curve, load current being measured horizontally and voltage drop vertically.

The curve in Fig. 68 shows results obtained in this way on a pair of 1 kw. transformers having a ratio of transformation of 100 to 50 volts.

Fig. 68 should be compared with Fig. 65, as both curves were taken on the same transformer. Full load corresponds to 20

amperes in Fig. 65, since the current is measured on the low-pressure winding, whereas in Fig. 68 the current is measured on the high-pressure winding, and full load is 10 amperes.

The reactance may be calculated exactly as in the previous case, except that equivalent resistance and reactance will now be referred to the high-pressure winding, and will have a higher value in consequence, although representing the same percentage loss of voltage as before.

The ratio of the resistances of the two windings is generally approximately that of the square of the number of turns, as in the present case.

The method of opposition may be carried out, using two equal windings on the same transformer, instead of two separate transformers, and loading one winding only.

No Load Current of a Transformer.—In order to produce the magnetic field in the core of a transformer, a certain amount of *magnetising current* is necessary. The amount of this current is calculated in the same way as the magnetising current of a direct-current electromagnet. At any instant the value of the current is given by the formula generally used for direct-current calculations.

$$I\,T = \frac{l\,B''}{\mu} \times \cdot 313$$

or

$$I\,T = \frac{l\,B}{\mu} \times 2{\cdot}02$$

or,

$$I\,T = \frac{\lambda\,B}{\mu} \times \cdot 8$$

where I = maximum current in amperes.
T = number of turns.
l = length of path in inches.
λ = length of path in cms.
B'' = maximum induction in lines per square inch.
B = maximum induction in lines per square cm.
μ = permeability of core.

It is usual to define the current in terms of *virtual* amperes, and magnetic flux in *maximum* number of lines. We must consequently introduce the relation between maximum and virtual amperes in the formulæ just given, which on the assumption of sine waves become :—

$$i\,T = \frac{l\,B''}{\sqrt{2}\,\mu} \times \cdot 313$$

i.e.,

$$i\,T = \frac{l\,B''}{\mu} \times \cdot 221$$

or,

$$i\,T = \frac{l\,B}{\mu} \times 1{\cdot}435$$

or,

$$i\,T = \frac{\lambda\,B}{\mu} \times \cdot 565$$

where i is in *virtual* amperes and B in maximum induction.

Thus, in order to produce a given alternating flux in the magnetic circuit of a transformer, there must be supplied a definite magnetising current, the actual value of which will depend on the number of magnetic lines and upon the permeability of the core. Actually the value of the permeability will not be constant, since it will vary during each period as the instantaneous values of the current and field fluctuate. For low values of B, such as are used in practice for transformers, no great error is introduced by taking the permeability as constant, and equal to its value at the maximum induction due to the current employed.

As in the case of a continuous-current electromagnet, no power is spent in the production of the magnetic field directly, although in both cases a loss necessarily occurs in the resistance of the windings. An additional source of loss exists with an alternating field, owing to the imperfect nature of the iron, which absorbs power in overcoming its molecular friction or hysteresis, and also has local eddy currents produced in it, which entail a loss of power. The power, which is thus always required in order to produce an alternating field in the core of a transformer, is really spent in overcoming incidental losses, and not in producing the alternating flux, which, as already stated, does not require an expenditure of energy. The magnetising current, which is given by the formula above, is an idle current, not requiring any power for its production. The no-load current of a transformer consists, consequently, of two portions, namely the true magnetising current, which being an idle current lags a quarter of a period behind the applied voltage, and the energy current required to overcome the hysteresis and eddy current losses, as well as the losses in the primary winding. This part of the current is in phase with the primary applied voltage.

As these two component currents are at right angles in phase, they may be represented as two sides of a right-angle triangle, of which the total no-load current forms the hypotenuse. Numerically,

$$\text{No-load current} = \sqrt{(\text{magnetising current})^2 + (\text{energy current})^2}.$$

The energy component of the no-load current is usually calculated from tables supplied by the makers of the sheet iron used in the core. The losses are stated at so many "watts per pound of iron at 50 cycles" for various inductions.

After calculation of the total watts from the weight of the core and value of induction to be employed, the energy, or iron-loss, current is obtained by dividing the total watts by the primary voltage, since

$$E_1 \times \text{energy current} = \text{watts}.$$

The iron losses are considered in more detail later (page 149).

Experiment XXIII.—Determination of No-load Current and Watts of a Transformer.

Diagram of Connections.

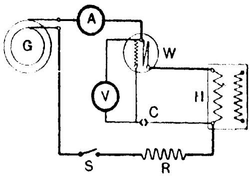


Fig. 69.

G Alternator supplying current.
I, *II* Primary and secondary windings of transformer.
V Voltmeter.
A Ammeter.
W Wattmeter.
*R** Variable resistance for adjusting voltage at transformer terminals.
C Voltmeter key.
S Switch.

Instructions.—Connect one coil of the transformer to a supply of alternating current. It will generally be found advisable to choose the low-tension winding or a part of this winding for the purpose, in order to get a magnetising current sufficiently large to be conveniently read. Connect a low-reading ammeter in series with the supply, and a wattmeter to read the power supplied. Connect a voltmeter to the same winding.

Keeping the periodicity of the current constant, vary the voltage at the terminals of the transformer connected to the supply, from a low value upwards, either by varying the excitation of the alternator supplying the transformer, or by employing an auto-transformer or potential regulator. For each value of the voltage take readings of the voltmeter, ammeter, and wattmeter. Enter the results in tabular form as shown below. The coil supplied with current is here called the "primary," although it may be intended to be used as secondary under normal conditions.

If the secondary winding is used as primary in the experiment, the primary no-load current under working conditions is obtained by dividing the observed current by the ratio of transformatio

* This resistance is not necessary for carrying out the experiment, and is only intended to enable fine adjustments of the voltage to be made. It must not absorb more than a small fraction of the applied voltage, lest the opening or closing of key *C* should produce an appreciable alteration in the voltage at the transformer terminals.

Determination of Transformer No-load Currents at Various Voltages.

Transformer No........ Type............
Output............kw..................periods per second.
Ratio of Transformation............to................volts.

Primary.				Energy Current $\frac{W_1}{V_1}$	Magnetising Current $\sqrt{I_1^2 - \left(\frac{W_1}{V_1}\right)^2}$
Volts. V_1	Amperes. I_1.	Watts. W_1	Back Volts. $= V_1 - I_1 R_1$		
15	·20	1	15	·067	·19
25	·31	3	25	·12	·286
35	·46	6·5	35	·18	·423
50	·86	15	50	·30	·807

From the results of this experiment a curve should be plotted comparing the primary voltage with the magnetising current. In practice the volts lost in the winding at no-load will almost always be negligible compared with the applied voltage, and column 4 may be omitted.

In addition to the total no-load current, the energy and idle currents should be plotted separately. The energy component is obtained by dividing the power supplied to the transformer (as read on the wattmeter) by the voltage of supply. The idle or magnetising current is then obtained by calculation, or from a diagram in which the total no-load current is the hypotenuse of a right-angle triangle, and the energy and idle currents are the sides. The method of drawing this triangle is to describe a semi-circle on a line A B representing the total current to a scale of amperes, and then to draw a circle from one end as centre, with radius equal the energy component. Joining the intersection of the circles to A and B, the triangle is complete.

The curve of magnetising current gives a magnetisation curve for the transformer analogous to the magnetisation curve, which is of such great importance in the case of generators.

If the number of turns in the winding to which the voltage is applied is known, the curve may be plotted with " Number of lines " instead of " volts " as the base, since the lines can be calculated from the formula already given on page 116.

$$V = 4{\cdot}44\, f\, T\, F \times 10^{-8}$$

The power spent in magnetisation, and represented by the iron-loss current, is converted into heat in the core, and plays an important part in the heating of the transformer.

In Fig. 70 are shown the results of an experiment carried out upon a small arc lamp transformer designed for a secondary voltage of 50. The thick line Curve I. shows the total no-load current observed. Curve III. is the curve of iron-loss current obtained by

dividing the watts by the voltage of the supply (the copper losses were too small to be taken into account). Curve II. shows the calculated magnetising current.

It will be noticed that the Curves I. and II. do not appear to pass through zero, although obviously the voltage and current must become zero together. The explanation is that the curve would bend downwards at the lower end, as shown by a thin dotted line, although the instruments available would not read sufficiently low to enable readings to be taken at this part of the curve. This bend is owing to the fact (referred to under Experiment VI.) that at very low saturation the permeability of iron is lower than at

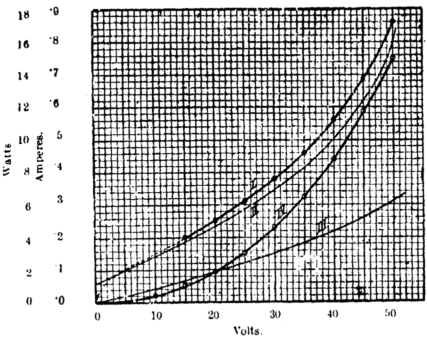


FIG. 70.—No-load Currents and Watts of a Transformer.

I.—No-load current.
II.—Magnetising current.
III.—Iron loss current
IV.—Watts (iron loss).

rather higher values of the saturation. At the beginning of the magnetisation curve, therefore, the increase of current is rapid compared with the magnetic field produced.

Curve II. is the true magnetisation curve for the transformer, and is similar in form to the magnetisation curve of iron, as will be at once recognised on viewing the curve with the figure turned so that the ampere scale becomes horizontal.

Transformers are always designed to work at low-core saturation (say, 4,000–5,000 lines per $cm.^2$), compared with the density employed in direct-current magnetic circuits.

This is necessary on account of the high iron losses which would otherwise result from the rapid alternation of the field. The "knee" of the magnetisation curve, which is so important in direct-current work, is never reached in the magnetisation curve of a transformer.

The experiment just given has a further important application, owing to the fact that the core losses of a transformer are very nearly the same at all loads as they are at no-load.

The wattmeter readings will indicate the core losses under working conditions, if the voltage applied is the normal working voltage.

From the results of this experiment it is consequently possible to calculate the efficiency of the transformer at all loads if the resistance of the windings is known, since the losses which occur in working consist of the iron losses and the I^2R losses in the windings.

A simple method of determining experimentally the copper losses (*i.e.*, the losses in the winding) is given in the next experiment.

Separate Determination of Copper Losses.—The method of calculating the voltage spent in overcoming the resistance of the windings of a transformer has already been given (see page 119). The power lost in the windings can be simply calculated by multiplying this voltage by the corresponding current, or by multiplying the equivalent resistance of the transformer, referred to either primary or secondary circuit, by the square of the current in that circuit.

The watts lost in resistance may thus be written in either of the forms :—

$$i_1^2 (r_1 + k^2 r_2) = i_1^2 R_1$$

or

$$i_2^2 \left(r_2 + \frac{r_1}{k^2}\right) = i_2^2 R_2$$

Where R_1 and R_2 are the "equivalent resistances" of the windings (see page 119).

The following experiment is a method for determining the copper losses by direct measurement, and depends on the fact that the iron losses in a transformer are very small when the magnetic flux in the core is low, since both eddy current and hysteresis losses decrease rapidly with decreased magnetic densities.

EXPERIMENT XXIV.—DETERMINATION OF COPPER LOSSES IN A TRANSFORMER.

DIAGRAM OF CONNECTIONS.

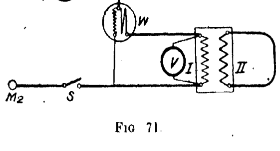


FIG 71.

M_1 M_2 Source of alternating current.
I, II Transformer windings.
W Wattmeter.
A Ammeter.
V Voltmeter.
S Switch.

Instructions.—Connect either primary or secondary winding of the transformer to the source of alternating current, employing a low-voltage voltmeter, an ammeter, and wattmeter to measure the power supplied. Connect the terminals of the other winding together by a thick copper conductor.

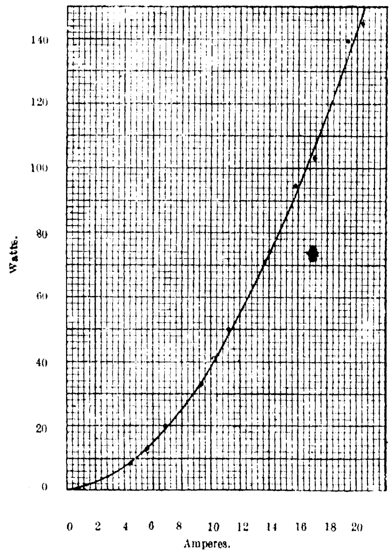


FIG. 72.—Copper Losses in a Transformer.

Arrangements must be made for varying the voltage supplied to the transformer, which throughout the experiment will receive only a few volts. The variation may be accomplished either by varying the voltage of the supply, or by resistance in series with the transformer winding.

Commencing with only a volt or two, gradually increase the applied voltage until the current read on the ammeter is the full-load current of the winding to which the ammeter is connected. For each value of the voltage applied take readings on the wattmeter, ammeter, and voltmeter.

On account of the very low induction used, the losses due to magnetisation of the iron may usually be neglected, and the wattmeter reading may then be taken to be equal to the power lost in the resistance of the windings. If it is found that the wattmeter gives a reading when the secondary circuit is open, this reading should in each case be subtracted from the reading taken with the circuit closed for the same applied voltage, since this will represent the iron loss.

It must be remembered that in this experiment the losses which are being measured are due to a high current flowing through a low resistance. Consequently the resistance of the connecting wires may be quite appreciable compared with the resistance of the transformer windings, and care must be taken that they do not introduce errors. The watts supplied to the transformer should be measured at the terminals of the transformer itself, so as to eliminate as far as possible losses in the leads. The resistance of the connection forming the secondary circuit may be measured, and the watts spent in it subtracted from the watts supplied, if a correction is found necessary.

In order to illustrate what has just been said, the following figures, taken from an actual test, are given, together with the curve showing the results of the experiment in Fig. 72. The curve is seen to be part of a parabola, because the watts are proportional to the square of the current.

DETERMINATION OF COPPER LOSSES IN A TRANSFORMER....

Transformer No. .... Type...............
Output.....................kw. ....cycles per second.
Transformation ....volts to ....volts.

Secondary Current	Watts	Volts
4·55	8·6	4·2
6·8	20·0	6·3
10·2	41·3	9·4
13·7	70·8	12·7
15·8	94·7	14·6
20·45	144·8	19·0

Predetermination of Efficiency under Load.—From the open-circuit test (Experiment XXIII.) and the short-circuit test (Experiment XXIV.) the approximate efficiency of the transformer under any load may be determined. This is done on the assumption that the iron losses remain constant at all loads at

the value corresponding to normal voltage in the open-circuit test. The copper losses are taken as those observed for the various load currents employed in the short-circuit test.

$$\text{Efficiency} = \frac{\text{Watts output}}{\text{Watts output} + \text{Iron losses and copper losses.}}$$

The approximate efficiency under load is thus determined without actually loading the transformer. The difficulty and expense of fully loading a large transformer makes this method of determining the efficiency very useful.

The efficiency may also be calculated directly from the open-circuit test (Experiment XXIII., page 131), together with separate measurements of the resistances of the windings.

Let W_i = watts spent in iron losses (determined as in Experiment XXIII.)

r_1, r_2 = resistances of primary and secondary windings

i_1, i_2 = primary and secondary currents.

W = output in watts

Then total losses $= W_i + i_1^2 r_1 + i_2^2 r_2$

$$\text{and efficiency} = \frac{W}{W + W_i + i_1^2 r_1 + i_2^2 r_2}.$$

$$\text{This may be written } \frac{W}{W + W_i + i_2^2\left(\frac{r_1}{k^2} + r_2\right)}$$

where k is the ratio of transformation (see page 119).

The value of W_i, the watts spent in iron losses, is the value of the watts taken by the transformer on no-load at normal voltage, as read on a wattmeter connected in the primary circuit. The iron losses are assumed constant at all loads, and the copper losses are calculated for each load for which the efficiency is required. The efficiency is thus determined by calculation from a single measurement made at no-load.

The copper losses may, however, be considerably different from those calculated in this way if the conductors are of large section. This is owing to eddy currents and "skin effects," which increase the resistance of the conductors when carrying alternating currents.

Predetermination of Regulation.—The open and short circuit curves are also of great value in enabling the drop in volts to be determined for any load and any power-factor.

When the transformer is short-circuited and supplied with current, the voltage applied to the terminals is entirely spent in overcoming the impedance of the windings (see Experiment XXIV.). Thus the "equivalent impedance" of the transformer

$$= \frac{\text{applied voltage}}{\text{current}} \text{ on short circuit.}$$

The applied voltage, which is the reading taken on the voltmeter V in Fig. 71, page 134, is partly spent in overcoming resist-

ance and partly in reactance. The relation between these voltages can, however, be obtained, since the resistance voltage will be

$$E_r = \frac{\text{watts}}{\text{current}}$$

The watts here referred to are obtained from the reading on the wattmeter in the supply circuit (Fig. 71, page 134).

The angle of lag of the current during the test can also be calculated, since

$$\cos \phi = \frac{\text{watts}}{\text{current} \times \text{volts.}}$$

We can, therefore, construct a voltage diagram for any value of the current observed, as in Fig. 72. *F E* is drawn horizontal, representing the calculated resistance voltage. From *E* the line *E D* is drawn equal to the impedance voltage, making the angle at *E* equal to ϕ. *D* is the point where *E D* meets a vertical drawn through *F*. *F D*, then, represents the reactance voltage of the transformer.

If the measurements were made on the winding of the transformer which is intended to be used as the primary, the voltages will be drawn to the scale of primary volts, and the equivalent resistance, reactance, and impedance are obtained by dividing the corresponding voltages by the observed current, which will

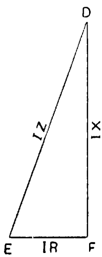


FIG. 73.—Transformer Voltages on Short Circuit.

be those referred to the primary circuit (see page 119). The converse will be the case if the measurements were made at the secondary terminals.

It must be remembered that

$$Z_1 = k^2 Z_2$$
$$R_1 = k^2 R_2$$
$$X_1 = k^2 X_2$$

(for meaning of symbols see page 122), so that it is immaterial upon which winding the measurements were actually made.

We may now show how these results are employed for calculating the terminal voltage of the loaded transformer. In any

loaded transformer the secondary voltage will be diminished by the impedance voltage, and will be the resultant of the $\frac{\text{primary voltage}}{\text{ratio of transformation}}$ and of the impedance voltage. Consequently, in a transformer working on non-inductive load, we shall have the conditions represented in Fig. 74, where *O D* represents the induced secondary voltage at no-load and *O E* is the terminal voltage, the line *D E* representing the impedance voltage, drawn to the scale of secondary volts, determined by experiment as stated above. *D F* is the reactance voltage introduced owing to the magnetic leakage of the transformer The triangle *D F E* is identical with that in Fig. 73, drawn to the scale of secondary volts.

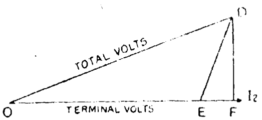


FIG. 74.—Secondary Voltages. Non-inductive Load.

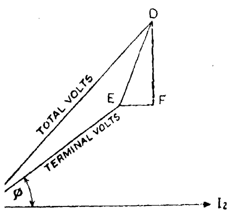


FIG. 75 —Secondary Voltages. Inductive Load

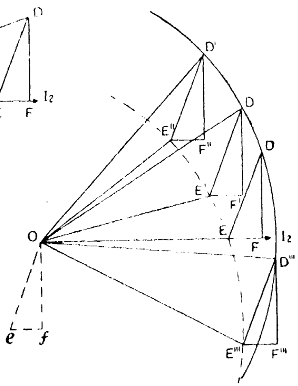
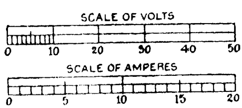


FIG. 76.—Secondary Voltages. Various Power Factors.



The drop in the transformer under these conditions is the difference between the length of *O D*, the no-load secondary voltage, and *O E*.

The actual figures used in Fig. 74 correspond to readings taken on the transformer, whose drop of voltage is shown in Fig. 65.

$O\ D$ represents 51·4 volts, the no-load voltage. The volts at the primary terminals on short-circuit were 35, consequently $D\ E$ is taken as $\frac{35 \times 128}{248} = 18{\cdot}1$ volts, $\frac{128}{248}$ being the ratio of the windings. The copper drop is taken as 7·3 volts, as already calculated on page 126. This is represented by $F\ E$. The terminal voltage $O\ E$ is thus seen to be 41½ volts, which agrees with the value shown from direct measurement in Fig. 65.

When working on inductive load, the diagram becomes that shown in Fig 75, where the current lags behind the terminal volts by an angle ϕ.

In this case the terminal voltage is again obtained from the no-load voltage by constructing the triangle $D\ F\ E$, the side $F\ E$ of which must be parallel to the current line, since the ohmic drop must be in phase with the current.

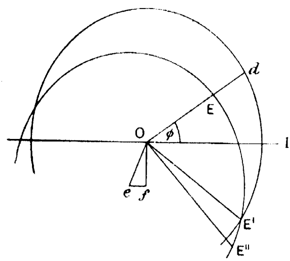


FIG. 77.—Construction for Finding Terminal Volts at any Power Factor.

Fig. 75 has been drawn for the same impedance voltage, and the full load current of 20 amps., as in the previous figures. The angle of lag is taken at about 34°, so that the value of the power-factor = cos 34° = ·83. At this power-factor, it will be seen that the terminal voltage is only 35 volts.

By drawing the diagram, Fig. 75, for a number of different power-factors, the regulation at any value of cos ϕ can be obtained.

Fig. 76 shows a diagram constructed in this way for a constant primary voltage (*i.e.*, a constant no-load secondary voltage of 51·4 volts), and constant load of 20 amps., but for four different values of the power-factor. It may be noticed that with a leading current the value of the terminal voltage $O\ E^{111}$ is greater than the voltage $O\ E$ on non-inductive load.

This diagram illustrates a useful approximate method of rapidly determining the drop in volts at any power-factor. By constructing the triangle $D\ F\ E$ in the position shown dotted as $O\ f\ e$ in Fig. 76, we obtain a point e such that a circle drawn from this point with the same radius as $O\ D$ will pass through all the points E. This

circle is shown as a dotted line. Thus any line $O\,E^1$ drawn to cut the dotted circle and the full line circle drawn from O, with radius equal to the no-load voltage, will show the terminal voltage as $O\,E^1$ and the drop in volts as the length cut off between the two circles on the line $O\,E'$ produced.

Fig. 77 shows the construction just described, the method of drawing it is as follows: From measurements made on the transformer with short-circuited secondary, construct the triangle $O\,f\,e$, corresponding to full load, as explained for Fig. 76. From O and e describe circles of radius equal to the no-load secondary voltage For any value of the power-factor, draw from O a line $O\,E\,d$, making the angle $d\,O\,I$ equal to the angle of lag or lead. $O\,E$ then gives the terminal voltage and $E\,d$ the drop corresponding to that power-factor.

The method of drawing a complete regulation curve for a given power-factor and varying loads will be easily followed from what has been said. The drawing of such a curve forms an excellent exercise for the student.

Efficiency Tests of a Transformer.—The most direct method of determining the efficiency of a transformer is to measure the power supplied by means of one wattmeter, and the power given out by a second wattmeter. The ratio of the two quantities measured, then gives the efficiency, since

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}$$

If the load on the secondary circuit is non-inductive, readings on a voltmeter and ammeter will be sufficient to give the output, since there will be no phase difference in the circuit, and consequently watts output = volts × amperes.

The properties of the transformer are in general more easily studied when the load is non-inductive, and consequently tests of efficiency are nearly always made on non-inductive loads. Transformers are not generally tested on inductive loads except to determine whether they are suitable for working under certain given conditions, or in accordance with a specification.

EXPERIMENT XXV.—EFFICIENCY TEST OF A TRANSFORMER ON NON-INDUCTIVE LOAD.

DIAGRAM OF CONNECTIONS.

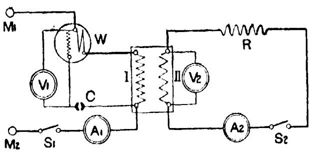


FIG. 78

M_1, M_2 Source of alternating current.
I, II Primary and secondary windings of transformer.
R Variable non-inductive resistance for altering load.
V_1 Voltmeter for reading primary voltage.
V_2 Voltmeter for reading secondary voltage.
A_1 Ammeter for reading primary current.
A_2 Ammeter for reading secondary current.
W Wattmeter for reading primary input.
C Voltmeter switch or plug.
S_1, S_2 Switches.

Instructions.—Connect the primary of the transformer to a source of current, having suitable voltage and frequency, in series with an ammeter. Connect a wattmeter in the primary circuit to read the watts supplied to the winding and join a voltmeter to the primary terminals.

Connect the secondary winding to a variable non-inductive load resistance in series with an ammeter and switch. Join a voltmeter to the secondary terminals of the transformer.

It is well to insert a voltmeter switch, as shown at C, in the circuit of the voltmeter and volt coil of the wattmeter, so that these instruments are only in the circuit when their readings are being taken.

Maintain the applied voltage constant during the measurement and take readings of primary watts and primary and secondary current and voltage, first on open circuit and then with gradually increased loads.

Tabulate the readings as in the following table :—

LOAD TEST OF A TRANSFORMER.

Transformer No.................... Type................
Output ....kw..................cycles per second.
Transformation.............volts to...............volts.

Primary.					Secondary			Efficiency.	Ratio of Transformation.
Current.	Voltage.	Watts.	Volt-Amperes.	Power Factor.	Current.	Voltage.	Watts.		

The following curves should be plotted on a load base, either of secondary current or watts.

(1) Efficiency.
(2) Secondary voltage.
(3) Power-factor at primary terminals.

Fig. 79 shows the results of an experiment carried out on a small 1 kw. transformer having a transformation ratio of 100 to 50 volts. The efficiency curve shows a characteristic which will be generally noticed in lighting transformers which are only occasionally required to work at full load, and are often connected to the mains

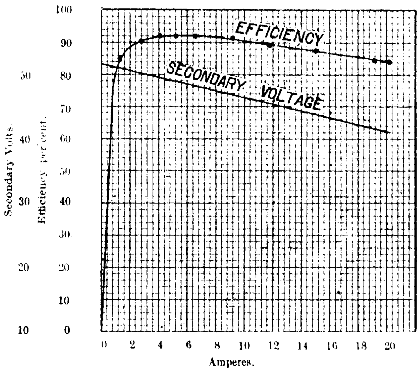


FIG. 79.—Efficiency of a Transformer.

for many hours to supply a light load only. This feature is the comparatively high efficiency at light loads, where the iron losses are almost the only ones felt, with a decided fall of efficiency towards full load on account of the comparatively large copper losses.

A transformer designed to have low iron losses (and consequently a high efficiency at light loads) and heavier copper losses will show a higher *annual* efficiency if working on a circuit which is only occasionally fully loaded than a transformer designed with smaller copper losses and greater iron losses. A transformer intended to work usually at nearly full load should have low copper losses.

In order to make a complete test of a transformer on inductive or capacity load, the method just described must be extended by measuring the output as well as the input by means of a wattmeter

EXPERIMENT XXVI.—DETERMINATION OF EFFICIENCY OF A TRANSFORMER ON INDUCTIVE LOAD.

DIAGRAM OF CONNECTIONS

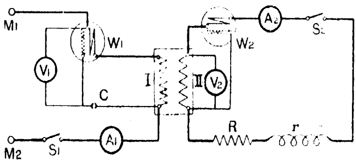


FIG. 80.

M_1, M_2 Source of alternating current.
I, II Primary and secondary windings of transformer.
R, r Variable resistance composed of inductive and non-inductive resistances.
V_1 V_2 Voltmeters for reading primary and secondary voltage
A_1, A_2 Ammeters for reading primary and secondary current.
W_1, W_2 Wattmeters for reading primary and secondary power.
C Voltmeter switch or plug.
S_1, S_2 Switches for breaking primary and secondary circuits.

The method of carrying out this experiment is practically identical with that given for the preceding test, except for the additional wattmeter reading in the secondary circuit. It is usually not possible in practice to obtain complete curves of efficiency and ratio of transformation at different power-factors, but isolated points are obtained at such power-factors as are required, and the curve can then be drawn in the neighbourhood of these points by comparison with the complete curve already taken on non-inductive load.

If a complete set of experimental curves were required for inductive loads, they might be obtained in a manner similar to that described in connection with Experiment XXXI. for the characteristic of an alternator.

As already stated, transformers are not very often tested on inductive loads, except to obtain the secondary voltage at certain loads to fulfil the conditions of a specification.

For an interesting paper giving information as to liquid load resistances, and a special form of inductive load in the form of a choking coil with very highly saturated core, see Morcom & Morris, Proc.Inst.E.E., Vol. 41, p 137.

Efficiency of Two Transformers by Double Conversion.—Sometimes it is found desirable to measure both input and output of transformers on the low-pressure side, so that both quantities can be measured with equal accuracy and possibly on the same

instruments. In this case two similar transformers may be connected with their high-tension windings together. The low-tension winding of one is then supplied with power and the low-tension side of the other is loaded. No measurement is made on the high-tension windings, and the ratio of low-tension input to low-tension output gives the joint efficiency of the two transformers. The efficiency of either is taken as the square root of this.

Sumpner's Method of Testing Two Transformers.—This is a method of testing transformers analogous to the Hopkinson method of testing dynamos, and founded on Hopkinson's suggestions for testing transformers. It has the advantage that each of the two transformers being tested supplies power to the other, so that, although both transformers are fully loaded, the power taken from the supply circuit is small, being only what is necessary to make up the losses in the two transformers. Also the losses are measured directly and are not obtained by subtracting the output from the input. In this way the result of a small percentage error in measurement affects the value of the total efficiency very little.

The method is an indirect one, *i.e.*, the input and output are not measured directly, but the losses in both transformers and the output of one of them are measured; the input is then calculated thus:

power supplied = output + losses in transformers.

The joint efficiency of the transformers is then

$$\frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}$$

Since the two transformers are taken to be similar, they may be assumed to have the same losses, and

$$\text{the efficiency of each} = \frac{\text{output}}{\text{output} + \frac{1}{2}\text{ total losses}}$$

or else it may be assumed that

$$\text{the efficiency of each} = \sqrt{\text{joint efficiency}}.$$

The two wattmeters employed in the test measure the iron and copper losses separately.

The method of carrying out the test is in outline as follows: Two similar transformers are connected to the same alternating circuit—usually the low-pressure winding is the one supplied with current, but this is only a matter of convenience. For the purposes of explanation the winding supplied with current will be called the primary. The other windings of the transformers are connected so as to be in opposition, *i.e.*, the pressure generated in each will tend to send a reverse current through the other. If the pressure applied to both transformers is the same, no current will flow in the circuit formed by the secondary windings. In order that a current may flow through these windings, an auxiliary transformer is put in series with the mains and the primary of one transformer, so as to make its voltage either higher or lower than that of the other. The secondary voltages will be unequal under these conditions, and a current will be produced, the amount depending on the

extent of the out-of-balance voltage. By varying the voltage in the auxiliary transformer any required current may be obtained from the secondary windings, *i.e.*, the transformers may be loaded to any desired extent. The current taken by the primaries will be *in opposite directions* relative to the supply circuit. If the auxiliary transformer assists the transformer to which it is connected, the secondary of this transformer will overcome the secondary of the other, and its primary will take power *from the mains*. The current in the secondary of the other transformer will be the reverse of that which its primary tends to send, and will consequently produce a reverse current in the primary, and this transformer will supply power *to the mains*. If the auxiliary transformer opposes the voltage of the first transformer, exactly the reverse conditions will arise. In either case, the power supplied to the mains is nearly the equivalent of the power taken from them, the difference depending on the efficiency of the transformers.

EXPERIMENT XXVII.—DETERMINATION OF EFFICIENCY OF A PAIR OF TRANSFORMERS BY SUMPNER'S METHOD.

DIAGRAM OF CONNECTIONS.

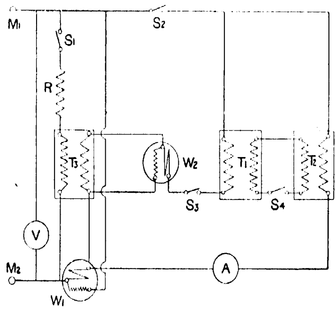


FIG. 81.

M_1, M_2 Source of alternating current.
T_1, T_2 Transformers to be tested.
T_3 Auxiliary transformer.
W_1 Wattmeter reading power taken from line, exclusive of power taken by primary of auxiliary transformer.
W_2 Wattmeter reading output of auxiliary transformer.
V Voltmeter reading supply voltage.
A Ammeter reading current of T_2.
S_1, S_2, S_3, S_4 Switches.
R Variable resistance.

Connections.—Connect the primary winding of the auxiliary transformer to the supply mains through a variable resistance and switch. The resistance should be sufficient to reduce the secondary voltage of the transformer to a few volts only

Connect one terminal of the secondary winding to one supply main, and the other through the series coil of a wattmeter and a switch to that winding of one of the test transformers which is to act during the test as primary The other end of this winding of the transformer under test is to be connected to the opposite supply main.

Connect the primary* of the second test transformer to the supply mains through an ammeter, and insert a switch where shown in the diagram at S_2, so as to break the supply to both test transformers independently of the auxiliary transformer.

Connect a voltmeter to measure the voltage supplied, and put in two wattmeters as shown, W_2, to read the power given out by the secondary winding of the auxiliary transformer, and W_1, giving the total power supplied to the transformers exclusive of that passing into the primary of the auxiliary transformer.

The secondary windings of the test transformers must now be joined together through a switch. If this connection is made correctly, the two primary windings will take current from the supply mains, and will generate approximately equal voltages in their secondaries which will act in *opposition*, each tending to send current through the other If the connections between the secondaries are the reverse, both will tend to send current round the circuit in the same direction, and a short circuit will result.

Before commencing the test, close only the switches S_1, S_2. Under these conditions transformer T_2 will receive current, and its secondary will excite the secondary of T_1, inducing a voltage in the primary of T_1 equal to that of the mains. If the connections are correct, the phase of this voltage will be *opposite* to that of the mains.

In order to test if this is so, connect a voltmeter or incandescent lamps to the terminals of the open switch S_3. If no voltage is found to exist between them the connections are correct, and the measurements may be proceeded with. If the connections have been reversed, there will be a voltage at the switch equal to twice the voltage of the mains. If this is found, the connections to the secondary of one of the test transformers must be reversed.

NOTE.—By employing two ammeters in place of the single ammeter shown in Fig. 81—one in series with S_2, and the other in series with S_3—the magnetising current and the load current may be read separately

Method of Measurement.—After making the connections as above, connect to an alternating supply of the required voltage and frequency.

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* Throughout the description of this experiment the winding which is chosen for connection to the mains is called the primary.

(1) Close all switches except S_1. Under these conditions T_2 will receive a magnetising current, and will in addition supply magnetising current to the secondary of T_1. The total current taken by T_2 will thus be the no-load current of the two transformers.*

The transformer T_1 will not take its full current, on account of the self-induction of the secondary winding of the auxiliary transformer which is in series with it. If this winding is short circuited, both transformers T_1 and T_2 will take equal currents, and there will be no current in their secondary windings (if their ratios of transformation are identical). and it will be immaterial whether the switch S_1 is open or closed.

In any case, the current taken from the mains will be equal to twice the no-load current of one transformer, and the reading of wattmeter W_1 will be twice the no-load losses. The no-load current nearly equal to that of both transformers is shown on ammeter A if the secondary of the auxiliary transformer is not short-circuited: if it is short-circuited, the current of T_2 only will be indicated.

(2) After reading the no-load current and losses on A and W_1 close the switch S_1. This will either add to, or subtract from, the primary voltage of transformer T_1, the voltage of the secondary of the auxiliary transformer. In either case, the equality of the voltages in the closed circuit formed by the two secondary windings of the test transformers will be destroyed, and a current will flow. By adjustment of the resistance R in series with the primary of the auxiliary transformer, vary the current passing through the ammeter A so that about six readings can be taken corresponding to loads varying by nearly equal amounts from light load to slightly above full load.

The load of the transformer T_2 is the product of the reading of the ammeter A and voltmeter V. The losses in the two transformers are the sum of the watts read on the wattmeters W_1 and W_2.

The wattmeter W_1 carries the magnetising and iron-loss current supplied from the mains. The readings of W_1 are thus the watts spent in overcoming the iron losses.

The action of the auxiliary transformer is to add to the primary voltage of transformer T_1 the voltage necessary to overcome the copper losses in both transformers. The power read on the wattmeter W_2 is consequently equal to the losses due to the resistance of the windings of both transformers. Included in the reading of W_2 will also be the losses in the resistance of the conductors joining the transformers and in the instruments (with the exception of W_1). To determine the losses in the leads and instruments alone another measurement should be made.

(3) Open all the switches and short-circuit the primary windings of the test transformers. Close switches S_1 and S_3. The power

* This is strictly true only when S_1 is open. The impedance of the open-circuited transformer T_1 is, however, so great that little change in the readings of ammeter A will result from opening S_1.

supplied by the auxiliary transformer, and measured by W_2 will now be spent in overcoming the circuit and instrument resistance, and is therefore to be subtracted from the readings of W_2 in (2) in order that these readings may give the copper losses of the transformers.

This test of the instrument and lead losses should be carried out with the voltage of the auxiliary transformer adjusted to give a fairly large current through the instruments, in order to make the losses sufficiently large to be easily read. If the instruments or connections are changed during the test, the losses should be read after each alteration.

In the final results, the losses in the instruments for each value of the load should be calculated from the readings. It must be remembered that these losses will be proportional to the square of the load current.

DETERMINATION OF EFFICIENCY BY SUMPNER'S METHOD.

Transformer No. .... Type..................
Output ....kw................cycles per second.
Transformation.......... .volts to ....volts.

Current.	Output Watts.	Wattmeter, W_2.	Iron Losses, Wattmeter W_1.	Instrument Losses $= W_3$.	Copper Losses, $W_2 - W_3$.	Total Losses, $W_1 + W_2 - W_3$.	Joint Efficiency Per Cent.	Efficiency of each Transformer. Per Cent.
3·02	151	8·0	22	0	8·0	30·0	83·5	91·4
5·08	254	28·5	23	5·0	23·5	46·5	84·7	92·0
7·2	360	48·5	25	10·5	38·0	63·0	85·1	92·3
10·6	530	112·0	25	25·0	87·0	112·0	82·5	90·8

Supply voltage was maintained constant at 50.

The preceding table shows the method of entering up results, and gives a few readings taken from an actual test by way of illustration. The complete curves obtained in the experiment are shown in Fig. 82, which shows the following curves: (1) Joint efficiency of the two transformers; (2) efficiency of each calculated from (1); (3) curve of copper losses; (4) curve of iron losses.

The transformers experimented upon were the two 1-kw. $\frac{100}{50}$ volt transformers for which a number of curves have already been given.

Iron Losses.—The iron losses occurring in the core of the transformer are due to hysteresis and eddy currents.

The power spent in hysteresis increases with the intensity of magnetisation of the core, but with a constant magnetic flux is proportional to the number of reversals in the polarity, *i.e.*, it is proportional to the periodicity of the current.

The eddy currents in the core, being induced in the conducting plates of the core by the varying flux across them, will also increase in direct proportion to the rate of alternation of the flux. The watts spent in producing the eddy currents will, however, vary as the square of these currents (since watts $= I^2R$). Hence the watts lost in producing eddy currents will vary in proportion to the square of the periodicity.

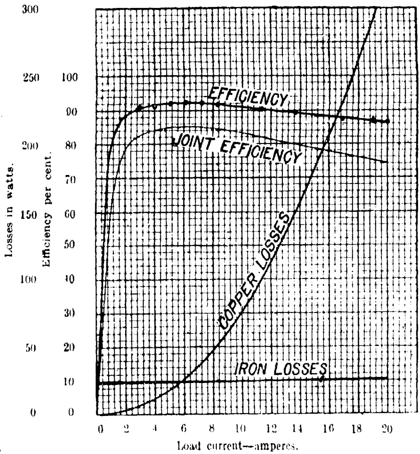


FIG. 82.—Sumpner's Test of Transformer.

Separation of Iron Losses.—The usual formulæ employed for the iron losses in the core of a transformer are as follows :—

Watts lost in hysteresis $= K\ f\ B^{1 \cdot 6}.\ 10^{-7}$ per cub. cm. of core.

Watts lost in eddy currents per cub. cm. of core
$= f^2\, t^2\, B^2\ 10^{-16}$.

Where K = a constant (between ·002 and ·004).
f = periodicity of current.
B = max. induction in lines per sq. cm.
t = thickness of core plates in mils. (thousandths of an inch).

From these expressions it appears that it would be possible, but somewhat difficult, to separate the two losses from observations made on a transformer at various magnetic densities, since the eddy-current losses increase as the square of the induction B, while the hysteresis losses increase as the 1·6th power of B. An easier

way of separating them experimentally is to keep the induction constant, and to measure the watts lost in the iron at different frequencies. By making a series of such observations, and plotting the results on a curve, as shown in Fig. 83, it will be found that at low frequencies the effects of the eddy currents are hardly seen, and loss and frequency have a constant ratio, *i.e.*, the curve is straight. If there were no eddy-current losses the curve would continue straight, *i.e.*, along the line $O\ M$, which is drawn as a tangent to the lower part of the curve.

The ordinates of the line $O\ M$ represent the losses due to hysteresis, while the ordinates between the watt curve and the line $O\ M$ give the eddy-current losses, which vary as the square of the periodicity.

EXPERIMENT XXVIII.—SEPARATION OF IRON LOSSES IN A TRANSFORMER.

DIAGRAM OF CONNECTIONS.

(Same as for Experiment XXIII., page 131).

Instructions.—Connect the transformer to a source of alternating current in series with a low-reading ammeter. Connect a wattmeter to this circuit so as to read the power supplied. Connect a voltmeter to the terminals of the same winding. Supply to the transformer current of different frequencies, commencing with the lowest periodicity, for which satisfactory readings can be taken, taking care that the transformer induction remains constant. This will be the case so long as the ratio of voltage to frequency is kept constant, *i.e.*, $\frac{V}{f}$ must be kept at the same value throughout. This is most easily accomplished by using as the source of supply an alternator having constant excitation and variable speed.

Take readings on the wattmeter and ammeter after adjusting the voltage to the value required to correspond to each value of the frequency. The readings should be entered in tabular form, and two curves plotted with frequency measured horizontally, and wattmeter readings and current vertically. A tangent drawn to the curve of watts at its lowest point, and passing through zero, divides the losses according to their causes, as already explained.

Results should be entered as in the following example.

SEPARATION OF IRON LOSSES IN A TRANSFORMER.

Transformer No. .... Type..........................

Output......................kw. ....cycles per second.

Ratio of transformation...............volts to...............volts.

Volts	Amperes	Watts	Speed Revs. per min.	Frequency Cycles per sec.
55	8·65	150	1725	57·5
44·2	8·45	114	1385	46·2
32·2	7·85	78	1010	33·7
11·1	6·65	26	348	11·6

The curve shown in Fig. 83 gives the results obtained from an experiment carried out not on a transformer, but on a choking coil.

The curve of voltage has been added in Fig. 83, and is a straight line passing through zero. In carrying out the test it is convenient to plot this voltage curve first, and afterwards to regulate the voltage at any speed to the value given by the curve.

This method of determining the iron losses may be applied to samples of iron in order to find the loss per cubic centimetre, and thus to judge of the quality. For this purpose samples are made

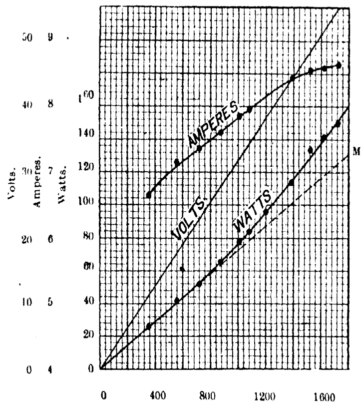


Fig. 83.—Separation of Iron Losses in Transformer.

up into a ring, or other form of magnetic circuit, and a number of turns of wire are wound on the circuit. The test is then carried out exactly as above described.

If the losses in the resistance of the winding are appreciable they must be calculated, and subtracted from the wattmeter reading.

The following modification of the experiment just given is interesting as a further illustration of the relation between the no-load current and periodicity of a transformer.

Experiment XXVIIIa. — Determination of No-load Current and Watts with Varying Frequency.

Diagram of Connections.

(As for Experiment XXIII., page 131.)

Instructions.—Make connections as for Experiment XXIII., connecting the circuit to an alternator, which can be made to supply

current of varying frequencies, while maintaining the terminal voltage of the transformer constant, either by adjustment of a resistance in series with it or by regulation of the alternator fields.

Measure the current and power supplied to the transformer at various frequencies on open circuit at constant voltage. Enter the results in columns with suitable headings, and plot curves of current and power supplied on a frequency base.

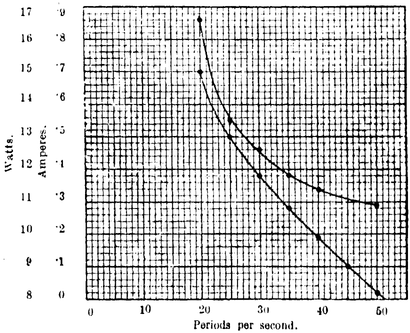


FIG. 83A.—No-load Current and Watts in Transformer at Various Frequencies and Constant Voltage.

Upper Curve — Current.
Lower Curve — Watts.

The curves shown in Fig. 83A were obtained in this manner, and show that both current and watts increase with a decrease of periodicity. This is owing to the fact that the induction in the transformer core must be increased at low periodicity for the terminal voltage to remain constant. The magnetising current increases with a decrease of periodicity in consequence of this. The current spent in overcoming eddy-current losses in the core remains practically constant in amount since these currents are proportional to the square of the periodicity and the square of the induction, and these two quantities vary in an inverse ratio in the experiment. The hysteresis losses, however, decrease with an increase of periodicity, since they are proportional to the product of periodicity and of the induction raised to the power 1·6. Consequently, an increase of induction will more than counterbalance a proportionate decrease in periodicity. The fall of power at increased periodicities is, therefore, chiefly owing to a decrease in the hysteresis losses.

Perhaps the most interesting point illustrated by the curves is the decrease in both current and power brought about by the higher frequency. It is easy to see from these results that an increase of frequency would enable a smaller core, and consequently a less expensive transformer, to serve a given purpose.

Measurement of Iron Losses with High-range Wattmeter.—It is often difficult to obtain a satisfactory reading on the wattmeter employed in measuring the iron losses of a transformer, due to the low current, high voltage, and low power-factor of the magnetising circuit. This difficulty may often be got over by employing a wattmeter intended for the measurement of much larger currents and powers, and connecting the wattmeter coils as shown in Fig. 84*

The magnetising current is supplied to the low-tension winding of the transformer, or preferably to a section of this winding, and the current coil of the wattmeter is connected so as to carry this current. The volt coil of the wattmeter is connected across the high-pressure winding, or a section of the winding giving a suitable pressure.

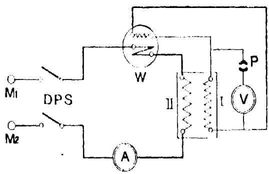


FIG. 84.—Measurement of Iron Losses.

Let T_1 be the number of turns carrying the magnetising current.

T_2 the number of turns connected to the volt coil of the wattmeter.

The voltage acting on the wattmeter volt coil is then $\frac{T_2}{T_1}$ times the voltage induced by the flux in the magnetising coil. The deflection of the wattmeter will therefore be $\frac{T_2}{T_1}$ times the power actually spent in magnetisation of the transformer core, and the readings of the wattmeter must therefore be divided by the fraction $\frac{T_2}{T_1}$.

A feature of this method of measurement is that copper losses in the magnetising winding do not affect the results. The wattmeter readings give the true core losses. The induction in the winding can be calculated from the readings of a voltmeter connected to the coils which do not carry the magnetising current.

By this method, a wattmeter suitable for relatively large currents and large amounts of power may be used for the measurement of only a few watts, with a corresponding gain in the sensitiveness of the measurement.

* Sumpner, Proc. Inst. Elec. Eng., vol. xxxiv., p. 153.

Heat Runs.—One of the most important tests to which a newly-constructed transformer is subjected is the test for ascertaining its temperature rise under full-load conditions of working. In order to economise power, such a test is usually carried out on a pair of transformers connected "back to back," the power supplied from the mains being reduced to the amount required to overcome the losses in the two transformers.

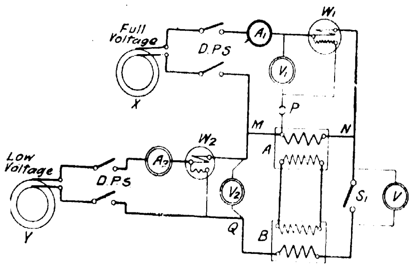


FIG. 85.—Heat Run on Two Single-phase Transformers.

One such method of testing has already been described Experiment XXVII., page 146.

We shall now indicate briefly the method of carrying out such a test when the power for supplying the iron and copper losses is supplied at different voltages, a method which is economical when testing transformers of large size.

Fig. 85 represents the connections for the test of two single-phase transformers, *A* and *B*. The low-tension windings of the transformers are connected in parallel to a supply at the normal voltage and frequency for these windings. The primary windings are connected in opposition, so that no current beyond the magnetising currents of the two transformers is taken from this source of supply.

In order to ensure that the transformers are connected correctly in opposition, the following sequence is followed in making the connections. Transformer *A* is connected to the supply through a D.P. switch in series with an ammeter and wattmeter (for measuring the transformer no-load current and iron losses). Transformer *B* then has its high-tension winding connected to that of *A*, and one terminal of its low-tension winding connected to the supply. Points *M*, *Q*, in Fig. 85 are first taken as directly connected together, and the connection to the low voltage supply at *Y* is assumed to be not yet made. The other low-tension terminal of *B* is then connected to the other pole of the supply in series with an open switch S_1, across which a volt indicator capable of taking twice the voltage of the supply is connected. On closing the D.P.

switch, with S_1 open, the voltmeter V will either read zero or double the supply voltage. If the reading is zero, S_1 may be closed and the connections proceeded with. If V indicates double the supply voltage, the connections between the transformers (either high-tension or low-tension) must be reversed. Wattmeter W_1 will measure the normal iron losses of the two transformers; the iron loss in either may be taken as one-half the watts registered. It is important to notice that the induction in the cores of both transformers has the normal working value, since each winding has the normal voltage between its terminals.

An independent source of current having a lower voltage (usually also a lower frequency) may now be used for supplying the copper losses. This possibility will be seen by regarding the main circuit through the low-tension windings of A and B from the following point of view. This circuit includes two very high inductances, and, as a result, carries a very small current. The resistance of the circuit is relatively insignificant, and is practically without effect on the amount of this current, or on the losses caused by this small current. The existing current and losses would be hardly affected at all if the circuit were broken between M and Q, and a small resistance (*e.g.*, that of a battery of secondary cells) were introduced between M and Q. The potential difference of this battery might, however, serve to send a current equal to the full-load current of the transformers through both the low-tension windings.

By these means the transformers would be developing simultaneously the normal iron losses (supplied from the source X) and the normal I^2R losses in the low-tension windings (supplied from the battery). The high-tension windings would be without current. Now let us imagine the battery between M and Q replaced by the armature of a low-voltage alternator, or by the secondary winding of a low-voltage transformer. This will not in any way interfere with the original supply from the former source, while a low voltage introduced between M and Q will be sufficient to send a large current through both primary and secondary windings of both transformers. The transformers may be regarded as connected in series with, and in opposition to, one another as regards a supply introduced between M and Q, so that a slight voltage introduced here will upset the equilibrium and produce a current. As regards the supply introduced between points M and N, the transformers are in parallel, each separately supporting the full applied voltage.

The instruments required to measure the copper losses are shown in Fig. 85, the ammeter A_2, voltmeter V_2, and wattmeter W_2 being used for this purpose. The current measured represents the load current common to both transformers. The watts registered are the copper losses of both transformers (half this power being expended in each). The voltage V_2 is the impedance voltage necessary to send the current through both transformers in series, and must be taken as spent half in each transformer.

Heat Runs of 3-phase Transformers.—The principles just given for the economical use of two sources of supply for the heating test of single-phase transformers are also adopted for 3-phase transformers.

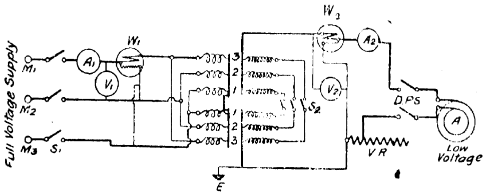


FIG. 86.—Heat Run on Two Three-phase Star-connected Transformers. Using a Single-phase Low-voltage Supply.

In Fig. 86 two star-connected transformers are shown connected in parallel to a 3-phase supply of normal voltage, which provides the iron-loss power. The high-tension windings are thrown out of balance by a source of low-tension single-phase voltage introduced between the star points of the two transformers. The

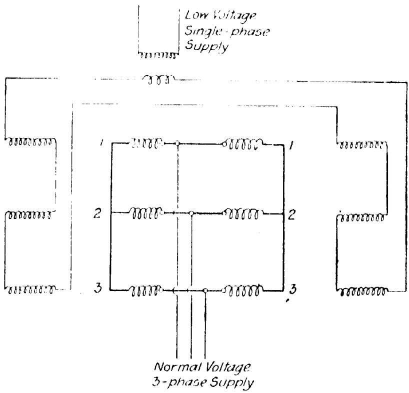


FIG. 86A.—Diagram of Connections for Heat Run.

current registered by A_2 must be adjusted to give three times the full-load current of the high-tension windings in order to correspond to load conditions. This is because the three phases are in parallel with respect to the low-voltage supply, and supplied with currents

mutually in phase. W_2 registers the total copper losses in both transformers. The earthing of a point in the low-tension circuit prevents any part of this circuit rising to a high potential, and causing danger to the operator. V_2 measures double the impedance drop per phase of the transformers (if the supply is at normal frequency), since two windings are connected in series. In order to obtain the impedance per phase, it is necessary to divide half the voltage, V_2, by one-third of the ammeter reading, A_2.

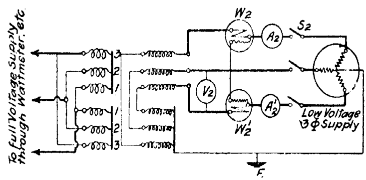


FIG. 87.—Heat Run on Two Three-phase Star-connected Transformers, Using a Three-phase Low-voltage Supply, the Star Point of which is Available.

The disadvantage of the test carried out according to the connections shown in Fig. 86 is that the conductors connected to the star points carry three times the normal phase current and must consequently be made very thick. If the transformer is a large one, this may be a serious objection. In order to avoid this feature, a series connection of the windings may be adopted, as shown in Fig. 86A.

In Fig. 87 are shown the connections for a similar test carried out with a low-voltage 3-phase supply for providing the copper

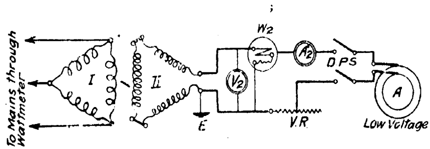


FIG. 88.—Heat Run on a Single Three-phase Mesh-connected Transformer, Using Single-phase Low-voltage Supply.

loss. The readings of A_2, $A^1{}_2$ will in this case give the actual current per phase. V_2 gives $2\sqrt{3}$ times the impedance voltage per phase. The sum of the readings of wattmeters W_2 and $W_2{}^1$ gives the copper losses in both transformers.

Fig. 88 shows the connections for a heat run on a single 3-phase mesh-connected transformer. The low-tension winding is supplied with iron-loss current from a 3-phase source at normal voltage. A source of single-phase current at low voltage is introduced into

the high-tension circuit for providing the copper losses. The ammeter A_2 reads the actual current per phase, while V_2 gives three times the impedance voltage per phase, if the supply is at normal frequency. W_2 gives the total copper losses of the transformer.

Graphic Representation of Phase-Relations in a Transformer.—It is important to form a clear idea of the relation existing between the several varying quantities in a transformer, and this may be best done by representing them graphically in vector diagrams.

Case I.—Transformer without load and without magnetic leakage.

Let the vertical line OF (see Fig. 89) represent the flux in the core passing through both windings. The induced voltage due to this flux will lag behind the flux in phase by 90°. The secondary voltage is, therefore, shown by the horizontal line $O\,E_2$. The induced back electromotive force in the primary winding would have the same phase as $O\,E_2$. The applied primary terminal voltage overcoming this back electromotive force is opposite to $O\,E_2$ in phase, and is shown by $O\,E_1$. A certain no-load current is necessary in order to maintain the flux $O\,F$. This current has two components, viz., a magnetising current which is an idle current, and lags 90° behind the applied voltage, and an energy current necessary to overcome the iron losses.

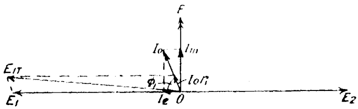


Fig. 89.—Transformer on No-Load.

In Fig. 89 the magnetising current is represented by the vertical lines $O\,I_m$ drawn to a scale of amperes.

The current $O\,I_m$ is the portion of the current which acts as magnetising current, and is in phase with the flux which it produces.

The energy component of the no-load current is at right angles in phase to the idle component, and is thus represented by a horizontal line $O\,I_e$ in phase with the applied voltage $O\,E_1$. This iron-loss current may be looked upon as overcoming the demagnetising action of the eddy currents in the transformer core, and of the hysteresis of the iron. In fact, the iron-losses produce a similar effect to currents in the secondary winding, and must be neutralised by an added primary current.

The total no-load current of the transformer is the sum of the idle and energy components, and is accordingly represented by the vector $O\,I_o$, which is the resultant of $O\,I_m$ and $O\,I_e$.

The method of calculation of the no-load currents has been given on pages 129 and 130, and their experimental determination has been considered in Experiment XXIII.

On account of the resistance of the primary winding, it will be necessary to apply to the terminals a voltage somewhat greater than the voltage $O E_1$ overcoming the back electromotive force. We must apply an additional voltage in phase with the current I_0 and having a value $I_0 r_1$, where r_1 is the resistance of the primary winding. This voltage is shown as the vector $O I_0 r_1$ in Fig. 89, and when added to the vector $O E_1$, we obtain the total applied terminal voltage $O E_{1T}$. The angle of lag in the primary circuit is evidently the angle between the current vector $O I_0$ and the vector of terminal voltage $O E_{1T}$. This angle is marked ϕ_1 in the diagram.

Note respecting Scales.—If the vectors of primary and secondary voltage were drawn to the same scale, they would appear as lines of very unequal length. This would make it difficult to measure the low voltages accurately, because they would usually be very short compared with the primary voltage vectors.

The usual plan is to draw the primary and secondary voltages E_1 and E_2 of *equal length*, and to employ a *different scale* for voltages

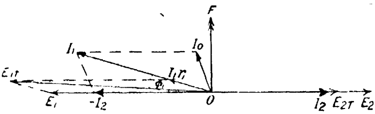


FIG. 90.—Transformer on Non-inductive Load.

in the primary and secondary windings. The scales are so chosen that a line which represents 1 volt on the scale of primary voltage will have the same length as the line denoting $\frac{1}{k}$ volt in the secondary winding. k denotes here the ratio primary to secondary turns.

A similar convention is adopted in regard to primary and secondary currents. One ampere in the primary winding is represented by a line of the same length as one indicating k amperes in the secondary circuit.

By this means all currents represented on the diagram can be considered equally effective as regards the magnetic circuit, since the secondary currents flow round the core in a less number of turns.

Case II.—Secondary circuit loaded non-inductively.—(See Fig. 90.) This diagram will be the same as the preceding, with the addition of the load current in the secondary and a corresponding current of $\frac{1}{k} I_2$ amperes in the primary, producing an equal and opposite number of ampere-turns. The added primary and secondary currents will appear on the diagram as equal and opposite current vectors.

The secondary current is shown by the line $O\,I_2$; its neutralising current in the primary is the line $O-I_2$. The total primary current is the resultant of I_0 and $-I_2$, and is shown by the vector $O\,I_1$.

The voltage overcoming the resistance of the primary winding is $I_1 r_1$, in phase with I_1. The total applied primary voltage is the resultant of E_1 and $I_1 r_1$, and is shown by the line $O\,E_{1r}$. The angle of lag between primary current and voltage is marked ϕ_1, as before.

The secondary terminal voltage is now no longer E_2, but is diminished by the amount $I_2 r_2$, due to the resistance of the secondary winding. This voltage must have the phase of the secondary current. The length $O\,E_{2r}$ gives the secondary terminal voltage, $E_{2r}\,E_2$ being equal to $I_2 r_2$.

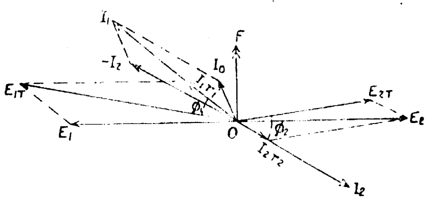


FIG. 91.—Transformer on Inductive Load.

Case III. Secondary circuit loaded inductively.—(See Fig. 91.) If the secondary circuit is inductive, the current, instead of coinciding with the line $O\,E_2$, will lag behind this as shown in Fig. 91, where $O\,I_2$ is drawn of the same length as in the previous case.

The remaining part of the construction is exactly the same as before, the primary current being again the resultant of $O\,I_0$, and of a current equal and opposite to $O\,I_2$. The primary voltage $O\,E_{1r}$ is the resultant of the voltage $O\,E_1$ overcoming the back electromotive force, and $I_1 r_1$ overcoming the primary copper loss.

The angle of lag of the primary circuit is again the angle $E_{1r}\,O-I_1$, which in this case is greater than before on account of the lagging current in the secondary circuit.

The secondary current will produce a drop of voltage in phase with the current and equal to $I_2\,r_2$, so that the terminal voltage is the resultant of $O\,E_2$ and $O\,I_2\,r_2$ drawn parallel to the current $O\,I_2$. The terminal voltage is consequently represented by $O\,E_2'$ and the angle of lag is the angle between $O\,I_2$ and $O\,E_{2r}$.

Case IV. Transformer with leakage.—(See Fig. 92.)

It has been shown on page 120 that magnetic leakage may be treated by regarding the winding in which it occurs as having reactance.

The diagram previously given may consequently be made to represent the conditions in a transformer with leakage by adding a reactance-voltage vector in the primary and secondary sides

to indicate the voltage absorbed in overcoming the leakage reactance of the primary and secondary windings. In each case this voltage will be 90° in advance of the current.

Fig. 92 will be seen to differ from Fig. 91 on account of the addition of the voltage I_1x_1 to represent the voltage lost in primary reactance, and the voltage I_2x_2 which diminishes the secondary terminal volts. Figs. 91 and 92 are similar, except that the voltage losses in the windings in Fig. 91 are only due to resistance, while in Fig. 92 they are due to impedances. The primary impedance voltage is the dotted line $O\,I_1x_1$. The secondary impedance voltage is $O\ I_2x_2$. The general effect of magnetic leakages is seen to increase the lag of the current in the primary circuit, so that in a leaky transformer the power-factor at the primary terminals is lower than that of the load circuit. In a well-designed transformer, when working at full load, the power-factor has practically the same value in both circuits. The reactance, at the same time, diminishes the secondary terminal voltage—especially when the load circuit is inductive.

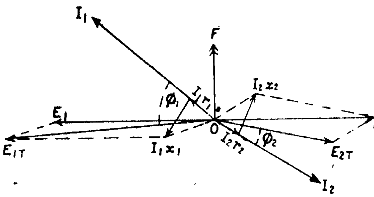


FIG. 92.—Transformer with Leakage on Inductive Load.

Simpler Form of Approximate Diagram.—(See Fig. 93.)—It will be noticed that in all the previous diagrams the primary terminal voltage E_{1T} is shown as varying with the load on the transformer. In actual working, the primary voltage is usually kept constant, and it is the secondary terminal voltage only which varies with the load.

By making use of the "equivalent" resistance and reactance (see pages 119 and 122) of the transformer, instead of treating the primary and secondary impedances separately, we may modify our diagram to represent the conditions at the secondary terminals when the applied voltage at the primary is kept constant.

Thus, in Fig. 93 it is assumed that no loss of voltage occurs in the primary winding, but that the whole of the voltage losses in the transformer may be accounted for by an "equivalent impedance" inserted in the secondary. With this modification, the diagrams in Figs. 92 and 93 show the same conditions.

The loss of voltage, due to the no-load current, is neglected in this construction.

The following table referring to the figures just given may be found convenient for reference :—

$O E_1$	Voltage overcoming induced primary back volts.
$O E_{1T}$	Primary terminal voltage.
$O I_1$	Primary current.
ϕ_1	Angle of lag at primary terminals.
ϕ_2	Angle of lag at secondary terminals.
$O I_2$	Secondary current.
$O I_e$	Iron-loss current.
$O I_m$	Magnetising current.
$O I_0$	No-load current.
$O E_2$	Secondary induced voltage.
$O E_{2T}$	Secondary terminal voltage.
$O F$	Flux in core.
r_1, r_2	Resistance of primary and secondary windings.
x_1, x_2	Reactance—due to leakage flux.
R_2, X_2	Equivalent resistance and reactance of transformer referred to secondary circuit.

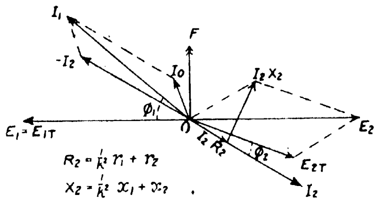


FIG. 93.—Approximate Diagram for Transformer with Leakage on Inductive Load.

Tracing of Curves of Primary and Secondary Voltage.—The method of determining experimentally the wave form of the voltage of an alternator (see page 82) may be extended to the determination of the wave form of the primary current of a transformer, either on open circuit, or when loaded. Simultaneous readings may be taken on the secondary winding by throwing over the voltmeter connections to the secondary circuit for each position of the contact. The two curves of primary and secondary amperes may then be traced on the same sheet of squared paper. Similar curves for a transformer loaded non-inductively and inductively should be obtained by the student.

The Auto-Transformer.—Transformers in which the primary and secondary windings are connected together, instead of being insulated from one another, are called auto-transformers.

Let A B (Fig. 95) be a single winding on a transformer core, and let an alternating difference of potential, V_1 be maintained between A and B. From the general principles of the transformer,

we know that a magnetic flux F will be set up in the core having a value determined by the applied voltage and periodicity. This flux will, in fact, have the value given by the equation on page 116:—

$$V_1 = 4{\cdot}44\, f\, T\, F\, 10^{-8}$$

The two following conditions must be fulfilled, whatever the value of primary or secondary currents: (1) The flux F will remain constant, so long as the applied voltage remains constant; (2) the number of ampere-turns required to produce this flux will

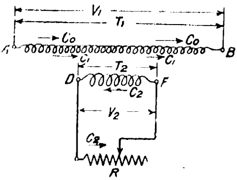


FIG. 94.—Ordinary Transformer (with load C_2).

be constant, *i.e.*, the magnetising current C_0 must remain constant. The magnetising ampere-turns will always be the vector sum of primary and secondary ampere-turns, so that this sum must remain a constant quantity, and any increase in current in one part of the circuit will always be accompanied by an equal and opposite increase in the ampere-turns of some other part of the circuit.

We may look upon each turn of the winding $A\ B$ in Fig. 95 as having applied to it an alternating difference of potential, tending

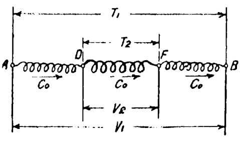


FIG. 95.—Auto-transformer (no-load).

to send an alternating current through it. On account of the alternating flux in the core, each turn of the winding is at the same time the seat of an induced alternating electromotive force which tends at any instant to send a current in the opposite direction to that in which the applied voltage acts. If the applied potential difference and the induced electromotive force were exactly equal, no current would result. The applied potential difference is very slightly greater than the induced electromotive force, and the magnetising current C_0 flows through the winding in consequence:

or, more strictly, the flux rises to such a value that the electromotive force induced by it is equal to the applied voltage except for the small difference required to enable the magnetising current to flow through the resistance of the winding.

Ratio of Voltages.—Since each turn of the winding in Fig. 95 has the same electromotive force induced in it, the secondary voltage obtained by connecting a load circuit to any portion, such as *D F*, will be proportional to the number of turns between *D* and *F*. The ratio of the primary to secondary voltage will accordingly be given by the relation :—

$$\frac{\text{volts between } A \text{ and } B}{\text{volts between } D \text{ and } F} = \frac{\text{turns between } A \text{ and } B}{\text{turns between } D \text{ and } F}$$

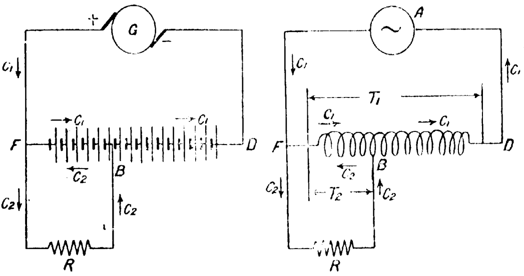


FIG. 96. Auto-transformer and Battery Analogy.

The ratio of voltages is thus seen to be the same as for an ordinary 2-winding transformer, if we consider the primary winding to consist of *all* the turns of the transformer and the secondary to be composed of the windings between the tap points connected to the load circuit.

Thus $$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

Where T_1 = total turns,
T_2 = turns supplying secondary circuit.

Ratio of Currents.—Perhaps the easiest way to get a clear idea of the current distribution in the windings of a loaded auto-transformer is to consider an analogous case in a continuous-current circuit, viz., that of a secondary battery in which all the cells are being charged in series, at the same time that some of them are being discharged. Such an arrangement is indicated on the left of Fig. 96, while the corresponding transformer circuits are represented on the right of the same figure.

Referring to the diagram of the battery, it is clear that there are two distinct electrical circuits. The upper circuit in the figure

consists of the generator, G, the complete battery and the connections between them. There are two electromotive forces in this circuit, viz., those due to the generator and to the battery respectively. The current in the circuit is due to the difference between these two opposing electromotive forces, and is numerically equal to the quotient obtained by dividing this difference by the resistance of the circuit. Since the generator is assumed to have the preponderating electromotive force, the current C_1 in this circuit flows in the direction indicated by the arrows, *i.e.*, in the *reverse* direction to that in which the electromotive force of the battery acts.

The second and lower circuit in the same diagram consists of the left-hand portion of the battery, the load resistance, and connections. The electromotive force acting in this circuit and producing current in it is the electromotive force of the left-hand cells of the battery. The resulting current will have the direction of this electromotive force, and a value, C_2, equal to the quotient obtained by dividing this electromotive force by the resistance of the circuit in which it acts.

On comparing the directions of the currents flowing in the two circuits, it is evident that part of the battery will carry a current which is the difference between the charging current C_1 and the discharging current C_2 of the load circuit. The remainder of the cells carry simply the charging current C_1.

The diagram of the alternating-current circuit (in which the arrows show the relative directions of the currents, and not actual directions) may now be compared with the one we have been considering. It is important to notice that the windings, F B, of the auto-transformer, which are common to the two circuits, A F D and F R B, carry a current which is the difference between the current of the alternator and the current in the load circuit.

In the alternating circuit, we have a fixed relation between the currents C_1 and C_2 which does not exist in the case of the battery currents. This relation arises from the condition governing the action of all transformers, that the sum of the magnetising and demagnetising ampere-turns remains constant, independently of the load. The transformer will be supplied with a small constant magnetising current, C_0, flowing through all its windings independently of the load. The total ampere-turns due to the currents C_1 and C_2, which vary with the load, must always be zero.

Neglecting the constant magnetising current, we may obtain the relation between the primary and secondary currents in the following way:—

Generally the windings between F and B in Fig. 96 will have a larger section than the remaining windings, since the current $C_2 - C_1$ will usually be much larger than the current C_1.

There will be $T_1 - T_2$ turns carrying the current C_1. These we may call the "thin" turns, since they will usually be of smaller section than the other windings.

There will be T_2 "thick" turns.

Then, in order that the resultant magnetising effect of all currents introduced by the load may be zero, the additional magnetising ampere-turns $C_1 (T_1 - T_2)$ must be equal to the demagnetising ampere-turns $(C_2 - C_1) T_2$.

Hence

$$C_1 (T_1 - T_2) = (C_2 - C_1) T_2$$
$$\frac{C_1}{C_2 - C_1} = \frac{T_2}{T_1 - T_2}$$

which gives the ratio of the currents in the windings.

From this it follows that

$$\frac{C_1}{C_2} = \frac{T_2}{T_1}$$

which gives the ratio of primary to load-circuit current.

In the above C_2 is the current flowing in the load circuit.

C_1 is the current flowing in the "thin" windings and supplied to the transformer.

$C_2 - C_1$ is the current flowing in the "thick" windings.

T_1 is the total number of turns.

T_2 is the number of "thick" turns.

The magnetising current C_0 has a practically constant value in all windings at all loads and is not included in the ratios given above.

It has been assumed that the auto-transformer is to be used for reducing the voltage in the diagrams just given. The formulæ which have been derived apply equally to the case where the auto-transformer is to be used for giving an output at a higher voltage and lower current than the source of supply.

Economy of the Auto-Transformer.—The greater economy of an auto-transformer, as compared with the usual type, is seen from the following comparison :—

Ordinary transformer :

C_1 flows through T_1 windings.

C_2 flows through T_2 windings.

Auto-transformer :

C_1 flows through $T_1 - T_2 (= T)$ windings.

$C_2 - C_1$ flows through T_2 windings. This current has a smaller value than the load current which flows in the secondary winding of the ordinary transformer.

As an illustration of the economy of an auto-transformer as compared with the ordinary type, let us take the case of a small 1 kw. transformer having a ratio of 200 : 50 volts, such as might be employed for supplying 50 volt lamps in a house connected to a 200-volt alternating supply system. Let us suppose that a 3 per cent. loss in the windings is allowed.

Taking first a transformer of the ordinary type, we shall probably allow half the loss in each winding.

Neglecting the magnetising current, the primary current will be

$$C_1 = \frac{1,000}{200} = 5 \text{ amps.}$$

The losses in the primary winding will be $1\frac{1}{2}$ per cent. of 1,000 = 15 watts.

The resistance of the primary winding will be

$$r_1 = \frac{15}{C_1^2} = \frac{15}{25} = 0{\cdot}6 \text{ ohm.}$$

Let us suppose, further, that the core employed is such that half a volt is induced in each turn.* The primary winding will then have 400 turns and the resistance per turn will be $\frac{0{\cdot}6}{400}$ ohm. If the length of mean turn were known, the gauge of wire could be now determined.

The secondary current is

$$C_2 = 20 \text{ amps.}$$

and the resistance of the secondary winding

$$r_2 = \frac{15}{20^2} = 0{\cdot}0375 \text{ ohm.}$$

the resistance per turn being $\frac{0{\cdot}0375}{100}$ ohm.

When wound as an auto-transformer, the core will only have 400 turns altogether, 300 turns being of small wire carrying the primary current alone, and 100 turns being of thicker wire and connected to the lamp circuit. We may now allow the same loss in the 300 turns carrying the primary current as we previously allowed in the 400 turns which had the same current flowing through them. Thus, the resistance of the 300 turns will be 0·6 ohm, and the resistance per turn $\frac{0{\cdot}6}{300}$, or $\frac{4}{3}$ of the previous value. We shall thus have $\frac{3}{4}$ of the previous number of "thin" turns, and each turn $\frac{3}{4}$ of the previous section, *i.e.*, we have reduced the weight of this winding in practically the ratio $\left(\frac{3}{4}\right)^2 = \frac{9}{16}$

In the 100 thicker turns the current will be 20 − 5 = 15 amps. instead of 20 amps. The resistance of this winding to give a loss of 15 watts will be $\frac{15}{15^2}$ ohm *i.e.*, $\left(\frac{4}{3}\right)^2$ or $\frac{16}{9}$ of its former value. We can thus choose the wire $\frac{9}{16}$ of the former section, so that the total weight of copper in the transformer has been reduced in

practically the ratio $\left(\frac{3}{4}\right)^2$ or rather more, since the length of the mean turn will also be less than before.

The principle illustrated by the example just given may be stated more generally by saying that on the assumption of equal losses and the same length of mean turn, the copper used in an auto-transformer will be $\left(\frac{k-1}{k}\right)^2$ of that required for transformers of the usual type, when k = ratio of transformation.

Two- to Three-phase Transformation. — From a single phase supply of voltage it is not possible to obtain a polyphase voltage by any system of direct transformation. From a 2-phase circuit it is, however, possible to obtain voltages of any relative phase and magnitude which may be desired, and by suitable connection of circuits to such voltages, any type of polyphase system may be supplied from transformers receiving a 2-phase input.

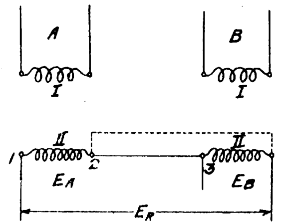


Fig. 97.—Resultant Voltage obtained from Secondaries of Two-phase Transformers.

We will first show that any voltage may be obtained from two single-phase transformers supplied from a 2-phase source, and then indicate how a 3-phase circuit may be supplied by a special arrangement of the transformers

In Fig. 97 let A and B represent two transformers supplied from separate phases of a 2-phase supply.

The electromotive forces of the secondary windings will be at right-angles in phase to one another.

Let us indicate these voltages by vectors, drawing the vector representing the electromotive force of transformer A as vertical and that of transformer B as horizontal.

By connecting the two windings in series as shown by the full line in Fig. 97 we shall obtain the resultant voltage E_R shown by a thick line in the vector diagram (Fig. 98). It is evident that E_R may be made to have any inclination between E_A and E_B by suitably choosing the relative magnitudes of E_A, E_B, *i.e.*, by suitably selecting the ratio of transformation of the two transformers.

Let the secondaries now be joined in series, as shown by the dotted line in Fig. 97, viz., 2 being connected to 4, instead of to 3. The result is to reverse the electromotive force of *A* relative to that of *B*, giving the resultant voltage between terminals 3 and 1

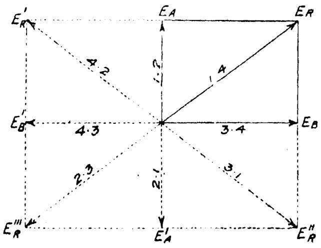


FIG. 98.—Combinations of Two-phase Voltages.

as $E_R{}^{II}$ in the vector diagram. The position of $E_R{}^{II}$ may be anywhere between *E* and $E_A{}^{I}$, depending only on the ratios of transformation adopted for the transformers. It is easy to see how, by other interconnections of the windings, the resultant voltage may be made to have a phase represented by positions of its vector in either of the other two quadrants of the diagram. The figures along the vectors indicate the terminals between which the voltage they represent is measured.

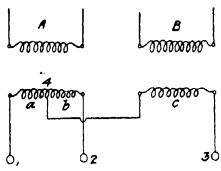


FIG. 99.—Two to Three-phase Transformation.

The Scott Transformer.—The principle just discussed has been embodied in a very simple form for converting from 2 to 3-phase. The connections of the secondaries are shown in Fig. 99, where the secondary of *A* is divided into two equal sections, *a*, *b*, the middle point of the winding being connected to the secondary of *B*. The ratio of transformation is so chosen that the voltage of $c = \sqrt{3} \times$ voltage of *a* or *b*. A true 3-phase voltage is then obtained at the terminals 1, 2, 3. This may be seen by considering in turn the composition of the voltages between the three terminals, 1, 2, and 3

Thus, from Fig. 99,

voltage between 1 and 2 is resultant of volts 1 to 4 and 4 to 2 giving vector 02, Fig. 100.

voltage between 2 and 3 is resultant of volts 2 to 4 and 4 to 3 giving vector 03, Fig. 100.

voltage between 3 and 1 is resultant of volts 3 to 4 and 4 to 1 giving vector 01, Fig. 100.

Because the voltage c is $\sqrt{3}$ times the voltage a or b it is easy to show that the angles between the vectors 01, 02, 03, are each 120°. The proof may safely be left to the student.

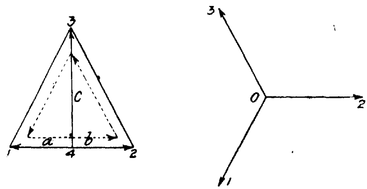


FIG. 100.—Secondary Voltages of Two to Three-phase Transformers.

It is essential to obtain the vector diagram by considering the voltages between the terminals in correct rotation, *e.g.*, 1-2, 2-3, 3-1; *not* 1-2, 1-3, 2-3, &c.

An excellent further exercise for the student is for him to supply a Scott transformer with 2-phase power and by actual measurements of the output to prove that this fulfils truly the conditions of a 3-phase circuit. He should prove not only that the three terminal voltages are equal, but that they differ by 120° in phase.

The Scott transformation is reversible and may be employed for conversion from 3 to 2-phase. Also, it may be carried out by means of a single transformer having two magnetic circuits (*i.e.*, with a 2 or 3-phase type of core) instead of by two separate transformers.

General Note to Chapter VI.—In all the discussions given in this chapter, the wave form of current and voltage has been assumed to be sinusoidal. In the case of transformer work, this is never accurately true, on account of the variation in permeability of the core corresponding to the varying magnetising currents. On this subject see pp. 400 to 402. It was considered advisable not to go into the question of distortion of wave forms, but to refer the student who wishes to study the question more completely to the more mathematical treatises on the subject. The results are to be interpreted in accordance with the explanations given at the beginning of the last chapter (see page 397). They are then both accurate, and in accordance with ordinary practice.

CHAPTER VII.

ALTERNATORS.

Generation of Electromotive Force in the Armature.—In an alternator, electromotive force is generated in the conductors by causing them to move across the lines of a magnetic field.*

The value of the electromotive force induced in each conductor is equal to $\frac{\text{lines cut per second}}{10^8}$ volts. The voltage induced in the armature at any instant = (the voltage induced in each conductor) $\times$ (number of conductors in series), if the voltages in the conductors are equal and similarly directed.

In a direct-current generator, the conductors supplying current to the external circuit are always similarly situated with respect to the magnets of the machine.

In an alternator, current is collected from the *same* and not from *similarly situated* conductors. Consequently the voltage varies with the strength of the field in which the conductors are moving at any moment, and reverses as the conductors move from a field of a given direction into an oppositely directed field.

Since the conductors move at a constant speed, the electromotive force induced in them will undergo exactly the same variations as the field strength when traced round the armature.

Wave Form.—Two factors, besides the speed, the number of conductors, and the value of the total flux, affect the character of the electromotive force generated in an alternator. These are the flux distribution in the air-gap and the system of winding adopted for the armature conductors.

It is not usually possible to get a field distribution giving a true sine curve as the wave form of the electromotive force induced in the conductors, although an approximation to this wave form is generally aimed at.

Although the field distribution determines the wave form of the voltage induced in the individual conductors of the armature, the voltage measured at the terminals will only have this form if all the conductors which are connected in series enter and leave the field under the poles simultaneously. This condition is only fulfilled when the armature is wound in one slot per pole per phase. In all other cases some conductors will enter the polar air-gap later than others; so that the armature voltage will be the sum of

* For simplicity in explanation it is assumed that the conductors move and the field is fixed. The same reasoning exactly applies to machines in which the armature is fixed and the field rotates.

a number of electromotive forces differing in phase. The difference in phase between the voltages of the conductors in two slots will be

$$\theta = \frac{d}{D} \times 180^\circ$$

where d is the length of arc between centres of slots,
and D is the length of arc between centres of poles.

Speaking generally, the effect of increasing the number of slots and thereby "distributing" the winding is to render a peaked wave more flat, and to make a flat wave more pointed, thus in either case making the resultant wave more nearly sinusoidal.

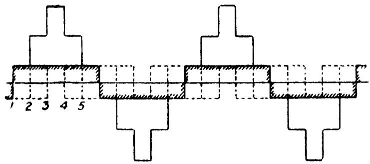


FIG. 101.--Resultant Voltage of Winding in Five Slots.

This effect is shown by the two diagrams, Figs. 101 and 102. Fig. 101 shows an absolutely flat curve of voltage, outlined by the shaded line. By adding the voltages generated in five slots, having successive displacements of 18°, we obtain the stepped curve which would be a fairly close approximation to a sine curve if the corners were rounded off and filled out.

A similar process of addition has been carried out in Fig. 102, where five triangular waves, each shaped like the shaded one, and

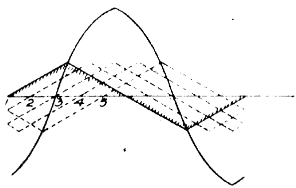


FIG. 102.—Resultant Voltage of Winding in Five Slots.

each displaced 18° relatively to the next one, have been added to give the full-line curve. This curve is again seen to be approximately sinusoidal.

As pointed out on page 88, the virtual or effective value of the alternating current depends upon the *average of the squares* of the instantaneous values, and not upon the *average* values. A voltage of a given *average* value will have a virtual value which depends

upon the wave form, being lower for a flat wave and higher for a wave form possessing a high maximum value or peak.

The ratio $\frac{\text{virtual value}}{\text{average value}}$ is thus determined by the shape of the curve and is termed the *form-factor*. Its value usually lies between 1·1 and 1·4.

Virtual Voltage of an Alternator.—The *average* electromotive force of an alternator is

$$E_{av} = \frac{2\,N\,n\,F\,p}{10^8 \times 60}$$

Where N' = Number of armature conductors in series,
F = flux per pole,
n = revolutions per minute,
p = Number of pole-pairs

The multiplier 2 is introduced because the armature conductors are taken as all connected in series, and not forming two parallel circuits, as they do with the closed circuit winding employed for direct-current generators. Hence for sine wave form the virtual voltage is

$$E_{virt.} = \frac{2\,N\,n\,F\,p}{10^8 \times 60} \times 1{\cdot}11$$

$$= \frac{N\,n\,F\,p}{10^8 \times 60} \times 2{\cdot}22,\text{*}$$

since 1·11 = the ratio of the R.M.S. value to the average value of the ordinates of a sine curve (see page 89).

The constant 2·22 will only give the value of the voltage when the ratio of virtual to average value of the wave form is 1·11, and when the winding is carried out in one slot per pole per phase. For other cases a slightly different constant must be used.

The usual plan of stating the voltage of an alternator is to put it in the form

$$e = \frac{N\,n\,F\,p}{10^8 \times 60} \times k = \frac{N\,F\,f}{10^8} \times k$$

where k is constant depending upon the nature of the wave-form of the machine and type of winding adopted and f is the frequency of the current generated.

Periodicity of an Alternator.—In most types of alternators the magnets are alternately north and south when counted in order round the armature. Thus each conductor passes under a north and south pole alternately, and, in doing so, experiences a complete cycle of changes in the electromotive force induced in it. In such a machine the periodicity of the current will be equal to the number

* It is worth noting that this formula is identical with that given for the transformer on page 116, when we substitute F for Fp (*i.e.*, the total flux), f for $\frac{n}{60}$ (*i.e.*, the frequency), and $2\,T$ (T = No. of turns) for N.

of revolutions of the armature (or field) per second × the number of pairs of magnet poles or

$$\text{periodicity} = f = \frac{n\,p}{60}$$

n = revolutions per minute.

p = number of pairs of poles.

In certain special machines the poles on each side of the armature are all of the same polarity, and the electromotive force induced in the conductors is due to the passage of the conductors through alternately weak and strong fields *in the same direction.* In this case the periodicity equals the number of revolutions per second × the number of poles.

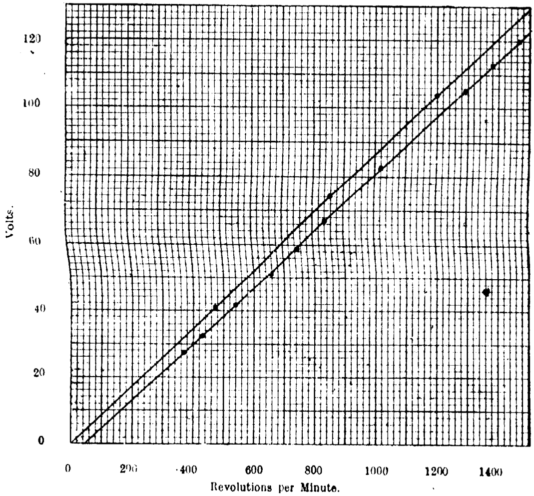


FIG 103.—Relation between Speed and Voltage of an Alternator.
Upper curve no load.
Lower curve load = 20 amperes
Excitation = 1·95 amperes.

Relation between Speed and Voltage of an Alternator.—The formula for the voltage of an alternator, given above

$$e = \frac{n\,N\,F\,p}{10^8 \times 60}\,k$$

shows that the voltage will vary in the same proportion as the speed if the remaining factors remain constant.

Hence, as in the case of a direct-current generator, if an alternator be constantly excited and driven at various speeds, the relation of speed to voltage will be constant. If plotted on squared paper with speed horizontal and voltage plotted vertical, the relation would be shown as a straight line, passing through zero, exactly as in the case of a direct-current dynamo.

This statement is illustrated by the two experimental curves shown in Fig. 103. The no-load curve passes through zero, while the curve showing the relation of speed to voltage with constant current lies below the no-load curve by a nearly constant amount.

The Magnetic Circuit.—The calculation of the excitation of the field system of an alternator is carried out in the same way as in the case of a direct-current dynamo and the magnetisation curve, showing the relation between excitation and magnetic flux, is of the same importance. The magnetisation curve is obtained experimentally by running the alternator at a constant speed and varying the excitation. The armature voltage corresponding to each value of the excitation is then observed. The curve is either plotted to show the relation between "ampere-turns" on the magnets and magnetic flux through the armature, or, more usually, the curve shows the relation between the actual quantities observed, *viz.*, exciting current and armature voltage.

The ampere-turns can be calculated by multiplying the exciting current by the number of magnet windings, while the useful magnetic flux can be calculated from the observed voltage and speed by substitution in the following formula, which has been previously given on page 174.

$$e = \frac{N\,n\,F\,p}{10^8 \times 60} \times k = \frac{f\,N\,F}{10^8} \times k$$

$$F = \frac{e \times 10^8 \times 60}{N\,n\,p\,k} = \frac{e \times 10^8}{N\,f\,k}$$

k being 2·22 for an open circuit alternator giving a sine wave form, and having a single winding.

EXPERIMENT XXIX.—DETERMINATION OF MAGNETISATION CURVE OR OPEN-CIRCUIT, CHARACTERISTIC OF AN ALTERNATOR AT NO-LOAD.

DIAGRAM OF CONNECTIONS

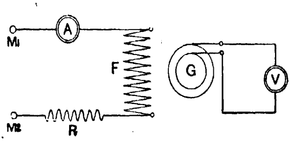


FIG. 104.

M_1 M_2 Source of direct current
G Alternator armature.
F Alternator field windings.
R Resistance for varying exciting current.
A Ammeter for measuring exciting current.
V Voltmeter for measuring alternator voltage.

Instructions.—Connect the field windings to a supply of direct current in series with a variable resistance and ammeter.

Connect the alternator armature terminals to a voltmeter.

Run the alternator at constant speed. Read the armature voltage first without excitation, and then with exciting current gradually increased to its full value. Then take a similar series of readings with gradually decreasing values of the current.

Enter the readings under headings as shown in the following table, and plot a curve in which exciting current is plotted horizontally and armature voltage vertically.

If the speed cannot be kept absolutely constant, a correction in the observed voltage must be made.

Let n = revolutions per minute at normal speed.
n^1 = revolutions per minute observed.
V = required voltage corresponding to normal speed.
V^1 = voltage actually observed at speed n^1.

$$\text{Then } V = V^1 \frac{n}{n^1}$$

DETERMINATION OF MAGNETISATION CURVE AT NO-LOAD

Alternator No.................. Type.......................
Normal output......volts......amps., at......revs. per min.

Exciting Current.	Revolutions per Minute.	Armature Voltage.		
		Reading.	Actual Voltage.	Corrected Voltage.

In the table of results given above, the column headed "Corrected Voltage" is obtained from the preceding column headed "Actual Voltage," by correcting for speed variations.

The readings obtained should form two curves, the curve with decreasing excitation being slightly higher than the curve with increasing excitation. The true magnetisation curve is the mean of the two.

It will probably not pass through zero on account of the residual magnetism of the magnets.

The curve in Fig. 105 is the magnetisation curve of a 6 kw.

4-pole alternator. In this case the rotating field showed no appreciable residual magnetism, and the alternator gave no measurable voltage when unexcited.

Nature of Curve.—From the curve may be obtained directly the absolute value of the magnet flux corresponding to any voltage.

This is illustrated by the following example in connection with the curve shown in Fig. 105.

The machine did not give a perfect sine wave form, but the virtual value of the curve only differed slightly from that of a sine

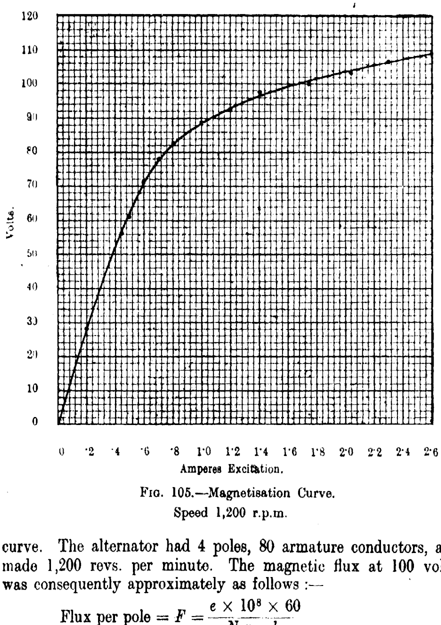


FIG. 105.—Magnetisation Curve.
Speed 1,200 r.p.m.

curve. The alternator had 4 poles, 80 armature conductors, and made 1,200 revs. per minute. The magnetic flux at 100 volts. was consequently approximately as follows :—

$$\text{Flux per pole} = F = \frac{e \times 10^8 \times 60}{N\, n\, p\, k}$$

$$= \frac{100 \times 10^8 \times 60}{80 \times 1{,}200 \times 2 \times 2{\cdot}22} = 1{,}407{,}500 \text{ lines.}$$

It will be seen on reference to Fig. 105 that the first part of the curve is approximately straight. This is because the permeability of the iron of the circuit and of the air-gap is fairly constant at low inductions, and the magnetic flux consequently increases in

the same proportion as the magnetising force. The slope is determined by the ratio between magnetic flux produced and magnetising force applied. Hence, the tangent of the angle which the straight part of the curve makes with the horizontal will be inversely proportional to the reluctance of the magnetic circuit, since

$$\text{Flux} = \frac{\text{Magneto-motive force}}{\text{reluctance}}$$

The steeper the inclination of the first portion of the curve, the less is the reluctance of the magnetic circuit. At a higher induction, the curve bends decidedly to the right, and then again becomes comparatively straight, making a much smaller inclination with the horizontal. When working upon this part of the curve, the effect of small variations in the excitation upon the voltage of the alternator is comparatively slight, and the ratio of magnetising current to magnetic flux is considerably greater, owing to the decreased permeability of the iron at the higher saturation.

Magnetisation Curve under Load.—The effect of a current flowing in the armature upon the terminal voltage is to produce a loss of voltage due to the following principal causes :—

(1) Armature resistance ;
(2) Armature reactance ;
(3) Armature reaction.

These factors are considered later ; their effect upon the terminal voltage of the alternator produces the difference between the magnetisation curves taken with and without load.

Experiment XXX. — Determination of Magnetisation Curve of an Alternator at Full Non-inductive Load.

Diagram of Connections.

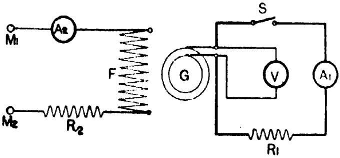


Fig. 108

M_1, M_2 Source of direct current.
G Alternator armature.
F Alternator field windings.
R_1 Variable non-inductive load resistance.
R_2 Field regulating resistance.
A_1 Ammeter for measuring load current.
A_2 Ammeter for measuring exciting current.
V Voltmeter for measuring alternator voltage.
S Switch for breaking load circuit.

Instructions.—Connect the alternator field windings to a source of continuous current through an ammeter and regulating resistance Connect the armature to an adjustable load resistance in series with an ammeter and switch. Connect a voltmeter to the alternator terminals.

Before exciting the alternator, close the switch in the load circuit and make R_1 as low as possible. Run the alternator at normal speed and then increase the excitation slowly until the full load current is reached. Maintain the speed constant and increase

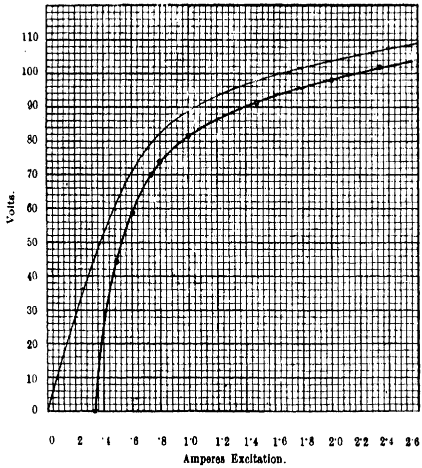


FIG. 107.—Magnetisation Curve with Load.

Load current = 20 amps.

the excitation step by step, at the same time increasing the resistance in the load circuit so as to maintain the load current at the same value as before. In each case take simultaneous readings of voltage and exciting current. Enter the readings in tabular form as in the preceding experiment. The results, when plotted with volts vertically and exciting current horizontally, give the full-load magnetisation curve of the alternator.

The lower curve shown in Fig. 107 is a load magnetisation curve taken on the same alternator as the no-load curve shown in Fig. 105. In order to enable an easy comparison to be made between the two curves, the curve of Fig. 105 has been re-drawn in Fig. 107 The vertical distance between the two curves shows

the amount of "armature drop" produced by the load. Thus with an excitation of 1·8 amps., the drop of volts is seen to be 6. Similarly, the horizontal distance between the curves shows the amount by which the excitation would have to be increased in order to bring the voltage at full-load to its no-load value. On referring to Fig. 107, if 93 volts are required, it will be seen that the difference in exciting current between the points corresponding to 93 volts on the two curves is ·38 ampere. The range of a field-regulating resistance to maintain this voltage at all non-inductive loads would have to be sufficient to produce this variation in the excitation. It must, however, be remembered that the loss of voltage will be greater on inductive loads. The direct experimental determination of this quantity is the subject of a later experiment.

The chief causes producing the armature voltage drop have been already enumerated; they are discussed in connection with the regulation test of the alternator, p. 192.

Approximate Determination of Armature Reactance.—The experiment just described suggests a simple method for determining the approximate armature reactance of an alternator.

When an alternator supplies a non-inductive circuit, the drop in voltage at its terminals is principally due to the voltage required to overcome the impedance of the armature. The armature reaction has chiefly a distorting effect on the field, and only a slight weakening effect, unless the resistance of the load circuit is low. If it is assumed that the electromotive force generated in the armature remains constant, it is easy to calculate the value of the armature reactance. To make the measurement, run the alternator at normal speed and excitation and note the voltage on open circuit. Close the circuit switch, so that the machine supplies current to a non-inductive load, and read the current and voltage. The second voltage is equal to $I \times R$, when

I = Measured current.

R = Resistance in external circuit.

This voltage will be in phase with the current, since the circuit is non-inductive. The voltage spent in the armature will be partly energy voltage in phase with the current, its amount being $I \times r$, when r is the resistance of the armature. The remainder of the voltage lost in the armature will be idle voltage 90° out of phase with the current ($= I\,x$) due to armature reactance x. The total voltage generated by the alternator is assumed to be the same as before closing the switch. Hence this total voltage will be equal to the resultant of the energy and idle voltages in the armature and that of the external circuit. The relation between the quantities is that indicated in Fig. 108 on the next page, where

$AB = E$ = no-load voltage = total voltage generated.
$DB = E_T$ = terminal voltage with load = IR.
$AC = Ix$ = idle voltage due to armature self-induction.
$AD = Iz$ = voltage overcoming armature impedance.
$CD = Ir$ = voltage overcoming armature resistance.

The base of the triangle is the sum of the energy voltages for the whole circuit, which is spent partly in the armature and partly in the external circuit.

The value of Ix is obtained by construction or by calculation, since $$Ix = \sqrt{E^2 - (E_T + Ir)^2}.$$

The value of the armature reactance is easily obtained by dividing this voltage by the current, I.

The following example, taken from Fig. 107, is given to show the method of calculation.

With an excitation of 1·64 amperes, the total voltage generated at no-load is seen to be exactly 100 volts. With a load of 20 amperes the terminal voltage is 93·7. We have for the whole circuit, including external resistance and armature,

$$(\text{Total volts generated})^2 = (\text{idle volts})^2 + (\text{energy volts})^2,$$
$$\text{or } 100^2 = (\text{idle volts})^2 + (93·7 + 20 \times ·14)^2,$$

since 93·7 is the energy voltage applied to the non-inductive external circuit, and the armature resistance was ·14 ohm.

$$\text{Hence idle volts} = Ix = \sqrt{100^2 - (96·5)^2}$$
$$= \sqrt{10,000 - 9,312} = 26·2.$$

The current in the armature was 20 amperes, the reactance is consequently

$$x = \frac{26·2}{20} = 1·31 \text{ ohms}$$

at the particular excitation and current taken.

The impedance of the armature

$$z = \sqrt{r^2 + x^2} = \sqrt{·14^2 + 1·31^2} = \sqrt{1·736}$$
$$= 1·317 \text{ ohms}.$$

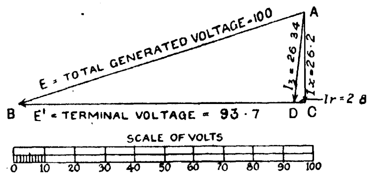


FIG. 108.—Diagram of Voltages in Armature and External Circuit.

It thus appears that the voltage spent in overcoming armature impedance is about 20 × 1·317 = 26·34 volts at normal excitation.

This value will vary with the armature current and field excitation on account of the varying permeability of the iron. The voltage just found must be carefully distinguished from the "voltage drop" which is the arithmetic difference between the volts at the terminals of the armature at no-load and full-load, and is seen from the curve to have the value 100 − 93·7 = 6·3 volts. The diagram, Fig. 108, shows the relation between the voltages of the example worked above

The value obtained for the reactance by this method is not strictly that of the reactance. It is greater than the true reactance, because it includes the effects of armature reaction, *i.e.*, the weakening of the field by the armature currents. It is to be remembered that the current will lag behind the *induced* electromotive force, although it is in phase with the terminal voltage. There will thus be a certain weakening component in the armature reaction. When carried out on a non-inductive load, as described, the value obtained for the reactance may be assumed to be approximately correct.

For many practical purposes it is not important to distinguish between loss of voltage due to weakening of the field, and that due to self-induction. Neither cause produces loss of power; both diminish the available voltage.

A further point to be noticed is that as the resistance of the external circuit becomes lower, a greater proportion of the total voltage generated is spent in overcoming the armature impedance, and the current will lag more behind the induced electromotive force in phase. These changes should be followed out in connection with the changes occurring in the shape of the triangle, Fig. 108, when drawn for various current outputs.

The point where the load magnetisation curve cuts the horizontal axis is given by the value of the excitation required to send the load current through the armature when short-circuited, and where the terminal voltage is consequently zero. In this case the whole of the voltage generated is spent in overcoming the armature impedance and reaction.

The voltage thus spent is seen to be 46 volts in Fig. 107. The difference between this voltage and the 26·3 volts calculated as lost in armature impedance with normal excitation is mainly due to the demagnetising action of the armature currents which now lag nearly 90° behind the induced voltage. This demagnetising action is explained later. (See page 195.)

Field Regulators.—As already pointed out on page 181, the load magnetisation curve shows the range of resistance necessary for the field regulator.

An examination of the magnetisation curve further enables the magnitude of the steps of the field-regulating resistance to be determined, in order that regulation may be accomplished with any desired degree of sensitiveness. Thus, if it is desired to regulate the voltage within one volt, the variation of exciting current corresponding to one volt difference of pressure may be at once obtained by an inspection of the curve. From this the exact value of each step of the regulator can be calculated for a fixed voltage of excitation. For instance, at the upper part of the curve at 2·4 amperes excitation in Fig. 107 each volt corresponds to an increased excitation of ·092 amperes. If the voltage of excitation is 100, the value of each step would be

$$\frac{100}{2 \cdot 4} - \frac{100}{2 \cdot 492} = \frac{9 \cdot 2}{5 \cdot 98} = 1 \cdot 53 \text{ ohms.}$$

Magnetisation Curves with Inductive Load.——The curve discussed in the foregoing section is the magnetisation curve obtained with the machine supplying the current to a non-inductive circuit. This curve is the most important of the load curves, which can be obtained from an alternator. In most cases, however, alternators have to supply circuits consisting partly of inductive resistances, and it is consequently important to determine in what way the self-induction of the external circuit affects the behaviour of the alternator. In order to do this, a partially-inductive resistance should be substituted for the non-inductive load resistance employed in Experiment XXX., and the voltage corresponding to the full-load current at various power-factors and with various excitations should be determined.

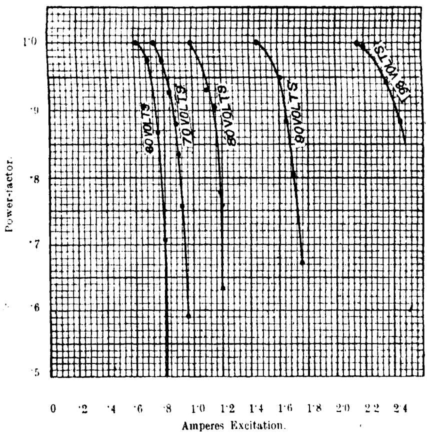


FIG. 109.—Curves showing Variation of Excitation with Power-factor.

Several curves obtained in this way have been drawn in Fig. 110, and show that the terminal voltage of the alternator is diminished by the lag of the current in an inductive circuit.

Before explaining the reason for this, the method of obtaining the magnetisation curves for inductive loads must be described.

The difficulty of getting a complete magnetisation curve on an inductive load arises from the fact that it is in practice very difficult to vary the impedance of the external circuit, while keeping the power-factor of the circuit constant, on account of the difficulty of varying the inductive and non-inductive portions of the circuit simultaneously in the required proportion. In the case of large machines, a single reading often is all that can be obtained on

full load at a given power-factor. The nature of the curve in the neighbourhood of the point can then be approximately traced by drawing a portion of the curve parallel to the non-inductive load curve. This often suffices for practical purposes, since the actual range in excitation employed is very small.

If more complete curves are desired, corresponding to several values of the power-factor, the following method may be employed. The description may be most simply given by referring to the readings, actually taken in order to obtain the curves shown in Fig. 110.

It was desired to obtain magnetisation curves for the three power-factors ·9, ·8 and ·7, the current being in all cases 20 amperes, as it was in the curve for non-inductive load.

From an inspection of the non-inductive curve it was considered that readings taken at voltages of about 100, 90, 80, 70, and 60 would be sufficient to enable the curves to be plotted. A preliminary curve was accordingly taken at each of these voltages in the following manner :—

The alternator was connected to a circuit composed partly of non-inductive resistances and partly of an inductive resistance

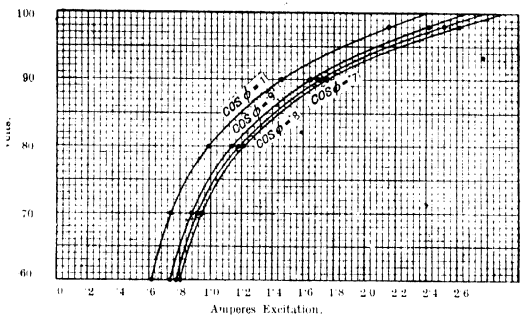


FIG. 110.—Magnetisation Curves at Full Load and Various Power-factors.

wound upon a magnetic circuit with a variable air gap. (It may in some cases be found most convenient to connect the inductive and non-inductive resistances in parallel and in other cases in series).

The inductive resistance was then given its maximum value, and the non-inductive part of the circuit was adjusted to give the correct current of 20 amperes. The excitation was then adjusted until the voltage had assumed each of the five values, 98, 90, 80, 70 and 60.

The circuit was then made less inductive, and a similar series of readings taken. This was repeated for five different values

of the inductive resistance, giving points corresponding to five different voltages on curves corresponding to five different values of the power-factor. These curves were plotted so as to show the variation of exciting current with power-factor. The curves actually obtained in this manner are shown on Fig. 109. From these curves points on the magnetisation curves corresponding to the desired power-factors were obtained, although actual readings for the exact value of the power-factor were not taken.

The excitations required to give respectively 98, 90, 80, 70 and 60 volts at the three values of the power-factor were read off from the curves in Fig. 109 and the complete curves obtained from these points, as shown in Fig. 110.*

With a leading, instead of a lagging, current in the circuit supplied, it would be found that the terminal voltage of the alternator is *increased* instead of decreased. Curves illustrating this are given later in connection with the determination of the characteristic curves of an alternator.

Load Characteristics of an Alternator.—The curve showing the relationship between the voltage and load current is called the "characteristic curve" or "regulation curve" of the machine, and forms the subject of the next experiment.

EXPERIMENT XXXI.—DETERMINATION OF THE LOAD CHARACTERISTIC OR REGULATION CURVE OF AN ALTERNATOR.

DIAGRAM OF CONNECTIONS.

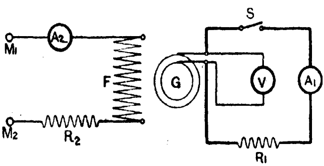


FIG. 111.

M_1, M_2 Source of direct current.
G Alternator armature.
F Alternator field windings.
R_1 Non-inductive resistance for varying load.
R_2 Resistance for maintaining excitation constant.
A_1 Ammeter for measuring load current.
A_2 Ammeter for measuring exciting current.
V Voltmeter for reading alternator voltage.
S Switch for breaking main circuit.

* The speed was not exactly the same during the readings taken for Figs. 107 and 110, which accounts for the slight difference between the load curve in Fig. 107 and the curve marked $\cos\phi = 1$ in Fig. 110.

Instructions.—Excite the fields of the alternator from a source of direct current in series with a regulating resistance and ammeter.

Connect the armature to a variable non-inductive resistance in series with an ammeter and switch. Connect a voltmeter to the armature terminals.

Run the alternator at constant speed and maintain the excitation (read on ammeter A_2) constant throughout the experiment.

Take readings of armature voltage and current, first with the switch S open, and then for gradually increasing values of the load up to the maximum output.

Enter the results in tabular form as shown below, and plot a curve in which voltage is measured vertically and current horizontally.

CHARACTERISTIC ON NON-INDUCTIVE LOAD.

Alternator No. .... Type .... Periodicity ....
Normal output.........amperes.........volts at......revs. per minute
Resistance of armature (hot).........ohms.

Load Current	Revolutions per Minute	Voltage

The voltage readings must be corrected, as shown on page 177, if the speed does not remain constant.

A curve obtained in this way on a 6 kw. 3-phase alternator at a constant excitation of 1·9 amps. is shown in Fig. 112. The curve is here drawn for the full working range of the machine, which is rated to give a full-load current of 35 amps., at 100 volts, when the voltage is regulated by the field rheostat. The field was, of course, kept constant during the experiment.

Drawing a horizontal line through the highest part of the curve (see Fig. 112) the drop of voltage at any load is represented by the vertical distance between the curve and this horizontal line.

The drop in voltage at full load is due to several causes, which are considered more fully later (see page 192).

Characteristic Curves with Leading and Lagging Current.—Since alternators have frequently to supply inductive circuits, and since the power-factor of the circuit affects the voltage, it is sometimes necessary to determine characteristic curves with the alternator working on inductive loads.

The difficulties already referred to in connection with magnetisation curves on inductive load (see page 184) arise in the case of the characteristic curve with even greater force. It would

thus be extremely difficult in practice to take a series of readings of current with the power-factor adjusted in each case to a constant value.

If complete curves at different power-factors are required, and not an isolated point only on each curve (which is often enough for practical purposes), the same process of plotting auxiliary curves comparing power-factor and voltage must be resorted to in this case also.

The following description of an actual test will indicate the method to be adopted in obtaining complete curves.

The machine experimented upon was a 1 kw. 4-pole Crompton alternator, with rotating armature.

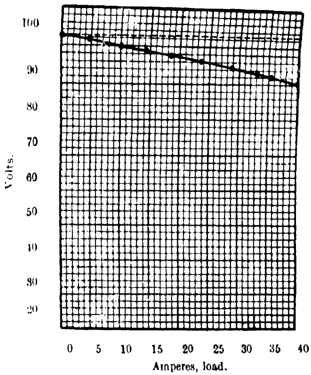


Fig. 112.—Characteristic of an Alternator

Excitation 1·9 amps. Speed 1,200 r.p.m.

(A) **Determination of Characteristic Curve on Inductive Loads.**—An inspection of the characteristic of the alternator on non-inductive load showed that four readings taken at about 4, 8, 12, and 16 amps. respectively, would be sufficient to enable the complete curves to be plotted.

Accordingly, a series of readings, each corresponding to a fixed value of the current, but each being taken with a different proportion of inductive and non-inductive resistance in the circuit, was taken.

The four series of readings were then plotted with voltage horizontal and power-factor vertical, as shown in Fig. 113. The actual values of the currents, for which these curves were drawn, were respectively 4·17, 8·28, 12·8, and 17·2, as these currents corresponded to scale readings of 4, 8, 12, and 16 amps. on the ammeter, which had a correction factor. From the curves obtained

in this manner, the lower characteristics shown in Fig. 115, for power-factors of ·9, ·8, and ·6, were plotted, each curve in Fig. 113 giving a single point on each characteristic. The general similarity between the curves in Fig. 113 and those drawn for a similar purpose in Fig. 109, comparing power-factor and excitation, will be noticed.

This was to be expected, since the variation in exciting current to maintain a given voltage must bear a fairly constant ratio to the drop of voltage, with a constant exciting current. The curves are, consequently, very similar in their significance.

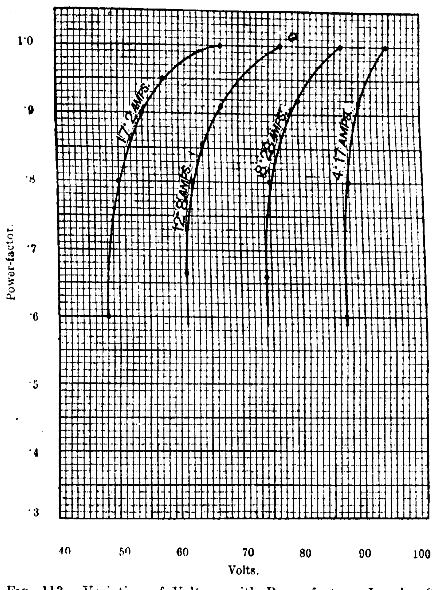


FIG. 113.—Variation of Voltage with Power-factor. Lagging Current.

(B) **Determination of Characteristic Curve with Leading Currents.**—The characteristic curves obtained in Fig. 115, which lie above the characteristic for non-inductive load, were obtained with currents *leading* instead of lagging behind the voltage. Such curves might be obtained by choosing as load a circuit possessing considerable capacity, such as a long concentric cable or a number of condensers, if it were possible to obtain sufficient capacity. The curves shown were determined in a manner which is usually more

convenient than this, viz., by connecting the alternator to a synchronous motor, the fields of which were excited so that the motor voltage was higher than the alternator voltage. It will be explained later in the section devoted to synchronous motors that over-excited synchronous motors produce a leading current in the circuit supplying them, and thus have an effect similar to that of condensers or cables with capacity. The amount of lead of the current can be adjusted by regulating the motor excitation, and the amount

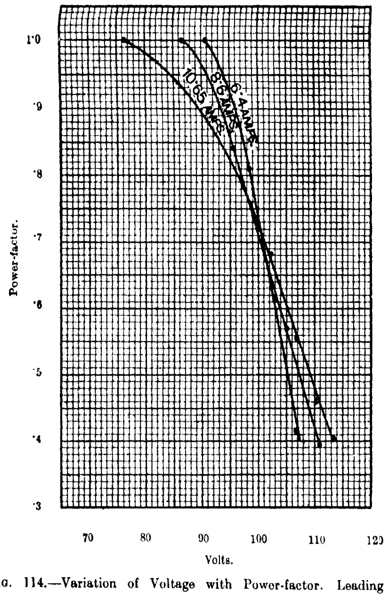


FIG. 114.—Variation of Voltage with Power-factor. Leading Currents

of current taken from the alternator can be easily varied by loading the motor by means of a brake upon its pulley or by making the motor drive a dynamo supplying current to a variable load circuit.

In taking the complete curves for several values of the power-factor, the same series of readings were taken as in the case of the inductive characteristics. It was comparatively easy to adjust the motor load and excitation to give a number of readings at different power-factors at a fixed current, whereas it would not

have been so easy to take a series of readings at a constant power factor but different load currents.

A number of readings were taken with different conditions of load and excitation at each of the following values of the current, 10·65, 8·6 and 6·4 amps. From the readings the curves comparing power-factor and voltage shown in Fig. 114 were plotted. The general shape of these curves is similar to that of those shown in Fig. 113; but in this case they cross each other.

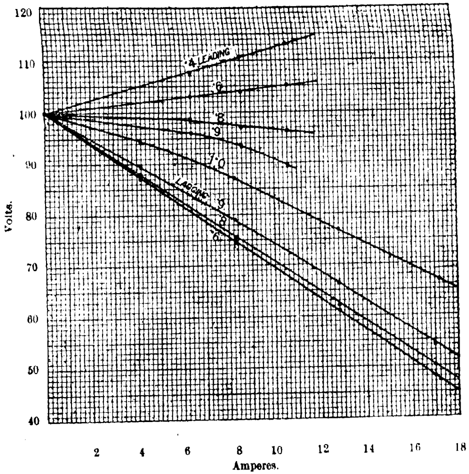


FIG. 115.—Characteristics with Various Power-factors.

One feature which will be noticed about the characteristics for lagging or leading currents is that the distance between the curves for $\cos \phi = 1·0$ and $\cos \phi = ·9$, is much greater than that between the curves for $\cos \phi = ·9$ and $\cos \phi = ·8$. In fact, the greater the angle of lag or lead, the less is the proportionate effect of decrease in the power-factor. This is owing to the fact stated on page 195, that the demagnetising action of the armature current is proportional to $\sin \phi$. The rate of increase of $\sin \phi$ is most rapid when ϕ is small and decreases as $\sin \phi$ approaches its maximum value of 1. Thus, a given increase in the value of ϕ produces a much greater effect when the actual angle of lag is small. Further, the angle whose cosine is ·9 is 26°, and the angle whose cosine is ·8 is 37°, so that by taking successive equal decrements of power-factor or $\cos \phi$, we are taking successively *decreasing* increments of ϕ

There is thus a double reason for the apparently rapidly diminishing effect of decrease in power-factor upon the slope of the characteristic, viz., each successive change in power-factor corresponds to a smaller change of angle of phase difference, and each successive change of angle corresponds to a diminished proportion of armature reaction.

Causes of Loss of Voltage under Load.—Briefly, the decrease in the voltage given out by the alternator when loaded is due to three causes :—

(1) Armature resistance, with which must be included eddy current losses in the conductors and in the iron in their vicinity.

(2) Armature reactance, due to the self-induction of the armature conductors.

(3) Armature reaction, by which is meant the magnetic field which is set up in the air gap by the armature current.

FIG. 116.—Armature Reactance Fluxes

It is important to distinguish clearly between the terms armature reactance and armature reaction.

When current flows in the armature conductors, a magnetic field is set up by the current. This field will consist partly of lines (*a*) forming closed loops round the armature conductors, completing their paths either across the face of the armature teeth or across the surface of the pole face opposite these teeth (see Fig. 116). The second part of the field due to the armature currents will be composed of lines (*b*) which form much wider loops, following the main path of the field magnetic circuit, and passing through the magnet iron carrying the magnet winding.

The important difference between these fluxes (*a*) and (*b*) is that the lines forming the smaller loops do not affect the amount of flux produced by the maget windings, whereas the longer lines (*b*) do affect the strength of the main field.

The *reactance* of the armature is due to the flux (*a*), while the *reaction* is caused by flux (*b*). ....

The reactance is unaffected by the power-factor of the load circuit. The effect of the armature reaction in strengthening or weakening the main field does depend on this power-factor.

(1) **Armature Resistance.**—The loss of voltage due to armature resistance may be determined as follows :—

While the machine is still hot from the run, send a direct current through the armature and determine the resistance, either by

accurately measuring the current with an ammeter and the voltage drop with a milli-voltmeter, or by comparing the drop with that in a standard resistance in the same circuit by means of a galvanometer.

To the resistance thus measured should be added a percentage depending on the type and speed of the machine, in order to allow for the power spent in producing eddy currents in the conductors, pole faces and armature core. It will be shown later that errors in computing the eddy current losses have a very small effect upon the calculated value of the armature drop.

Since it is not possible to measure directly the amount of the losses due to eddy currents, they can only be estimated from the results of experience in machines of similar type, unless a special experiment can be made to determine them.*

In the case of polyphase windings, it is desirable to make the measurements on one phase and to base calculations on the voltage lost per phase. In this connection see also page 301 dealing with the measurement of resistance in 3-phase windings.

(2) **Armature Reactance.**—The most accurate method of determining the armature reactance at various loads has been explained in Experiment VII., page 56, where current was passed through the stationary armature and the impedance voltage measured at its terminals. Unfortunately it is not usually possible to apply this test to any except small alternators. The alternating flux produced in the poles by current sent through the armature is capable of inducing a high voltage in the field windings which may become dangerous to the insulation of these windings, since these are only insulated sufficiently to withstand the relatively low continuous voltage employed for the excitation.

An approximate value of the reactance may be obtained indirectly from the open and short-circuit characteristics (see pages 204 and 205), or from a load curve (see page 181). In these cases armature reaction and reactance are measured together.

If t = turns per coil of the armature winding,

ϕ = virtual flux produced in one turn per ampere,

$L = \dfrac{\phi \times t^2}{10^8}$ is the coefficient of self-induction of the coil.

= number of lines produced by 1 amp. flowing through whole coil $\times$ turns which flux threads through $\div 10^8$.

The reactance of the coil is $2\,\pi\,f\,L$

$$= 2\,\pi\,f\,\frac{t^2\,\phi}{10^8}$$

* In small machines with low linear velocity the eddy currents are not likely to affect the behaviour of the machine seriously. They may therefore be neglected in comparison with the loss in armature resistance, or taken into account by adding a percentage to the losses in the resistance. An approximate rule sometimes used for larger machines is to assume the eddy current losses to be equal to the armature resistance loss.

The value of ϕ can be calculated with a fair degree of accuracy from the dimensions of the slots and coils. This calculation is given in books dealing with alternator design.

(3) **Armature Reaction.**—This term applies to the flux in the air gap of the alternator which is produced by the armature currents. A most important feature of the armature reaction is its dependance upon the power-factor of the circuit supplied by the alternator.

We shall first consider the case of a single-phase alternator.

Effect of Power-factor upon Armature Reaction.—As being the simplest case for purposes of explanation, the Fig. 117 has been drawn to represent part of a 2-pole single-phase alternator with ring-wound armature.

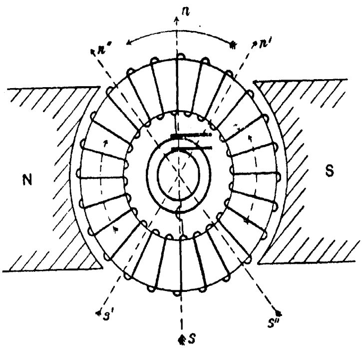


FIG. 117.—Diagram Illustrating Effect of Armature Reactions.

Case I.—Current and Voltage in Phase.—If current is supplied to a non-inductive circuit, there will be no phase difference between current and volts, and both will pass through their maximum and minimum values simultaneously, if the effect of the self-induction of the armature is neglected for the present.

With the armature in the position shown, the voltage will be at its maximum value. If the current is also at its maximum value (*i.e.*, if there is no difference of phase between the current and volts) the current will produce a field perpendicular to the main field, as indicated by the symbols *n s*. This armature reaction field will distort, but will neither strengthen nor weaken the main field. Earlier in the revolution, the armature field would be weaker, since the current would be weaker, and would be inclined to the

right, partly distorting and partly assisting the main field, as indicated by the axis $n'\ s'$. Later in the revolution the field will also be less, but will be directed so as to partly *oppose* the main field, its direction being, for instance, $n''\ s''$.

The average effect of the armature currents is thus to distort, but neither to strengthen nor weaken the main field.*

Case II.—Current Lagging.—In this case the current will only attain its full value after the position of maximum voltage has been passed, when, for instance, the conductors connected to the slip rings have moved into the position $s''\ n''$. The maximum current will then produce a field in the direction $s''\ n''$. Positions of the armature on either side of this will correspond to weaker fields, which will be of equal value at equal angles on either side of $n''\ s''$. Hence the average effect of the armature currents will be the production of a field along $n''\ s''$, which will partly *weaken* and partly distort the main field. This may be briefly stated thus: A lagging current tends on the whole to weaken the main field.

Case III.—Current Leading.—The current will in this case attain its maximum value before the armature reaches the position of maximum voltage, *e.g.*, when the conductors connected to the slip rings are at $n'\ s'$. The maximum armature field will be induced in the direction $n'\ s'$, and the average magnetic reaction will be such that the main field is both *strengthened* and distorted. Hence a leading current will on the whole strengthen the main field of an alternator.

Exactly similar reasoning applies to the case of a multipolar alternator, whether with fixed or rotating armature. This action necessarily makes it more difficult to regulate the voltage of an alternator with varying load when the power-factor of the circuit is low, *i.e.*, when there is considerable phase difference between current and voltage.

The foregoing reasoning may be put into more exact form by the statement, that if the ampere turns of the main magnetic circuit be $T\ I$ and the product of current and windings in the armature be $t\ i$, the effective (R.M.S.) value of the magnetising action of the armature currents will be $\cdot 7\ t\ i$, and the total effective magnetisation of the fields is then due to

$$T\ I - \cdot 7\ t\ i \sin \phi \text{ ampere turns,}\dagger$$

* The effect of decreased permeability owing to the concentration of the lines under one pole tip is here neglected.

† A multiplying factor must be introduced if the relative positions of windings and pole make the armature windings not all equally effective. The value of t may usually be taken to be

$$\frac{\text{total number of armature conductors}}{\text{number of poles}}$$

i.e., the armature current diminishes (or increases if ϕ be negative) the effective ampere turns of the field by the amount

$$\cdot 7\, t\, i \sin \phi,^{*}$$

where ϕ is the angle of lag of the current in the armature.

Armature Reaction of a Polyphase Alternator.—The armature reaction field of a polyphase alternator has a *constant value*, instead of being an alternating flux, as in the single-phase alternator. Also, with a given power-factor, it has a *fixed position* relative to the main poles, instead of rotating relatively to them, as was seen to be the case with the single-phase machine. We shall not attempt to prove these statements here, as the proof is the same as that given in connection with the rotating field of a polyphase induction motor on page 290. With a stationary armature, the currents produce a constant reaction field which rotates at the same speed as the main poles and in the same direction. If the armature revolves, the reaction field rotates relatively to the armature conductors in a reverse direction, and so remains stationary in space in the same way as the armature reaction field of a direct current generator.

The position of the armature reaction flux relative to the field poles depends on the phase difference between the armature current and induced electromotive force. If current and electromotive force are in phase, the reaction flux will have its axis midway between the poles as $n\ s$ in Fig. 118. This produces distortion, but neither strengthening nor weakening of the main field. If the current makes, with the electromotive force, a phase angle ψ, the reaction flux will be moved in the direction of rotation of the armature relative to the field by an angle ψ as indicated by $n'\ s'$. The reason for this change of axis has been already explained fully in connection with the single-phase alternator. If the current lags, there will be a definite number of armature ampere-turns tending to demagnetise the poles of the field, depending on the angle of lag. Similarly, if the current leads the electromotive force in phase, the reaction flux will strengthen the field. The converse of these statements is true for a synchronous motor, where a lagging

---

* This is obtained as follows :—

The actual current in the armature conductors may be written $i = I \sin(\theta - \phi)$ where ϕ is the angle of lag of the current behind the voltage, and θ is the angular distance of the conductors from the point corresponding to minimum electromotive force, I being here maximum value of current in conductor.

The demagnetising action of the current when the conductors are in any positon is proportional to the resolved component of the current parallel to main field, *i.e.*, to $\cos\theta$, since the magnetic field produced is perpendicular to the main field. Thus the field due to the conductors completely opposes the main field, when $\theta = 90°$, and assists it when $\theta = -90°$

Hence the demagnetising armature ampere-turns at any instant are

$$\begin{aligned} m &= t\, i \cos\theta = t\, I \sin(\theta - \phi)\cos\theta \\ &= t\, I\,(\sin\theta\cos\phi - \cos\theta\sin\phi)\cos\theta \\ &= t\, I\,(\sin\theta\cos\theta\cos\phi - \cos^2\theta\sin\phi) \end{aligned}$$

the average value of $\sin\theta\cos\theta = 0$

" $\cos^2\theta = \frac{1}{2}$.

Hence the average value of $m = -\frac{1}{2}\, t\, I \sin\phi$

$$= -\tfrac{1}{2}\, t\sqrt{2}\; i \sin\phi$$
$$= -\cdot 7\, t\, i \sin\phi.$$

current strengthens, and a leading current weakens, the main field In the case of a multipolar alternator, the armature reaction poles are equal in number to the field poles.

Let the alternator have N armature conductors per phase,

p pairs of poles,

m phases,

and let i virtual amperes flow in it.

Then the armature ampere-turns per phase per pole producing the reaction field

$$= i \times \frac{N}{2} \times \frac{1}{2p} \times k$$

where k is a factor depending on the arrangement of the conductors.

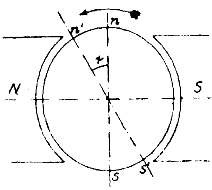


FIG. 118.—Armature Reactions in Polyphase Alternator

The resultant ampere-turns due to m phases will be (see page 293)

$$\frac{\sqrt{2}\, m}{2} \times \frac{i N k}{4 p} \text{ ampere-turns.}$$

The ampere-turns weakening the main field will be

$$\frac{\sqrt{2}\, m\, i\, N\, k}{8 p} \sin \psi$$

where ψ is the angle of lag of the current behind the total induced voltage.

In order to counteract this, and to maintain the same voltage as at no-load, the ampere-turns to be added to the field spools will be

$$\lambda \frac{\sqrt{2}\, m\, i\, N\, k}{8 p} \sin \psi$$

where λ is the leakage factor for the magnetic circuit.

As explained on page 210, ψ is nearly equal to ϕ, the angle of lag of the current in the load circuit, and may usually be considered as equal to it.

Experiment XXXII. — Determination of Excitation Required to Maintain Constant Voltage.

Diagram of Connections.

As for Experiment XXXI.

(See Fig. 111, page 186.)

Instructions.—Make connections as in Experiment XXXI. Run the alternator at normal speed with the load circuit open, and adjust the excitation to give the full voltage. Note the exciting current. Close the main-circuit switch and gradually increase the load to its full value. For each value of the load current adjust the field regulator so that the machine gives its normal voltage. Then read the load current and exciting current. The speed of the machine must be kept constant throughout the experiment.

If the machine is required to work on inductive loads, a few readings should be taken on inductive load, having the minimum power-factor on which the machine will be required to work.

The results of the test should be entered in tabular form as indicated below, and a curve plotted with load current horizontal and exciting current measured vertically.

Determination of Excitation for Constant Voltage.

Alternator No. .... Type ....

Normal output ...... amps. ...... volts at ...... revs. per minute.

Load Current	Voltage	Exciting Current	Speed
0	95	1·33	1,200
15	95	1·68	1,200
25·7	95	2·05	1,200
35·0	95	2·445	1,200

The readings obtained on a 6 kw. 3-phase generator with rotating field when loaded non-inductively on one phase only are plotted in Fig. 119. The voltage was kept constant at 95, and the speed regulated to give a periodicity of 40 cycles per second. The machine employed was the same as that for which the magnetisation curves, Figs. 105 and 107, and the characteristic, Fig. 112, were drawn. On referring to the magnetisation curves, it will be seen that the lowest excitation employed in the present experiment, viz., 1·33 amps., is well above the knee of the magnetisation curve, and that the ratio of increase in voltage to increase of excitation is fairly constant. Consequently, if the characteristic were straight, indicating a uniform drop of voltage as the load increased, the curve in Fig. 119 would also be practically a straight line. The characteristic in Fig. 112 begins, however, to bend downwards at about 20 amps., and consequently with currents above this value the

excitation must increase more rapidly in order to keep up the voltage to its original value. This is indicated by the upward bend in the excitation curve, Fig 119, occurring at 20 amps.

A curve of the kind just described shows the range ,which a field-regulating resistance must have in order to maintain constant voltage, and also shows what variation of resistance will be required for any given variation of load which may be contemplated.

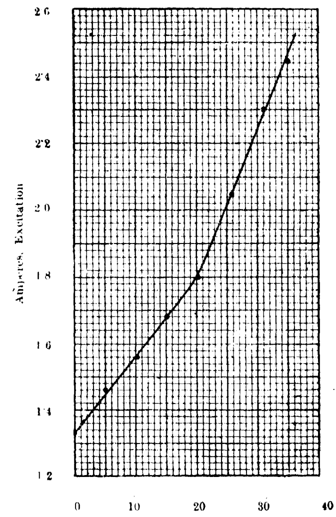


FIG. 119.—Variation of Excitation to Maintain Constant Voltage.

In making use of curves such as that shown in Fig. 119, it must be remembered that a different curve showing a greater increase in excitation would be obtained on an inductive load circuit, since the armature drop in this case would be greater.

Since the curve of variation of excitation may be approximately obtained from the results of the determination of the characteristic and magnetisation curves, it will in most cases be better to make use of these rather than to obtain a separate curve of regulation for inductive load by direct experiment.

The method of deriving the curve of regulation for an inductive load from those already taken would be approximately as follows

the method of procedure depending somewhat upon the information required, and the curves already obtained.

Suppose that the loss of voltage at a certain load, either inductive or non-inductive, has been obtained by experiment or calculation. On referring to the magnetisation curve, Fig. 105, page 178, the point on the curve corresponding to the excitation under the assumed conditions, must be found. A line is then drawn vertically upwards through this point, its length being equal to the loss of voltage on the volt scale. A horizontal line drawn through the upper end of this vertical line to meet the magnetisation curve will represent by its length the approximate increase in excitation required.

Short-circuit Characteristic.—If an alternator is only partially excited, and its terminals are short-circuited through an ammeter, the whole of the electromotive force generated in the armature is spent in overcoming the armature impedance.

The curve, comparing armature current on short circuit with excitation, forms a companion curve to the "open-circuit characteristic." From these two curves many important deductions may be made as to the behaviour of the alternator. These are discussed on page 203 *et seq.*

EXPERIMENT XXXIII.—DETERMINATION OF SHORT-CIRCUIT MAGNETISATION CURVE, OR SHORT-CIRCUIT CHARACTERISTIC.

DIAGRAM OF CONNECTIONS.

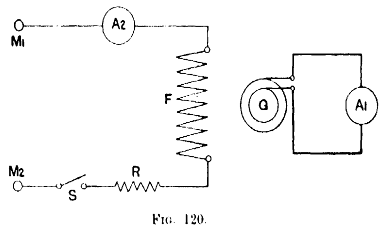


FIG. 120.

M_1, M_2 Source of direct current.
G Alternator armature.
F Alternator field windings.
R Field regulating resistance.
A_1, A_2 Ammeters.
S Switch.

Instructions.—Excite the alternator from a source of direct current through a regulating resistance, switch and ammeter. Connect the armature terminals together through an ammeter. Run the alternator at normal speed, exciting the field at first with

a small current. Read the ammeters, and while maintaining the speed fairly constant,* gradually increase the excitation and observe the readings of the ammeters for various values of the armature current, until the normal load current is reached.

Results should be entered in tabular form as indicated below.

DETERMINATION OF SHORT-CIRCUIT CHARACTERISTIC.

Alternator No. .... Type ....
Normal output ...... amps. ...... volts at ...... revs. per minute.
Frequency .... Normal excitation .... amps.

Exciting Current	Speed	Amperes on Short Circuit
·12	1,200	8·6
·23	1,200	15·45
·43	1,200	27·5
·55	1,200	35·0

The resistance of the armature, ammeter, and connecting leads should be determined by measurement with direct current or other means.†

A curve comparing exciting current and short-circuit armature current should be plotted from the readings. For moderate values of the armature current this will be found to be practically a straight line, but bends upwards rather sharply if continued for higher excitations.

Fig. 121 shows the results of a test carried out in the manner just described on a 6 kw. 4-pole Schukert alternator with rotating field, the speed being 1,200 revs. per minute, and the normal maximum current 35 amps. The curve is a straight line cutting the vertical axis at a point corresponding to 1·3 amps. This shows that the residual magnetism is sufficient to send a current of 1·3 amps. through the short-circuited armature.

A useful extension of the short-circuit test enables a measurement of the copper losses to be made. For this purpose, the generator is driven by a direct-current motor and observations are made of the power taken to drive it. By plotting values of the driving power against values of the alternator armature current the losses (which are almost all "copper losses") at any current loading may be obtained. The driving losses at zero current in the alternator armature, must be regarded as constant and must be subtracted from the driving power observed, to give the "copper losses" in the armature.

Effect of Speed Variation in Short-circuit Currents.—Since the reactance of the alternator armateur ($= 2\ \pi\ f\ L$) varies

* From the results shown in Fig. 122, it will be seen that when running at approximately normal speed, the short-circuit current remains constant for considerable speed variations. The importance of exact speed regulation does not therefore exist in this experiment.

† See remarks on page 193 for correction to be made for eddy currents.

directly with the frequency, *i.e.*, with the speed of the machine. variations in speed will alter the voltage generated in the same ratio as the reactance It follows that if the armature resistance is small, so that the armature impedance is composed almost entirely of reactance, the short-circuit current will vary very little with a change of speed, since the current will then be the quotient of the volts divided by the reactance, while both voltage and reactance vary proportionally to the speed.

The curve shown in Fig. 122 gives the result of a test carried out on a short-circuited alternator at varying speed. The curve shows that the short-circuit current is practically independent of speed over a wide range in the neighbourhood of the normal speed.

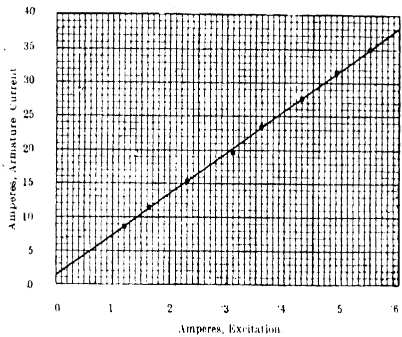


FIG. 121.—Short Circuit Characteristic.

The great importance of the short-circuit characteristic curve is its use in conjunction with the open-circuit characteristic for predetermining the pressure variation of an alternator under load. We must now explain the use of these two curves for this purpose.

Regulation of an Alternator.—There are two definitions employed in practice for the regulation, or pressure variation, of an alternator, viz. : (*a*) The fall in terminal voltage which results when the machine is loaded up to its full output, speed and excitation being kept constant, the excitation being that necessary for producing full voltage at *no-load*, (*b*) the rise in pressure at the alternator terminals which occurs when the full-load current is switched off the machine, the excitation having been previously adjusted to give normal voltage at *full-load*. This definition is the one adopted by the British Standards Committee.

The second definition gives a lower value for the voltage drop

than the first, since the magnets are more highly saturated in the second case.

Both values of the regulation are directly obtainable from tests already detailed. The regulation according to definition (*a*) is given directly by the characteristic curve (Fig. 112, page 188), since it is simply the length of the ordinate between the curve and the horizontal line drawn through the point of no-load voltage. The regulation is usually expressed as a percentage of the normal full-load voltage.

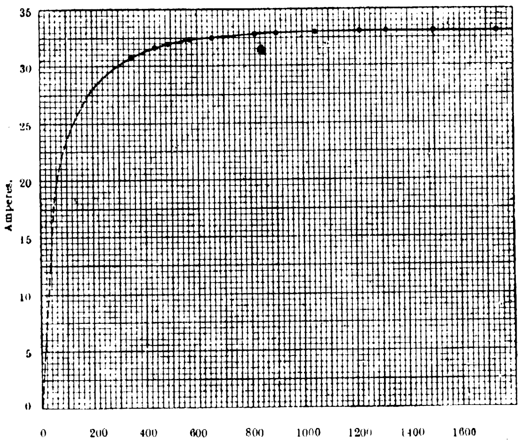


FIG. 122.—Variation of Short-circuit Current with Speed.

Excitation ·55 ampere.

The regulation by definition (*b*) is obtained from the curve Fig. 107, page 180, by noting the full-load excitation to produce normal voltage, and then seeking the no-load voltage corresponding to this excitation on the no-load curve (Fig. 105, page 178). The difference between the no-load and full-load voltage with this excitation is the required value, and may be expressed as a percentage of the full-load voltage.

Graphical Methods of Predetermining the Regulation of an Alternator from Open-circuit and Short-circuit Readings. —In addition to the drop due to resistance, the loss of voltage which occurs when the alternator is loaded is due partly to armature *reactance* and partly to armature *reaction*. It is difficult in practice to separate the voltages lost due to these causes. No graphic

construction can be accurate which does not make the distinction, since the load does not affect the reactance and reaction in the same manner. An accurate construction being beyond the scope of this book, we shall consider two simple approximate constructions in which the distinction between reactance and reaction is not drawn, and then refer the student to other sources where more advanced treatment of the problem is given. The two constructions given are only approximate, but are of importance because of their simplicity, and because they form a convenient basis on which more accurate, but more complicated, methods may be founded.

The two constructions are each based on one of the assumptions, that the idle voltage lost in the armature is due (*a*) entirely to armature reactance, or (*b*) entirely to armature reaction. The value of the regulation based on assumption (*a*) gives a loss of voltage greater than it should be, and this method is sometimes called the "pessimistic method" in consequence. The assumption (*b*) leads to a value of the drop which is less than the true one, and is therefore known as the "optimistic method." The reason for these terms will become clear from what follows.

(*a*) **Pessimistic or Reactance Method.**—In this case the armature current is assumed to produce no reaction flux, and the voltage drop is assumed to be entirely due to resistance and reactance of the armature, the main field maintaining its value constant. The reactance employed in the construction will be larger than the true armature reactance since it is made to account for the loss of voltage which is really due to armature reaction, *i.e.*, to the weakening of the main field, in addition to the loss which is due to the true reactance of the armature winding. This is called the *synchronous reactance* of the alternator. As the field is assumed to remain constant at all loads, the total voltage generated will also be constant, and will be the vectorial sum of the terminal voltage and of the voltage overcoming the armature impedance. The variation in terminal voltage is consequently obtained by the construction of a diagram of voltages as shown in Fig. 124, exactly as described for the transformer in a previous chapter (see page 139).

The reactance of the alternator is obtained by dividing the open-circuit voltage by the short-circuit current corresponding to the same field ampere-turns. It is evident that we should obtain different values for the reactance for different values of the excitation, because the curve of short-circuit current is practically a straight line (see Fig. 123), whereas the open-circuit volt curve is not.

In Fig. 123 are reproduced the open-circuit and short-circuit curves of the 6 kw. 3-phase alternator which have already been given in Figs. 105 and 121. The short-circuit curve is produced beyond the values observed, so as to extend over the full range of excitation. If we divide the open-circuit voltage generated at an excitation of 0·55 amp. (= 66·3 volts) by the full-load current of 35 amps., which is the short-circuit current at this excitation,

we obtain a reactance of $\frac{66 \cdot 3}{35} = 1 \cdot 9$ ohms nearly. If, however, we divide the voltage at normal excitation of 1·9 amps. (say 102·5 volts) by the short-circuit current obtained by producing the current curve to the same excitation (118 amps.) we find the reactance to be $\frac{102 \cdot 5}{118} = 0 \cdot 87$ ohms, or less than one-half the previous valuo. The latter figure is the best to adopt, since it corresponds to the normal saturation of the field.

We are now in a position to determine the regulation of the alternator at any load and power-factor. Taking first the case

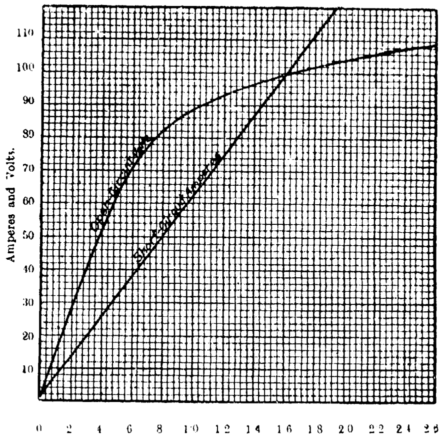


FIG. 123.—Open- and Short-Circuit Curves.

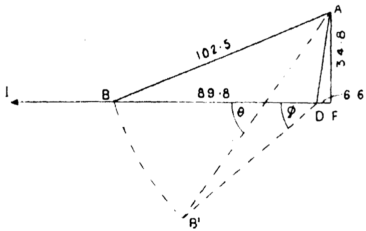
of unity power-factor and a load of 40 amps., the Fig. 124 shows the construction, where the triangle $A\ F\ D$ is first constructed so that

$A\ F = 40 + 0 \cdot 87 = 34 \cdot 76$ volts.

$F\ D = 40 \times 0 \cdot 156 = 6 \cdot 6$ volts, 0·156 being the armature resistance.

$A\ B$ is then drawn equal to 102·5 volts, cutting off a length $D\ B$ on the horizontal line representing the terminal volts. This length is found to be 89·8 volts. Referring to the experimental curve, Fig. 112, page 188, we see that the actual voltage at 40 amps. was 90·1 volts, so that our value is practically correct. Tho

discrepancy would be greater in most cases, especially with more fully saturated fields, the value then obtained being considerably less than the true value. We must now explain why any discrepancy between the diagram and the actual conditions should exist. The assumption is made that both reactance and reaction of the armature remain the same at all loads. Actually, both of these quantities are diminished as the saturation of the fields and the armature current increase. Since the value of the reactance employed in the diagram was calculated from readings taken on the alternator at low saturation,* its value is higher than it would be under full-load working conditions. For correct values of the regulation a different value of the reactance would have to be taken for each load and power-factor.



A F = Armature reactance volts.
F D = Armature resistance volts.
A D = Armature impedance volts
A B = No load voltage.
D B = Terminal volts under load of 40 amps. cos ϕ = 1.
D B' = Terminal volts under load of 40 amps. cos ϕ = 0·8.

FIG. 124.—Voltage Regulation Diagram.

In the case of a transformer, the magnetisation curve is practically straight within the working limits, and the reactance remains practically constant, so that the method just described will give satisfactory values for the regulation

For power-factors other than unity, the diagram becomes similar to that shown in dotted lines in Fig. 124. The student will have no difficulty in following this from the description already given, and a reference to the analogous case of the transformer The terminal voltage when supplying 40 amps at a power-factor of 0·8 is seen from the length of $D\,B^1$ to be about 73 volts

(*b*) **Optimistic or Ampere-turn Method.**—In this case the armature reactance is assumed to be zero, and the drop in volts of the alternator is assumed to be entirely due to resistance and to

* If the excitation in the short-circuit test could be carried up to its normal value, the current curve would be found to bend upwards as the point of saturation of the iron is reached. The actual straight-line curve is the result of measurements made with low excitation.

the effect of armature reactions on the main field in the air gap. The armature reactions are, therefore, taken greater than their true value, in order to account for the effects of reactance which are assumed not to exist. It is assumed that the effect of both armature reaction and reactance may be represented by a number of reactive ampere-turns, proportional to the armature current, and producing a magnetomotive force along the axis of the armature reaction field. The assumption made is that the total field ampere-turns may be looked upon as performing two functions, viz., (1) some are taken up in balancing the armature reaction ampere-turns, while (2) the remainder produce the air-gap flux which determines the value of the terminal voltage. The armature is assumed to be non-inductive, since the reactance is included in the reaction ampere-turns, and a correction for armature resistance is made subsequently.

The reactive ampere-turns of the armature are obtained directly from the short-circuit curve. It is assumed that, when on short circuit, the armature current lags 90° behind the induced electromotive force, and the axis of the armature reaction is the same as that of the main flux, so that the ampere-turns on short circuit directly oppose the field ampere-turns. Since there is then no terminal voltage, the field and armature ampere-turns are taken as being equal and opposite. Thus, for any value of the load current, the armature reactive ampere-turns are taken to be those actually supplied to the field in the short-circuit test at this current.

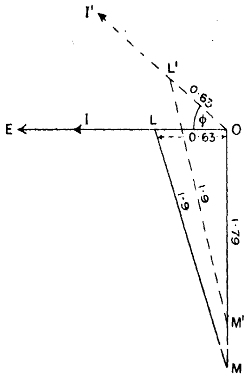
When working on non-inductive load, the armature reaction field and the main field are perpendicular to one another. We proceed as follows to determine the terminal voltage of the alternator, of which the curves are given in Fig. 123, for a non-inductive load of 40 amps.

Measure off $O\ L$ horizontally along the line $O\ E$ to represent the excitation required to overcome the armature reaction flux. This is the excitation corresponding to 40 amps. on the short-circuit curve, and is seen to be ·63 amp. from Fig. 123. From L describe a circle with a radius equal to the total field excitation (1·9 amps.) to cut the line drawn vertically through O at M. The triangle $O\ L\ M$ is then a triangle of ampere-turns, and $O\ M$ (=1·79 amps.) represents the excitation producing the terminal voltage. By reference to the magnetisation curve (Fig. 123), it is seen that an excitation of 1·79 amps. corresponds to a voltage of 101·8. We must, however, subtract the voltage lost in armature resistance, in order to obtain the terminal voltage. The armature resistance being 0·165 ohm, the volts to be subtracted are $40 \times 0{\cdot}165 = 6{\cdot}6$, and the terminal voltage is consequently found to be $101{\cdot}8 - 6{\cdot}6 = 95{\cdot}2$ volts. A reference to the experimental curve (Fig. 112, page 188), shows that this value is about 5 per cent. too high.

When the load circuit is inductive, the construction is carried out in the same way, except that the excitation overcoming the armature reactions is set off along a line $O\ C^1$, making an angle ϕ

This is shown by dotted lines in Fig. 125, where the alternator supplies a current of 40 amps. to a circuit having a power-factor of 0·8 The correction for armature resistance is made by subtracting $I\ R\ \cos\ \phi$ from the voltage which corresponds to the excitation $O\ M^1$, thus, in the present instance $O\ M^1$ represents 1·45 amps., corresponding to a voltage of 97·2. Subtracting $6{\cdot}6 \times 0{\cdot}8 = 5{\cdot}28$ volts, we obtain 91·92 volts at the terminals. This value may be compared with that obtained by the pessimistic method.

In the case of large alternators, the armature resistance must be taken from 50 per cent. (for closed slots) to 100 per cent. (for open slots) greater than the resistance as measured by direct current



O L = Excitation overcoming armature reaction.
O M = Excitation giving terminal volts. $\cos\ \phi = 1$.
M L = Excitation in field winding.
O M¹ = Excitation giving terminal volts, $\cos\ \phi = 0{\cdot}8$.

FIG. 125.—Ampere-turn Regulation Diagram.

in order to allow for eddy current losses in the poles and conductors. Further, a correction for change in the leakage factor of the magnetic circuit should be made.

The same form of diagram may be used to give the excitation necessary to maintain a certain terminal voltage under load. In this case, $O\ M$ is taken to represent the excitation necessary to give the required terminal voltage on open circuit. $O\ L$ is the excitation for overcoming armature reactions (given by the short circuit curve). $M\ L$ will then show the required excitation, which must, however, be increased so as to produce the additional voltage for overcoming armature resistance.

We must now explain why this method gives optimistic results. The assumption is made in the construction shown in Fig. 125 that by adding together the ampere-turns required to produce the no-load terminal voltage and those required to overcome armature reactions at zero terminal voltage, the resultant ampere-turns will be able to produce both effects simultaneously. This can only be true if the ampere-turns are always equally effective, independently of the saturation of the field, *i.e.*, if the magnetisation curve is a straight line. We know, however, that the higher the excitation, the less will be the effectiveness of the ampere-turns. Consequently by adding together the two component ampere-turns, as just described, we shall obtain a less resultant magnetisation than the sum of the magnetising effects due to the component ampere-turns acting singly. On this account the actual terminal voltage of the alternator when working under load will be rather less than the value obtained from our construction. This is what is meant by saying that the method is optimistic.

Conclusion.—It is evident that both of the methods just given would lead to the same result if the alternator had a straight line magnetisation curve. Since this is never the case, neither method is entirely reliable, although with large and well-designed machines the ampere-turn method is found to give fairly close results in practice. The divergence between the results of the two methods is more marked at low power-factors. For completely satisfactory results, the effects of reactance and reaction of the armature must be separately allowed for. For a more advanced discussion of the matter, the student may be referred to the following articles: Potier, Eclairage Electrique, vol. xxiv., 1900, page 133; Blondel and Fischer Hinnen, Elektrotechnische Zeitschrift, vol. xxii., 1901, pages 474 and 1,061; "Electrical Engineer," vol. xxxiii., 1904, page 979; Henderson and Nicholson, Proc. Inst. El. Eng., vol. xxxiv., 1905, page 465. See also Miles Walker, Specification and Design of Dynamo-Electric Machinery, p. 278 *et seq.*

Phase Difference between Electromotive Force and Load Current. Even when working on a non-inductive circuit, the phase difference between the current and the total voltage generated in the armature of the alternator will vary considerably with a varying load. This is perhaps brought out more clearly by the diagram Fig. 124, where the line *A B* represents the induced armature electromotive force and *D B* shows the phase of the current in the external circuit (since the external current and terminal voltage will be in phase with a non-inductive load circuit). The angle *D B A* shows the angle of phase difference between current and voltage generated in the armature itself.

The general result of an increase of load is an increase in the angle of lag between current and *total* volts,* until on short circuit it approximates to 90°.

* The student must note the distinction between "total volts generated in the armature" and "terminal volts." The terminal volts will, of course, always be in phase with the current in the case of a non-inductive circuit.

From the characteristic curve of an alternator on load, or from the short-circuit characteristic, the angle of lag between current and total volts can be approximately determined by the construction given on page 205, on the assumption that the total voltage generated remains constant at all loads.

It follows that the phase difference between current and armature electromotive force is not the same as that between current and voltage in the load circuit. This distinction is made in Fig. 124, where ϕ is the angle of lag in the load circuit and θ the lag of current behind electromotive force in the armature. In most practical cases the angles are sufficiently nearly equal for a distinction between them to be unnecessary.

Variation of Exciting Current.—Since the armature reaction flux is subject to pulsations due to the rotation of the field poles, it will induce a pulsating E.M.F. in the field windings, which will, in turn, produce current variations superposed on the continuous-current excitation. This effect may be observed by the oscillograph or ondograph. Another method of showing the effect of the armature current on the field is to trace the variation of the exciting current by a rotating contact as described in Experiment XIV.

Curves obtained in this way show that the excitation of a single-phase alternator, although supplied from a uniform source of electromotive force, is subject to regular pulsations. These pulsations have the peculiarity that they occur with exactly twice the frequency of the fluctuation of the armature current.

Efficiency Tests of Alternators.—The direct method of determining the efficiency of an alternator, by measuring the power supplied to drive it and the power given out in the form of electrical horse-power, is not generally adopted on account of the difficulty of measuring accurately the mechanical power supplied. The nearest approach to this method of measurement, which is frequently adopted in testing small machines, is to drive the alternator by means of a motor whose efficiency is known at various loads. By measuring the power supplied to the motor, and multiplying this by the motor efficiency, the power transmitted to the alternator is obtained.

In the case of large alternators the power necessary to drive the machine at full load is costly, and the absorption of the load current becomes a difficult matter. Also the measurement of the input and output with a sufficient degree of accuracy is by no means an easy matter.

It is consequently usual to calculate the efficiency of alternators from measurements of the losses occurring in the various portions of the machine, and to divide the output by the calculated quantity (output + losses) at various loads in order to arrive at a curve of efficiency.

Losses in an Alternator.—The chief losses in an alternator may be summarised under four headings: (*a*) Armature resist-

ance loss, (*b*) iron losses (hysteresis and eddy currents), (*c*) excitation losses, (*d*) friction losses (air and bearings friction).

The losses (*a*) can be calculated for any load by multiplying the armature resistance by the square of the current at that load, care being taken in the case of polyphase alternators to take the value of the current actually flowing in the windings, which is not always the same as that in the external circuit (*e.g.*, in a delta-connected 3-phase winding). In large machines, a value of the resistance must be taken greater than that which would be measured with continuous current (see page 208).

The losses (*b*) should be determined by a special experiment. Some of the most usual methods are given below.

An approximate calculation of the iron losses can be made in the same way as that described in the case of transformers, from the volume and induction of the iron subjected to the alternating flux.

The losses (*c*) can be directly calculated from the resistance of the field windings and voltage of the excitation.

The losses (*d*) can usually be determined at the same time as the iron losses.

Determination of Losses in Unloaded Alternator.—For the copper losses (*a* and *c*) the only determination to be made is that of the resistance of the windings, which is usually performed by sending a measured direct current through the windings and measuring the fall of potential in them. The resistance thus measured must then be multiplied by a factor depending on the size of machine and type of winding in order to make allowance for skin effect and eddy currents. In each case watts lost = $(\text{current})^2 \times$ resistance. These losses may also be obtained by observations of the power required to drive the alternator on short-circuit. (Experiment xxxiii., page 200.)

In order to determine the frictional and iron losses, the simplest method is to run the alternator as a synchronous motor. After running the machine up to normal speed and switching on the supply, the field is adjusted until the current taken is at its least value, and the power-factor of the driving circuit is consequently high. The power taken by the armature under these conditions is read on the wattmeter, and is the sum of the losses due to friction, hysteresis, and eddy currents. The copper losses in the armature due to the driving current will probably be too small to be worth subtracting. Allowance can easily be made for them from the known resistance of the armature and the value of the current, which must in any case be observed in order to ascertain whether the excitation is correct. Assuming (which is nearly though not accurately true) that the friction and iron losses remain the same at all loads, the total loss is obtained for any load current by adding together the watts thus measured and the calculated copper losses in the armature and field.

Another method which is easily carried out, if a direct-current motor of known efficiency is available, is the following :—

The motor is coupled to the alternator, and made to drive it at the full speed of the alternator. If the alternator fields are unexcited, there will be practically no iron losses, and the whole of the power supplied by the motor will be due to mechanical friction. If the normal excitation of the magnets is subsequently applied, the increase of power given by the motor in order to maintain the same speed will be equal to the iron losses of the alternator.

If the alternator is provided with a direct-coupled exciter, it is convenient to employ this exciter as the motor in the test, after having made a careful determination of its efficiency. The quantities to be measured are small and liable to variation, great precautions must therefore be taken to run the alternator as nearly as possible under working conditions as regards temperature, &c., and only to take readings after the machine has been rotating for some time, so that the bearings may be thoroughly lubricated. The first readings should be repeated several times between the other readings, so as to ensure that the conditions have remained constant.

This test is often carried out in the following way :—

The motor is made to drive the alternator unexcited. The power supplied to the motor after subtraction of the armature copper losses W_1 is noted. The alternator fields are then excited, and the power taken by the motor (again corrected for armature resistance loss) to maintain the same speed is noted ($= W_2$). The motor is disconnected from the alternator, and the power required to drive it when running light is noted ($= w$). Then the friction losses of the alternator are taken to be ($W_1 - w$), and the iron losses ($W_2 - W_1$). The excitation of the motor should be maintained practically constant during the three measurements. It is assumed that the losses in the motor (except armature resistance loss) are the same at no-load and when driving the alternator. The motor should be large enough for these assumptions to be justified.

The Retardation Method of Measuring Iron Losses.—This is a method which is especially applicable to alternators of the flywheel type. The machine is brought up to speed by means of a motor and belt, or by its own exciter. The belt is then slipped off, or current shut off from the exciter, as the case may be, and observations are taken of the rate at which the machine slows down, by reading a tachometer at equal intervals during the retardation. Curves plotted from these readings give the information required to enable the iron losses to be determined. If the machine is not of the flywheel type, it can sometimes be conveniently tested by the same method after mounting on the shaft a flywheel of sufficient weight to render the time of retardation long enough for readings to be taken. A curve of retardation (plotted with time horizontally and the tachometer readings vertically) is first obtained from the machine when allowed to slow down from full speed to rest with the fields unexcited, *i.e.*, under the action of air and bearing

friction only. A second curve is then taken in a similar manner but with the fields excited. The higher rate of retardation which will occur in this case is due to the effect of hysteresis and eddy current.

The principle of this measurement is as follows :—

The energy of rotation possessed by a rotating body is

$$W = \tfrac{1}{2} I \left(\frac{2 \pi n}{60} \right)^2$$

where I is the moment of inertia of the body,

n is its speed in revolutions per minute.

If the moment of inertia is measured in units of the absolute C.G.S. system, *i.e.*, in gramme-cm.^{2}, the energy is expressed in

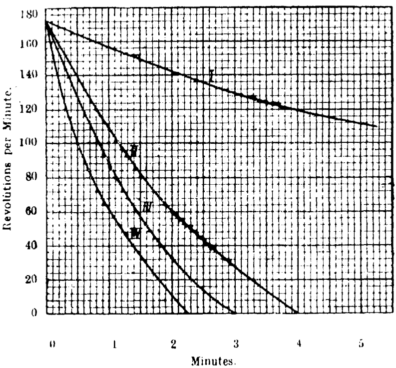


FIG. 126.—Retardation Curves of an Alternator.

I.—Unexcited.
II.—Excitation 27 amperes.
III.— ,, 34 ,,
IV.— ,, 45 ,,

the same system as dyne-centimetres or ergs. In order to bring the ergs to kg. meters we must divide by $9{\cdot}81 \times 10^7$, or if to watt-seconds by 10^7, or if to foot-pounds by $1{\cdot}356 \times 10^7$.

Thus the energy of a rotating body in watt-seconds or joules is

$$W = \tfrac{1}{2} I \left(\frac{2 \pi n}{60} \right)^2 \times 10^{-7} = {\cdot}545\, I n^2\, 10^{-9}.$$

If the speed of the alternator when slowing down under the influence of the frictional and other losses is observed at successive intervals of time, the difference in the energy possessed by the

machine at two periods is a measure of the work done in overcoming the losses. The amount of this work divided by the time between the observations is the mean power thus absorbed. The power absorbed will, of course, grow less as the speed diminishes. By plotting a curve of mean power for a number of speeds, obtained as the machine gradually slows down, the losses at full speed may be obtained by continuing the curve of losses backwards to cut the ordinate corresponding to full speed.

Thus, for example, readings were taken every half-minute in obtaining the upper curve in Fig. 126 ; the reading at full speed was 172 revs. per minute. The next reading after half-minute was 164. The stored energy at full speed was $\cdot 545\, I\, 172^2 \times 10^{-9}$ watt-seconds. The energy lost during the first half-minute was $\cdot 545\, I\, (172^2 - 164^2) \times 10^{-9}$. The average power absorbed by the losses at a speed whose average value was 168 revs. per minute, was therefore

$$\frac{\cdot 545\, I\, (172^2 - 164^2)}{30 \times 10^9} \text{ watts.}$$

From a series of such values a curve was plotted with watts vertical and speeds horizontal, showing the watts lost at various speeds, the watts lost at full speed being then taken from the point where the curve would cut the ordinate corresponding to full speed. It is to be noted that such a set of readings gives the watts in terms of I, the moment of inertia of the rotating parts. In order to eliminate this unknown factor, a separate observation is made to determine the total friction and iron losses at full speed. This is usually done in the manner already alluded to, by running the alternator as a synchronous motor and measuring the power absorbed by means of a wattmeter. This observation then gives the value of the total losses at full speed, and enables the actual value of all the other proportional readings to be assigned. The iron and friction losses can thus be obtained at any speed. By taking readings with several values of the excitation, the iron losses at varying induction are obtained.

For a full discussion of the determination of losses by the method of retardation, the reader may refer to C. F. Smith, " Experimental Determination of Losses in Motors," Proc. Inst. Elec. Eng., Vol. 39, page 437.

Determination of Losses under Load Conditions.—The losses measured under no-load conditions will always be somewhat less than when under load, the iron losses always increasing somewhat with load. The difficulty of correctly assigning the value of the copper losses has already been spoken of.

Various methods of testing alternators with the full current flowing, both for the purpose of finding the temperature rise and also for measuring the losses under full-load conditions, but with small expenditure of power, have been proposed, a few of the simpler methods are now to be described.

Method I.: Alternator Run as Synchronous Motor.—It has already been stated that the iron and friction losses may be simply determined at no-load by running the alternator as an unloaded synchronous motor and measuring the power taken by means of a wattmeter. If under the same conditions, the voltage applied to the machine is gradually increased, by raising the excitation of the generator providing the supply, the alternator under test will still continue to run at the same speed and with the same excitation, but will take an increased armature current at a continually diminishing power-factor. If the voltage applied is increased until the machine takes its full-load current, the losses occurring in it will be practically those which would exist if the alternator were supplying full load as a generator, and may be measured on the wattmeter as before. A temperature run may consequently be taken in this way without unnecessary expenditure of power. In practice, it may be difficult to obtain satisfactory readings for the losses on account of the low power-factor of the circuit in which they are read.

Method II.: Two Similar Alternators not Coupled.—The difficulty mentioned in reading the alternating power supplied to the synchronous motor may be overcome if two similar alternators are available. One machine is employed as a generator, and is driven by a continuous-current motor having known losses. The other alternator is run from the first one as a synchronous motor at normal excitation. By raising the excitation of the generator, any desired current may be made to circulate between the machines. The increased loss in the two alternators will be indicated by the increase in power supplied to the continuous-current motor. If, as is preferable, the synchronous motor excitation is decreased by about the same amount as the generator excitation is increased in order to produce full-load circulating current, one-half of the additional power given to the alternator by the continuous-current motor may be taken to be the losses due to the load on one alternating machine. The losses are in this way measured in a direct-current circuit.

Method III.: Two Similar Alternators Coupled Together.—If the machines are arranged as in the preceding method, but a rigid coupling is introduced between the alternators, the test may be carried out as already described, but with the additional advantage that the power-factor of the circuit connecting the machines may be maintained at a constant value. If the alternators are coupled in direct opposition, the power-factor will be practically unity, and the machine having the greater excitation will be the generator. By altering the angle of coupling the power-factor is altered, while at the same time either machine may become the generator irrespective of the relative excitation. The generator will be the machine which is more than 180 electrical degrees in front of the other in direction of rotation. This relation will be understood by referring to the chapter on synchronous motors, page 235.

Method IV.: Differential Connection of Armature Winding in a Single Alternator.—Mordey has suggested that a differential test similar to the ones already described may be carried out on a single alternator, by dividing the armature winding into two unequal sections and joining them together so that their electromotive forces oppose each other, the entire winding being then short-circuited through an ammeter. A circulating current is obtained because of the difference between electromotive forces in the two sections, and its value may be adjusted by means of a choking coil when the fields have their full normal excitation. For further particulars of this method the reader may be referred to Proc. Inst. Elec. Eng., Vol. XXII., page 116; Elec. World and Eng., Vol. XLII., page 715; Elektrotechnische Zeitschrift, Vol. XXII., page 682, also S. P. Smith, Proc. Inst. E.E., Vol. 42, page 190.

Running Alternators in Parallel.—Alternators can be run in parallel if brought to the same speed and voltage, and if switched together on to the supply circuit when exactly coincident

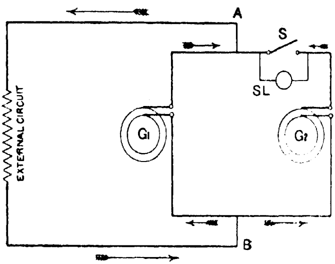


FIG. 127.—Use of Lamp as Synchroniser.

in phase. This is owing to the fact that if one machine tends to lag behind the other, it will receive current from the other tending to drive it as a motor, and it is thus prevented from falling further out of step. If the machines are not driven with exactly the same regularity they will vary slightly in phase, and in doing so the leading machine will always supply current to the lagging one, which will have the double effect of slightly retarding the leading machine and accelerating the lagging machine. The difference in phase between the two machines must never exceed ¼ period, or they will fall out of step.

The condition for parallel running is, therefore, that with a small angular displacement between the machines, the current sent by the leading machine must be sufficient to prevent the displacement becoming greater.

Synchroniser.—In order to ascertain when two machines are running at the same speed and are in phase, so that they may

be switched into parallel, a synchroniser is employed. The principle of its action is now to be discussed.

Fig. 127 represents two alternators G_1 and G_2 connected in parallel to an external circuit A B, of which A represents one main conductor and B the other.

Obviously, in order that the machines may both supply current to the circuit A B, the terminals, which are simultaneously of the same sign, must be joined to the same conductor. This is shown in the diagram, where, at the instant represented, the upper terminals are both +, and are connected to A.

The conductors joining the alternators form a second closed circuit distinct from the main external circuit. The two machines generate electromotive forces tending to send currents in opposite directions round this smaller circuit. At the instant represented on the diagram, the directions of these electromotive forces will be those represented by the arrows. Consequently, when the machines are in correct phase for working in parallel, there will be equal and opposite electromotive forces acting in this local circuit, having a resultant electromotive force equal to zero. If the machines were wrongly connected together (*i.e.*, + to — instead of + to + and — to —) the resultant electromotive force given to the external circuit A B would be zero, while the electromotive force in the local circuit joining the machines would be twice the voltage of either machine.

The condition for switching in parallel is that of *no resultant voltage* in the local circuit joining the machines—*i.e.*, the machines must have:—

(1) Equal voltage.
(2) Equal periodicity.
(3) Equal phase.

The simplest method of determining the condition is indicated in Fig. 127, which shows an incandescent lamp S L connected in parallel with the switch S employed for connecting the machines together.

This lamp serves to show whether there is any resultant voltage in the circuit joining the alternators, since it glows when the machines are in series (*i.e.*, joined + to —), and ceases to glow when they are truly in parallel, and when the resultant voltage is consequently zero.

If the two machines run at different speeds, the lamp will appear alternately bright and dull, the changes in the lamp occurring less frequently as the difference in speed becomes less, and as the time taken for the faster-running machine to catch up the slower machine becomes greater.

A voltmeter with a range equal to double the normal voltage of the alternators may be used instead of the lamp, if it is provided with a damping device to prevent the needle swinging too much, and if its moving parts are light and free enough to follow rapidly the variation of voltage.

It is important to notice that in the direct form of synchroniser

just described, parallelism is indicated by zero reading on the voltmeter or darkness of the lamp. A single-pole switch and single lamp are shown in Fig. 127. Usually a double-pole switch with a lamp across each break in the circuit would be employed. The lamps would, in this case, be in series with one another, and would, consequently, only receive half as much voltage, *i.e.*, the full voltage of one alternator as a maximum.

Synchronisers Employing a Transformer.—If the pressure of the alternator is too high to make direct connection to a lamp possible, a transformer may be used, the primary winding taking the place of the lamp shown in Fig. 127, and the lamp being connected in series with the secondary, in order to reduce the pressure applied to the lamp. The action is exactly the same as that just described, since the resultant voltage of the lamp will be nil when the alternators are in phase, and the lamp will not glow under these conditions.

The following modification (see Fig. 128) is of more general use, as a 2-pole switch may be used with a single lamp or voltmeter,

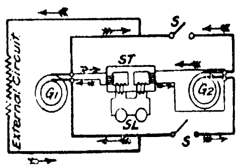


FIG. 128.—Diagram of Synchroniser.

and the lamp can be made to glow at coincidence or at opposition in phase, as desired.

The generators are connected to separate equal windings on a small transformer. Separate transformers are frequently used instead of the single transformer with double magnetic circuit shown in Fig. 128. The lamp or voltmeter is connected in series with the secondary windings of the transformers which have fewer turns than the primary windings, and consequently giving a lower voltage.

The voltage given to the lamp will be the sum of, or the difference between the voltages which would be produced by the windings acting singly, according to the relative direction of the voltages in the windings. It depends how the alternators are connected to the primary windings, whether the lamp in the secondary will be bright or dull when the machines are in parallel. It is usual to choose the connection so that the voltmeter reads its maximum, or the lamp burns most brightly, when the machines are in phase.

When more than two alternators have to be connected so as to be capable of being run in parallel, as is usually the case at a large generating station, the connections have to be modified somewhat from those given above. Since it may be necessary to

run any machine with any other, or with several other machines, a simple means must be provided whereby the low-pressure winding of the synchronising transformer of any machine may be readily connected to that of any other. The usual method adopted is to have synchronising bus-bars extending across the back of all the generator switch-board panels, as shown in Fig. 129, where G_1 G_2 represent the armatures of two of the machines, of which there may be any number connected to the switchboard in a similar manner.

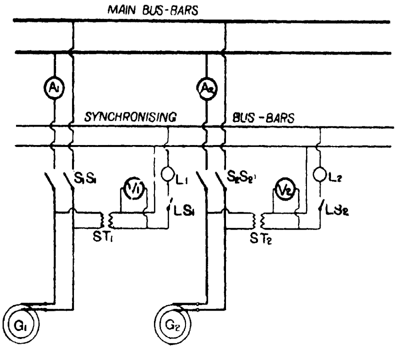


FIG. 129.—Connections for Synchronising Alternators.

The letters given on the diagram, Fig. 129, have the following meanings :—

G	Alternator armature.
S S	Main 2-pole switch.
A	Alternator ammeter.
V	Alternator voltmeter.
S T	Synchronising transformer.
L	Synchronising lamp.
L S	Synchronising lamp switch.

It will be noticed that the generator voltmeter is shown connected to the secondary of the synchroniser transformer. This arrangement has the advantage that an ordinary low-pressure voltmeter may be used, while its dial may be graduated so as to indicate the full pressure of the alternator, since the actual voltage at the voltmeter terminals is always a definite fraction of that generated by the alternator.

The alternative connections for synchronising with lamps bright and dark are indicated in a slightly different form in the annexed Figs. 130 and 131. In Fig. 130 the alternators are synchronised with lamp dark. In Fig. 131 the machines are in synchronism when the lamps are bright.

Connections for Polyphase Alternators.—Figs. 132 and 133 show connections for two and three phase alternators, drawn so as to correspond with Figs. 130 and 131, the switches and bus-bars being omitted. In a polyphase machine there is a possibility that the relative rotation of the machines may be reversed ; this is consequently a matter to be carefully watched. When lamps are connected as shown between a pair of 2-phase alternators the

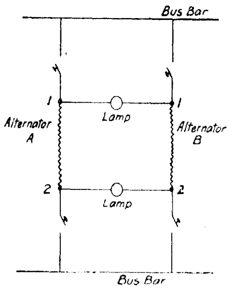


FIG. 130.
LAMPS DARK.

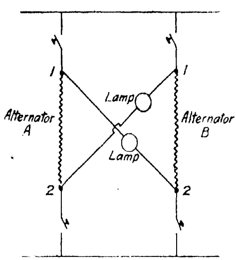


FIG. 131.
LAMPS BRIGHT.

Connections for Synchronising.

lamps will glow and grow dull simultaneously If the machines are symmetrically connected together and rotate in the same direction. lamps are dark at the correct moment for synchronising. If the lamps are to be bright when the machines are in phase they must be connected as follows, viz., between

Terminal	1	of machine A	and terminal	2	of machine B
,,	2	,,	,,	1	,,
,,	3	,,	,,	4	,,
,,	4	,,	,,	3	,,

In this case, also, the lamps will light up simultaneously.

Fig. 133, for 3-phase alternators, shows the connections of synchronising lamps in which they will vary in brightness simultaneously, and synchronising is accomplished with all lamps out.

In order to get simultaneous variation in brightness and correct conditions for synchronising with the lamps all bright, a symmetrical change of the connections of the lamps must be made, which is best shown by tabulating the connections as follows :—

Machine A	1	to	Machine B	2
,,	2		,,	3
,,	3		,,	1

It is to be observed that the figures in each column are in the same order, and the connections are thus symmetrical.

The lamps will now always be equal in brightness, and will glow with maximum brightness at the correct moment for switching in parallel.

When running in parallel, terminals indicated by similar numerals will be joined to the same bus-bars.

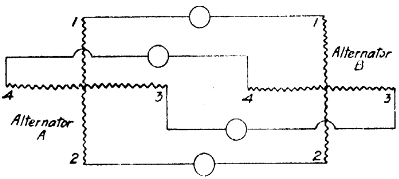


FIG. 132.—Connections for Synchroniser. 2-phase.

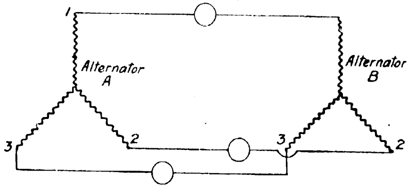


FIG. 133.—Connections for Synchroniser. 3-phase.

Any unsymmetrical arrangement of connections, such as

Machine A	1	to Machine B	1
,,	2	,,	3
,,	3	,,	2

will cause the lamps to glow successively, instead of simultaneously. An advantage of this connection is that the order of lighting up depends on the relative speeds of the machines to be synchronised, so that by watching the lamps it is possible to tell whether the machine which is about to be connected to the bus-bars is running too fast or too slowly, and thus the engine attendant can see at once whether he should give more or less steam to the driving engine.

The maximum voltage across any lamp is equal to the alternator terminal voltage, *i.e.*, $\sqrt{3}$ times the phase voltage in the case of a star-connected alternator.

The diagrams discussed above are shown with the synchronising lamps connected directly to the alternator terminals. In principle the introduction of synchronising transformers to reduce the voltage of the lamps makes no difference, the various combinations described being carried out in exactly the same way as with the directly connected lamps shown.

In order to synchronise an alternator with another which is already running at normal speed and voltage, run up the current to about the correct speed, and adjust its field current until the voltages of the machines are equal.

The lamp will probably blink rapidly. The speed must now be adjusted in the direction giving less rapid flickering. The lamp should then go completely out and light up brightly more and more slowly. When the action is sufficiently slow and marked, so that in the case of a small machine two or three seconds elapse between each period of brightness, close the double-pole switch as the lamps are brightest (if this is the condition when in synchronism). Then note the sudden throw of the ammeter in the circuit between the machines, and after this has assumed its steady reading, regulate the field of the alternator until the ammeter reading is as low as can be obtained. For large machines it is essential that the speeds should be practically identical before switching in.

After this, the switch in the load circuit may be closed, and the machines tested under load.

The machines should divide the load between them in the proportion to their rated output for considerable variations in the load without subsequent regulation of the excitation.

It may be desirable to test this by increasing the load gradually, and noting the current given by each machine without alteration of the field regulators.

Special forms of synchronisers, indicating whether the alternator runs too fast, or too slow, are employed in practice where the alternators are permanently installed, but their description hardly comes within the scope of this book.

CHAPTER VIII.

SYNCHRONOUS MOTORS

A Synchronous Motor is in construction similar to an alternator. It requires consequently a supply of direct current for exciting the magnets, and a supply of alternating current to the armature.

Mode of Operation.—Let Fig. 134 represent the armature of a 2-pole alternator or synchronous motor. If current is supplied at the brushes so as to flow in the direction indicated, the action will be like that of a direct-current motor. The armature

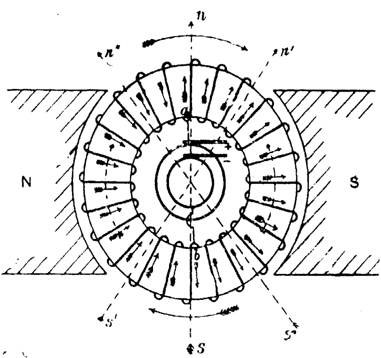


FIG. 134.—Diagram of Armature Current in Synchronous Motor.

will exhibit the polarity indicated in the Figure by the arrow n s, and rotation will take place in the direction shown. If current flowed in the opposite direction through the armature, rotation would take place in the opposite direction. A rapidly alternating current supplied to the armature while at rest would produce a series of impulses tending to drive the armature first in one direction and then in the other. The result would be a rapid vibration of the armature, but no rotation, the inertia of the moving part being too great to allow it to move appreciably from the neutral position between each alternation of the current. Suppose, however, the

2-pole armature shown in Fig. 134 were made to rotate at exactly such a speed that it made half a revolution in the time taken by the current to change from its maximum value in one direction to its maximum value in the opposite direction. In that case the direction of the current would be reversed each time the position of the conductors *a*, *b* connected to the rings was reversed. Consequently, the polarity of the armature after half a revolution would be the same as before.

When followed out in detail, the action of the currents would be as follows: Beginning with the armature in the position shown, suppose that the current is at its maximum value, and flows as shown by the small arrows. The poles induced in the armature core will be in the positions indicated by *n s*. As the armature rotates in a clockwise direction the current dies down, because it is an alternating current. After a quarter of a revolution the current will have become zero, and the armature poles will at the same time have gradually diminished in strength to zero while approaching the centre of the field poles. After the conductors *a*, *b* have passed the centre of the field poles, the armature current gradually increases, but flows in the opposite direction, so that the points in the armature marked *n* and *s* are now respectively of south and north polarity. With increasing armature current and the simultaneous movement of the conductors connected to the slip rings towards the mid-position between the poles, the turning moment increases, until the newly-formed south pole reaches the position *s* on the diagram, formerly occupied by the opposite point of the armature. The cycle now repeats itself.

The revolution of the machine would under these conditions be maintained by the current. The continuance of the rotation depends upon coincidence of the reversal of the direction of the current and the reversal of the position of the armature. If the current reversed too soon or too late, the polarity induced by the current would oppose the rotation of the armature during part of the revolution. If the *rate of alteration* of the current were changed relatively to the speed of the armature, the current would sometimes assist and sometimes oppose the rotation, and the motor would probably cease to rotate.

The essential point about a synchronous motor is that it can only operate when rotating *synchronously*, or in step with the current supplied to it.

It will be well to mention here that in the case of a *polyphase synchronous motor*, the combined effect of the armature currents is to form a *constant* (not alternating) armature polarity. This polarity has a *fixed position* relative to the fields, which is again unlike the single-phase motor. Consequently, whereas the single-phase motor has a rapidly varying torque, the polyphase motor develops a practically constant torque.

The torque of the polyphase synchronous motor is thus produced in a manner corresponding closely to that of the direct

current motor, so far as the relation between the polar field and armature field is concerned.

We shall not here attempt an explanation of this action. It will be understood after the student has read pp. 288 to 293, which explain the production of a rotating field by a polyphase winding. (See also p. 196, dealing with armature reactions of a polyphase alternator.)

Effect of Increase in Number of Poles.—We have seen that a 2-pole synchronous motor will make exactly one revolution for each cycle of the alternating current supplied to it. Thus a 2-pole machine supplied with current having a periodicity of 25 would make $25 \times 60 = 1{,}500$ revs. per minute.

If the motor has a greater number of poles, the speed will be such that an armature conductor passes from one pole to the next of the same character in the time of each period of the current. Hence, if the machine has 12 pairs of poles, it will take the time of 12 periods to make one revolution. Thus,

$$\text{Speed of motor} = n = \frac{f \times 60}{p} \text{ revs. per minute.}$$

When f = periodicity of current supplied.
p = number of pairs of poles.

Method of Starting a Synchronous Motor.—As just explained the action of a synchronous motor depends on its running at the speed of synchronism, and the machine will not rotate if switched on to an alternating supply while stationary. It must, therefore, be started by same external means from rest, and, when it has been brought up to the correct speed, it may be switched on to the alternating circuit, and will then continue to rotate.

Synchronous motors are often started by means of direct-current motors either intended for that special purpose, or intended as direct-current generators for supplying the exciting current required by the synchronous motor. The use of a direct-current motor necessitates a supply of direct current which is sometimes not available before the machine is started, *e.g.*, in cases where the motor drives its own exciter. Very frequently small alternating current induction motors are installed for the purpose of starting the synchronous motors.

Sometimes a synchronous motor can be started from rest by the action of the eddy currents formed in the pole faces of the magnets. This can only be done in the case of 2- or 3-phase machines, and will be refered to in discussing them.

In cases where only a single motor is to be driven by an alternator, as is frequently the case in testing, the motor may be made to start by running the alternator very slowly, and then giving the motor a few sharp turns by hand to bring it to the reduced speed of the alternator. In starting the motor in this way it will be found easiest to turn the motor by hand without excitation, and to switch on the field when sufficient speed is attained for it to

fall into step with the alternator. If the alternator speed is then gradually increased, the motor will speed up and keep in step.

If a synchronous alternating-current motor is to be connected to an alternating circuit, it must first be run up to the *speed* of synchronism, its *field* must be adjusted to the correct amount, and the switch connecting it to the supply must be closed when the *phase of the motor* corresponds to that of the circuit.

The process is consequently similar to that adopted in synchronising two alternators (see page 217).

The following experiment shows the effect of varying the excitation of a synchronous motor, which is a matter of much importance.

EXPERIMENT XXXIV.—DETERMINATION OF THE EFFECT OF VARIATION OF THE EXCITATION OF A SYNCHRONOUS MOTOR.

DIAGRAM OF CONNECTIONS.

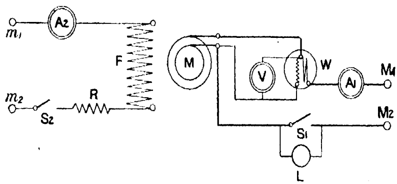


FIG. 135.

M_1, M_2	Source of alternating current.
m_1, m_2	Source of direct current.
M	Motor armature.
F	Motor field windings.
A_1	Ammeter reading armature current.
A_2	Ammeter reading exciting current.
V	Voltmeter reading armature voltage.
W	Wattmeter reading power supplied to armature
R	Field regulating resistance.
S_1, S_2	Switches.
L	Synchronising lamp.*

Instructions.—Excite the motor field from a source of direct current through a regulating resistance, ammeter, and switch. Connect the armature to the source of alternating current through a switch, ammeter, and series coil of a wattmeter. Connect a synchroniser in parallel with the switch, for synchronising the motor before switching the armature on to the supply. Connect a voltmeter and the volt coil of the wattmeter across the terminals of the armature.

Run the motor, close the switch in the exciting circuit, and

* The synchroniser will frequently be of the more complete type indicated in Fig. 128. A double-pole switch will in this case generally be employed.

synchronise as described in the previous chapter, closing the switch in the armature circuit when synchronism is obtained.

Next determine the most favourable excitation by varying the exciting current until the armature current indicated by A_1 reaches its lowest value.

The experiment should now be continued as follows: With the motor running on no-load, vary the excitation, first decreasing and then increasing the exciting current, and for each value of the excitation note the current and power taken by the motor armature. The variations of excitation should be taken as far as possible in both directions.

The voltage applied to the motor and the periodicity of the current must be maintained constant throughout the experiment. If the current is derived from an alternator which is not very large in comparison with the motor, it will be necessary to adjust both the speed and excitation of this generator as the current taken by the motor varies.

After taking a complete set of readings as described with the motor unloaded, several similar sets of observations should be taken with the motor loaded, the load being kept constant for each complete series of readings. The load may be applied either by a band or other form of brake, on the motor pulley, or the motor may be made to drive a dynamo, the current of the dynamo being kept constant during each set of readings, and varied when a change of load is desired. If the efficiency of the dynamo is known, the actual load on the motor can at once be determined by measuring the output of the dynamo. If the efficiency is not known, it is sufficient for the purpose of the present experiment to assume an approximate value of the efficiency in order to show the difference between the curves obtained. The direct-current motor employed to start the synchronous motor in the first place may often be conveniently employed as the dynamo for this purpose.

If the excitation of the direct-current machine is maintained constant its armature current will be directly proportional to the torque exerted by the motor.

The results should be entered in tabular form as indicated below.

EXCITATION CHARACTERISTIC OF SYNCHRONOUS MOTOR.

Synchronous Motor No. .... Type..............

Output.........h.p. at.........volts and.........revs. per minute.

Periodicity........................cycles per second.

Excitation Amperes	Terminal Volts	Armature Current	Armature Watts	Load	Cos Φ

The results of each set of readings should be plotted on squared paper, excitation being plotted horizontally and amperes and watts vertically, giving one curve of current and a curve of watts for each load applied to the motor (see Figs. 136 and 137).

The significance of the curves obtained in this and the following test is discussed later, in connection with the general theory of the synchronous motor (see page 235, *et seq.*).

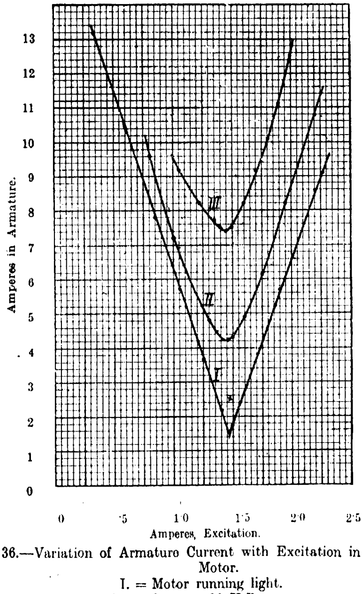


FIG. 136.—Variation of Armature Current with Excitation in Synchronous Motor.

I. = Motor running light.
II. = Output ·38 H.P.
III. = „ ·81 H.P.

Figs. 136 and 137 show curves of current and watts obtained by varying the excitation of a small 4-pole 1 h.p. motor having a ring-wound rotating armature and a speed of 1,500 revs per minute. The three curves correspond to three sets of readings taken, each at a constant output. The loads were: No load, ·38 h.p., and ·81 h.p. respectively.

It will be seen that the curves of current consist of two inclined portions, which in the case of no load are nearly straight lines, meeting at the lowest portion of the curve corresponding to 1·47 amps. excitation.

With 1·47 amps. excitation the armature current is most nearly in phase with the back voltage, and the current supplied to the motor is consequently a minimum. With a lower excitation the current lags behind the back electromotive force, and the amount of current necessary to maintain the rotation of the shaft increases for a double reason: (1) It has to increase in order to make up for the weakening of the field by the lessened excitation; (2) the total current supplied must increase in a greater ratio than the weakening of the field, because only the portion $I \cos \psi$ is active, and the angle ψ increases as the current lags behind the back electromotive force.

The curve consequently rises to the left of the minimum point, due to these causes.

The curve also rises, although rather less steeply, for values of the excitation higher than 1·47 amps. This is due to the fact that the armature current in this case leads the back electromotive force in phase, so that a smaller and smaller component of the current supplied to the armature reacts on the fields so as to produce the rotation. Thus, in spite of the fact that the field is strengthened, the current supplied must be increased in order to keep the component of the current which is in phase with the back electromotive force at a sufficiently high value to produce the required constant torque.

It will be seen that the right-hand part of the current curve is somewhat less steeply inclined than the left-hand branch, since, as just explained, in this case the field is strengthened by the change of excitation, and makes it unnecessary for the value of $I \cos \psi$ to increase so rapidly as is the case for points on the left-hand portion.

From the slight difference between the inclination of the two branches of the curve, it is evident that the variation in strength of the field produces only a small direct change in the armature current, and that nearly the whole of the variation is produced indirectly because of the alteration in the lag of the motor armature which follows the change of field. The lack of symmetry in the two limbs of the curves will depend also on the magnetisation curve of the machines, since excitation, and not flux, is plotted horizontally. In the present instance the magnetisation curves were practically straight lines within the range here employed. The upper curves follow generally the form of the no-load curve, but are more curved and show a more gradual bend at the lowest point, indicating a less sudden transition from lagging to leading current, and consequently more stable running.

The curves of watts in Fig. 137 will be seen to follow generally the shape of the curves of current, but show a less proportional variation. At first it might be thought that the power supplied to the motor would only vary very slightly, since the motor runs at a constant speed and exerts the same turning effort. From the curve it is evident that this is not the case, at any rate when the motor is lightly loaded, the power supplied varying from a minimum of 165 watts to a maximum of 625 watts.

This variation is specially high in the present case, since the motor was small and had a high armature resistance and low efficiency. A variation in driving power would, however, be found with any motor, and would be similar in its nature although smaller in extent in the case of a more efficient machine. An increase in armature current necessarily gives rise to increased losses in the armature.

The curve of watts shows very forcibly the importance of choosing the most favourable excitation for the motor, since the *useful* work

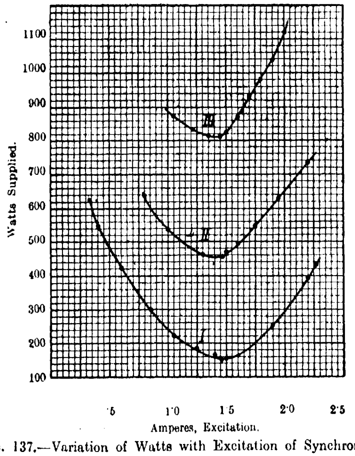


FIG. 137.—Variation of Watts with Excitation of Synchronous Motor

I. = Motor running light.
II. = Output ·38 H.P.
III. = „ ·81 H.P.

is the same for all points on the curve, although the power supplied to the motor varies so greatly.

The shape of the curves obtained in Experiment XXXIV. depends upon the self-induction of the armature of the motor. If the armature were without self-induction, the armature current would be in phase with the resultant voltage, and would increase in proportion to it. There would then be only one possible value of the exciting current for a given load, and the two limbs of the curve would coincide, giving a single line.

The greater the self-induction of the armature the more divergent will the two limbs of the curve be, and the greater will be the difference between the two values of the excitation at which it will operate with a given load and current. This will be explained

more fully in connection with the graphic representation of the conditions (see page 237).

Fig. 138 shows the curves of power-factor obtained from the same readings as the curves of current and watts given in Figs. 136 and 137. The no-load curve shows how rapidly the power-factor changes with the excitation in the case of an unloaded motor. This curve shows the curious feature of a power-factor rapidly falling and then slightly increasing, with further divergence from the most favourable excitation shown by the upward bend of the

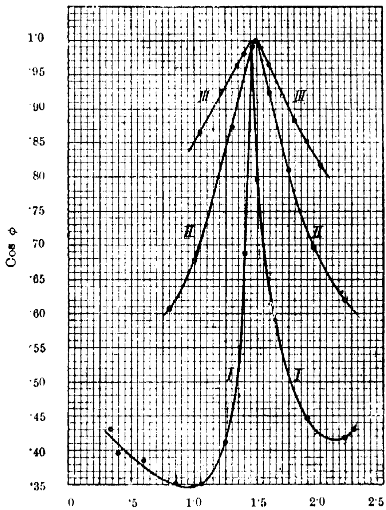


FIG. 138.—Curves of Power-Factor and Excitation.

I. = No load.
II. = ·38 H.P.
III. = ·81 H.P.

lower branches of the curve. This must be explained by the fact that the rapid increase of armature current when the excitation is increased above or decreased below the correct value produces a considerable loss of energy in the armature, due to its resistance and the iron losses in it. These losses ultimately become so great that the power spent in the motor armature increases more rapidly than the current. Although, therefore, the current increases and only does the same work as the smaller current in *driving* the armature, it does so much work in heating the armature that

after a certain point the power-factor begins to increase, instead of decreasing further, as more current is supplied.

It is important to observe how the curves just described are affected by an increase in load applied to the shaft of the motor. In each case the figures show three curves obtained from the same motor when unloaded and when loaded by a brake, and giving out ·38 and ·81 h.p. respectively at the same speed as before. It was found impossible to obtain as great a range of excitation with the motor loaded as when running light. In each case the extreme readings shown are the extreme values of the excitation for which the motor would run.

The curves of current and power-factor are much flatter at the apex when the motor is loaded. This is especially noticeable in the case of the curves of $\cos\phi$ From this it is evident that, in the case of a loaded motor, a small variation in exciting current has only a very small effect when this variation is near the point of most favourable excitation. This has a most important practical result in the steady running of a synchronous motor under load. The extremely sharp point in the curves of the unloaded motor at the point of best excitation shows that very small variations in the exciting current on either side of the best value will produce considerable changes in the armature current and in the phase angle at which the motor will run. This illustrates one of the chief difficulties to be met with in the running of rotary converters from the alternating current side. A rotary converter is really driven as an unloaded synchronous motor by an alternating current, and the "hunting" of these machines is mainly due to the want of stability of the conditions under which such machines work, as shown in the curves in Figs. 136 to 138.

Effect of Change of Load at Constant Excitation.—Having found the effect of an alteration in exciting current at various constant loads, it is of interest to ascertain how a change of load will affect the performance of a motor excited with a constant current. This forms the subject of the next experiment.

EXPERIMENT XXXV.—DETERMINATION OF EFFECT OF VARIATION OF LOAD UPON A SYNCHRONOUS MOTOR HAVING CONSTANT EXCITATION.

DIAGRAM OF CONNECTIONS.

Same as Fig. 135 for Experiment XXXIV.

Instructions.—Make the same connections as described for Experiment XXXIV., page 226. After exciting the field and synchronising the motor, close the main switch in the armature circuit, and disconnect the starting motor. Vary the excitation until it gives minimum armature current with the motor unloaded. Keep this value of the excitation constant, and gradually load the motor by a brake, or by making it drive a generator and increasing the output of this machine. For each value of the load take readings

of the current and watts supplied to the motor. Read also the terminal voltage, which should be kept as constant as possible.

A series of similar readings should then be taken for several different values of the excitation both above and below the most favourable value, the excitation being kept constant through each series.

The results should be entered in tabular form. The table given for the preceding experiment gives suitable headings. Three sets of curves should be plotted, to show armature current, watts input, and values of cos ϕ respectively, plotted on a base of load.

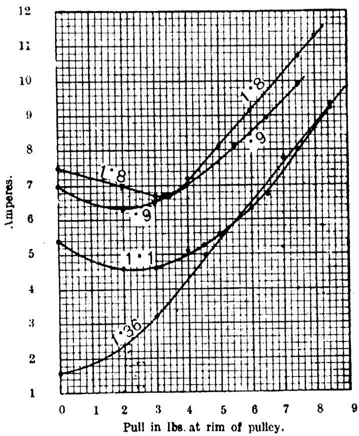


FIG. 139.—Variation of Armature Current with Load in a Synchronous Motor at Various Excitations.

Pulley 8 in. diameter.

The curves in Figs. 139 to 141 show the results of a test, made on the same motor as that from which the curves in Figs. 136 to 138 were obtained. The curves were taken for four values of the excitation, viz., ·9, 1·1, 1·36, and 1·8 amps.

The most favourable excitation at the voltage employed in this experiment was about 1·36 amps., hence the curves show the relation between the load and the watts, current, and power-factor for normal excitation, for two values of the excitation below the normal, and for one value above the normal.

The current curve (Fig. 139) show that with normal excitation the current increases almost in direct proportion to the load, in much the same way as it does with a direct-current shunt-wound motor. With a lower excitation of 1·1 amps. the current is much greater at light loads, on account of the low power-factor, but

approximates to the same value as for normal excitation at higher loads. With the excitation still further diminished to ·9 amps. the initial current is still higher, and although the current curves tend towards that obtained with normal excitation, it does not reach it for the range of loads shown.

With excitation 1·8 amps., considerably above the normal, values of the current are obtained which are greater than for any of the other curves, and the curve indicates little tendency to approach that taken at normal excitation.

The curves of watts (Fig. 140) show that the difference between the power given to the motor when normally and when under-excited is comparatively slight; indeed, the excitation of 1·1 amps.

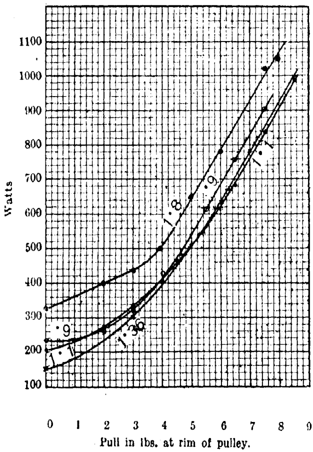


FIG. 140.—Variation of Power taken by Synchronous Motor with Load at Various Excitations.

Pulley 8 in. diameter.

shows rather better results than the normal 1·36 at higher loads. The curve for over-excitation indicates a considerable amount of waste power as being supplied.

The curves in Fig. 141, showing the variation in power-factor, illustrate in a very decided manner the advantage at light load of correct excitation. The power-factor in the curve for 1·36 amps. excitation is high throughout. With the other excitations the power-factor only becomes fairly high at the higher loads. The explanation of the curve for 1·8 amps. being higher than the ·9 amp. curve

is to be found in the greater armature waste power in the motor when over-excited.

Factors Determining Armature Current. — If no electromotive force were induced in the armature of the synchronous motor, the current taken by it would be numerically equal to the applied voltage divided by the armature impedance. The induced "back" voltage of rotation, however, acts in opposition to the terminal applied volts, so that the current actually taken is due to the difference between these opposing voltages. Since the "back" voltage is not exactly opposite in phase to the terminal

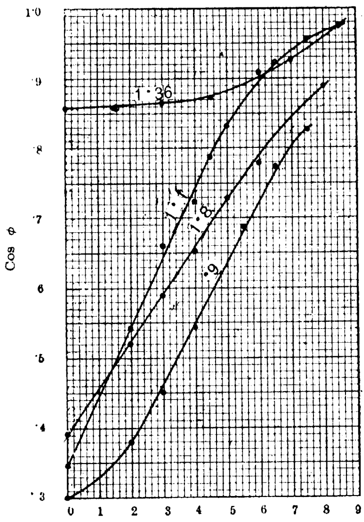


FIG. 141.—Variation of Power-Factor with Load in a Synchronous Motor. Pulley 8 in. diameter.

volts, the difference between them must be obtained vectorially. This vectorial difference we shall call the "resultant" voltage, its value being obtained graphically, as in Fig. 142, where the applied and induced voltages are shown as acting in almost direct opposition.

We thus have the armature current

$$= I = \frac{\text{resultant of applied and back voltage}}{\text{armature impedance}}.$$

in which the resultant voltage is obtained by the parallelogram law from the applied and back volts, as shown in Fig. 142.*

The back electromotive force of a synchronous motor depends only on the excitation, since the speed of the motor is constant for a given frequency of the supply. The induced electromotive force may, therefore, have a large range of values, and may be either greater or less than that of the source of supply.

The current taken by the armature depends only on the magnitude of the resultant voltage (E_r in Fig. 142) and the constant impedance of the armature. With a given excitation of the motor

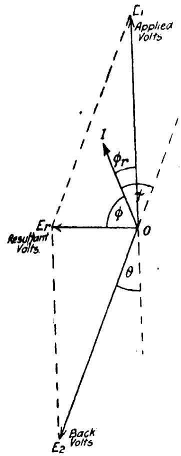


FIG. 142.—Diagram of Synchronous Motor.

and voltage of supply, the resultant voltage E_r, will depend upon the angular displacement θ of the motor electromotive force behind its position of opposition to the applied voltage. A change in the load on the shaft of the motor alters the value of θ, and consequently affects the resultant voltage and the amount of the current taken by the armature, just as in a continuous current motor the current varies in consequence of a change of speed caused by any variation in load.

The synchronous motor cannot run with the phase of the induced electromotive force and of the applied voltage in exact opposition,

* The diagrams and reasoning here assume that the motor has a constant synchronous reactance, which includes the effects due to armature reaction. Armature reaction is not, therefore, regarded as affecting the strength of the main field. Another method of regarding the matter is given on page 242.

if the excitation is so adjusted that these voltages are equal. In such a case the resultant voltage would be zero, and there would be no armature current. The effect of the load is to pull the armature back through a small angle θ, thereby producing a resultant voltage of such a value as to produce the amount of current which will give the requisite torque. An increase in load would produce an increase in the angular displacement and an increase in current.

It will be seen from Fig. 142 that the induced electromotive force E_2 may be greater than the supply voltage without producing any essential change in the conditions. This is in marked contrast to the case of the continuous-current motor.

It might be thought at first that if a greater voltage were induced in the motor armature, this would overcome the voltage of the line and produce a reversal of the current. It will be shown, however, that it is the phase relations of the two voltages, and not their relative magnitude, which determines whether the machine operates as a motor or generator.

Graphic Representation of Synchronous Motor. — We shall first make the assumption that the armature reaction and reactance may be considered equivalent to a constant synchronous armature reactance (cf. p. 204). Let the applied voltage be represented by a vector $O\ E_1$, as in Fig. 142, and let the induced electromotive force of the synchronous motor be $O\ E_2$ making an angle θ with the position of opposition to the applied voltage. The resultant voltage available for producing current in the armature is $O\ E_r$, the diagonal of the parallelogram drawn with $O\ E_1$, $O\ E_2$ as sides.

The numerical value of the current is obtained by dividing the voltage represented by $O\ E_r$ by the impedance of the motor armature. The current will lag behind the voltage E_r because the armature circuit is inductive. The angle of lag ϕ will depend on the relative magnitude of the resistance and synchronous reactance of the armature; its value must fulfil the condition $\tan \phi = \frac{X_a}{R_a}$ where X_a and R_a are the synchronous reactance and resistance (calculated so as to include eddy losses, &c.) of the armature. For a given machine the angle ϕ will be constant, and the current will lag behind the resultant voltage of the circuit by a constant phase angle. This angle ϕ must not be confused with the angle of phase difference between current and voltage at the motor terminals, which is the angle $E_1\ O\ I$ (marked ϕ_r) which determines the power factor at the motor terminals.

After calculating the value of the current due to the resultant voltage, it is easy to construct the voltage diagram $O\ P\ E_r$ if either R_a or X_a is known, since this triangle may be inscribed in a semicircle having $O\ E_r$ as its diameter. The current (to a scale of amperes) may then be marked off along $O\ P$, as shown by $O\ I$ in Fig. 143.

The power supplied to the motor from the mains = current $\times$ E_1 $\times$ cosine of the angle $E_1 O I$. This is the same as the product of the voltage E_1 $\times$ the component of the current, $O\ I_{e_1}$, in phase with it.

The output of the motor is similarly equal to the product of the induced voltage $O\ E_2$ multiplied by the current and by the cosine of the angle between the current and voltage. This, again, is equivalent to the product of E_2 and $O\ I_{e2}$, the component of the current along the line $O\ E_2$. The fact that the current I_{e2} and the voltage E_2 are drawn in opposite directions from O indicates that

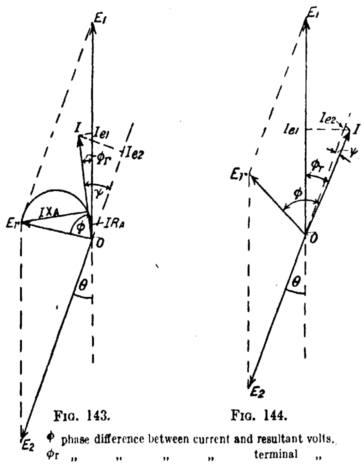


FIG. 143. FIG. 144.

ϕ phase difference between current and resultant volts.
ϕr " " " " terminal "
ψ " " " " induced "
θ angle of displacement of motor behind opposition to applied volts.

DIAGRAMS SHOWING EFFECT OF CHANGE OF EXCITATION IN SYNCHRONOUS MOTOR.

their product must be considered to have an opposite sign to the product of E_1 and I_{e1}. Since the latter represents power supplied to the motor, the former represents negative power supplied to the motor, *i.e.*, positive power given out.

If there were no losses in the motor armature, the power supplied to the motor and the power given out by it would be equal. The difference between them is accounted for by the watts lost in armature resistance, and is numerically equal to $I^2\ R_a$.

Effect of Variation in Excitation.—Fig. 143 shows the conditions in the circuit when the motor is excited so as to make its generated voltage equal to the applied terminal voltage; *i.e.*, E_2 is drawn equal to E_1. The effect of a decrease in the excitation of the motor would be to diminish the value of the induced voltage

E_2. The conditions in the circuit after such a diminution in the motor excitation are shown in Fig. 144. The resultant voltage E_r is seen to have altered in phase, so that both E_r and I lag behind their previous positions, while both quantities have increased in value. In actual working, a decrease in motor excitation would have the effect of at once increasing the amount of power taken by the motor from the line. This would momentarily accelerate the motor; that is, it would cause the voltage E_2 to advance in phase, so as to reduce the angle θ, thereby reducing the power taken, until this falls to the same value as before the change of excitation.* The conditions would then become those indicated in Fig. 144.†

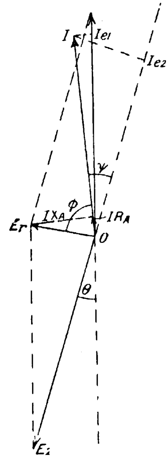


FIG. 145.—Synchronous Motor Diagram.

On comparing Fig. 143 with Fig. 144, where the motor excitation has been reduced, we see that the effect of this change in excitation has been to cause an alteration in the magnitude and phase of the current supplied to the motor. The current is always advanced in phase by an increase of excitation, and may be either increased or decreased in magnitude.

So long as the load on the motor remains at a constant value, the power supplied to the motor must have a constant value (except for $I^2 R_a$ losses, which we shall neglect for the present), and the

* Except for the small change in armature heating.

† The decrease in the angle θ in Fig. 144, as compared with Fig. 143, is so small as to be hardly measurable when reproduced to a small scale.

current must therefore have a constant *energy component*. Consequently for all values of the excitation, the point I in Fig. 143 will lie on a definite horizontal line for a given load. An increase in excitation will cause the point I to move to the right, and *vice versâ*. The most favourable excitation will be that which brings the current into coincidence of phase with $O\ E_1$, thereby enabling the motor to perform its work with a minimum of current and with unity power-factor at its terminals. Over-excitation produces an increase of current leading the terminal voltage in phase. Under-excitation also produces an increase in the current taken, but lagging in phase behind the voltage. These points are of the greatest importance.

We are now in a position to explain the shape of the Vee curves shown in Fig. 136 as a result of the tests in Experiment XXXIV. For this purpose we shall take a simple numerical example.

Example.—Draw curves of current and power-factor for a single-phase synchronous motor having a variable excitation, when supplied at 250 volts and taking a constant power of 5 kw. The motor has an armature resistance of 0·3 ohm and a synchronous reactance of 1·2 ohms.

The general form of the diagram for this motor is shown in Fig. 145.

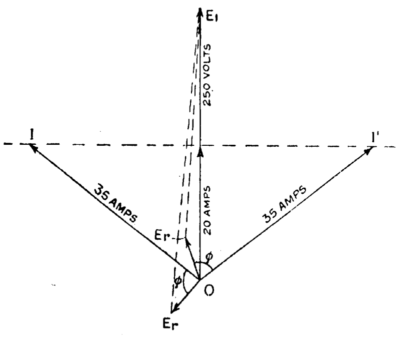


FIG. 146.—Diagram showing Construction of Vee Curves.

The armature impedance is

$\sqrt{(0{\cdot}3)^2 + (1{\cdot}2)^2} = 1{\cdot}24$ ohms.

The angle of lag of the armature current behind the resultant voltage E_r is given by the relation

$$\cos\phi = \frac{0{\cdot}3}{1{\cdot}24} = 0{\cdot}24.$$

The energy current taken by the motor is

$$\frac{5000}{250} = 20 \text{ amps.}$$

Let us now draw a diagram for the motor, beginning by drawing $O\,E_1$ as a vertical line to represent the applied electromotive force of 250 volts. Whatever the value of the current taken by the motor, it must have a component in phase with $O\,E_1$ of 20 amperes (see Fig. 146).

Choosing a scale of amperes, we next draw a horizontal dotted line at a height above the point O equal to 20 amperes. The current

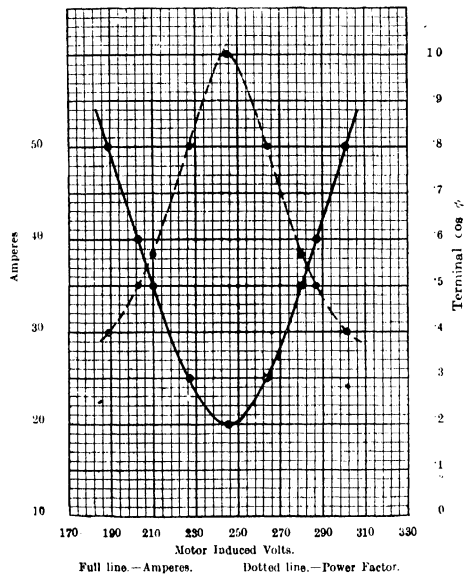


FIG. 147.—Calculated Curves for Synchronous Motor.

vector for any excitation will be a line, drawn from O to meet the dotted horizontal line, since such line will have an energy component of 20 amperes. This line may therefore be considered to be the locus of the current vector for an input of 5 kw.

Drawing any such line $O\,I$ at random, we can from it determine the voltage necessary to send this current through the armature impedance.

Let $O\,I = 35$ amps. be the value chosen for the current. The resultant voltage producing this current

$$= E_r = 35 \times 1{\cdot}24 = 43{\cdot}3 \text{ volts nearly.}$$

This voltage is shown on the diagram, making an angle ϕ with the current vector such that $\cos\phi = 0{\cdot}24$.

The resultant voltage vector $O\,E_r$ forms the diagonal of the parallelogram which has the applied voltage $O\,E_1$ and the motor-induced voltage $O\,E_2$ as sides. It is evident, therefore, that the line $E_1\,E_r$ is equal to the vector of motor-induced voltage. This length when measured off on the diagram will give the motor-induced volts corresponding to the armature current I. The value of $\cos\phi_r$ is obtained by dividing the energy current (= 20 amperes) by the total current (= 35 amperes) and is in this case 0·57.

A leading current of 35 amps. is also shown in the diagram, with the corresponding voltage E_r and dotted line of induced motor volts.

By taking successively a number of positions of the current vector, and proceeding as indicated above, we obtain the curves of current and $\cos\phi_r$ indicated in Fig. 147.

The following table gives the corresponding values as measured from the diagram :—

Current.	Resultant Voltage.	Motor Voltage.		Power-factor.
		Current Leading.	Current Lagging.	
20	24·7	245	245	1·0
25	30·9	264	227	0·8
35	43·3	280	210	0·57
40	49·5	287	203	0·5
50	61·8	301	189	0·4

Ampere-turn Diagram.—Instead of assuming a constant armature reactance, and no reaction, we may look upon the armature as having reactions, but no reactance (cf. p. 206). On this assumption, the magnet excitation is partly spent in overcoming armature reaction flux and partly in producing the air-gap flux which gives rise to the induced armature voltage. The resultant excitation is regarded as constant, and due to the combined action of the field ampere-turns and the armature reactive ampere-turns. The armature is regarded as non-inductive, and the loss of voltage in armature resistance is small and may be neglected for the present. The back voltage induced in the armature must always be equal and opposite to the applied terminal volts under these assumptions, and is consequently independent of the excitation of the motor field. An increase in magnet excitation has the result that the armature reactive ampere-turns increase in

such a way that they exactly neutralise the increase in ampere-turns on the magnets. Similarly, if the field excitation alone is insufficient to produce the air-gap flux necessary to enable the induced armature voltage to balance the terminal volts, the armature currents will increase with a lagging phase so as to supply as many ampere-turns as are necessary for producing the necessary flux.

We have another aspect of the reason why in a synchronous motor over-excitation of the field gives rise to leading armature currents and *vice versa*.

In exact analogy with the diagram given for the regulation of an alternator, Fig. 125, page 208, we may construct excitation diagrams for the synchronous motor.

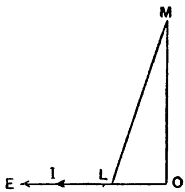


FIG. 148.

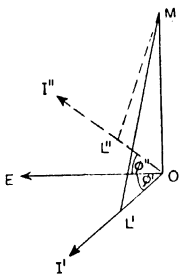


FIG. 149.

Ampere-turn Diagrams of Synchronous Motor.

In Fig. 148 let the vertical line $O\ M$ represent the excitation necessary to induce a voltage equal to the terminal applied volts (as obtained from the open-circuit characteristic). Let $O\ L$, drawn horizontally, be the excitation corresponding to the current I on the short-circuit characteristic. Then the line $M\ L$ will represent to the same scale the field excitation, which will produce unity power-factor at the motor terminals. For any other value of the armature current and terminal power-factor, the excitation corresponding to the current as derived from the short-circuit characteristic is drawn at an angle ϕ to the horizontal (see Fig. 149), such that cos ϕ is the power-factor at the motor terminals. For a given number of amperes, the current may either lag behind or lead the terminal volts. As seen from page 240, in one case the field excitation has an increased value, and in the other case is diminished below its value for a power-factor of unity. The dotted lines show the conditions for under-excitation and lagging current

while the full lines in Fig. 149 show the case of over-excitation and leading current.

If necessary, the excitation diagram may be corrected for armature resistance loss, as already explained on page 208 for the alternator.

It will be seen that Fig. 149 resembles the excitation diagram for an alternator (Fig. 125, p. 208), but is inverted. This is because of the opposite relative action of the armature reactions in a generator and motor.

CHAPTER IX.

THE POLYPHASE CIRCUIT.

Generation of 2- and 3-phase Currents.—In the armature of an alternating-current generator, the electromotive force generated in conductors which are situated in similar positions with regard to the poles at any instant will be identical in phase, while conductors situated in intermediate positions will have induced in them electromotive forces which are intermediate in phase. As already explained, it is usual to connect in series coils which are formed of conductors lying in several slots, and which are, therefore, not identical in phase, although differing but slightly. Since it does not add much to the output of a generator to connect in series conductors which are not approximately in phase with each other, the armature winding of a single-phase alternator is composed of groups of conductors, the groups being so spaced as to have the same pitch as the poles. There are thus portions of the armature between these groups which cannot be advantageously wound with conductors connected to the same circuit.

Two-phase Current.—Let an armature be wound with conductors situated in slots having a pitch equal to that of the poles. The electromotive force in all the conductors will have the same phase variation. If an exactly similar set of conductors is wound in the intermediate positions between the first set, and these are independently connected together, they will give an alternating electromotive force similar in voltage and periodicity to that induced in the first winding, and only differing from it in phase.

An alternator having two armature windings with the conductors of one winding situated in advance of the other by half the pitch of the magnet poles, *i.e.*, the coils of one exactly half-way between the coils of the other, will supply a 2-phase current at the four terminals with which the armature is then provided.

Thus, a 2-phase circuit gives two independent alternating currents, each carried by conductors forming a separate circuit. The two circuits have the same voltage and periodicity, and a fixed relative difference of phase of a quarter period, so that one current passes through its maximum value as the other is at zero and *vice versa*.

The relation between the currents or voltages of the two circuits may be represented by two equal rotating lines at right angles to each other, as in Fig. 150. The lines are not shown joined together, since the quantities represented by them are in two separate

circuits. The power supplied by the generator will be twice the power given to either circuit, if the circuits are equally loaded.

Three-phase Current.—If three windings are applied to the armature with equal spacing between the conductors of each winding, the phases of the electromotive forces induced in the windings will differ by one-third period, so that there will be a point of maximum electromotive force induced in each winding in rotation.

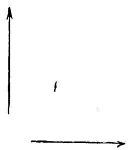


FIG. 150.—Relation between Voltages in a 2-phase Circuit.

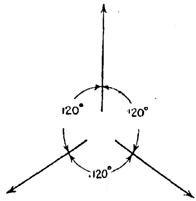


FIG. 151.—Relation between Voltages in a 3-phase Circuit.

If the three windings form completely separate circuits, the armature will have six terminals, and the current and voltage relations may be represented by three rotating lines, each making an angle of 120° with the other two. (See Fig. 151). The power developed by the machine will be three times the power of each winding. A system of this kind is termed a 3-*phase* system, and consists of three alternating circuits, having equal voltage and periodicity, and a fixed phase relation of such a kind that there is one-third period phase difference between the voltages of any pair of circuits.

From what has just been said, it appears that 2-phase currents require for their transmission four wires, and 3-phase currents similarly require six wires, and an *n-phase* circuit would have $2n$ wires. This is true in general; but for a 3-phase circuit the special relations existing between the currents in the three circuits make it possible to employ in this case three conductors instead of six. The special conditions which make this possible must now be explained.

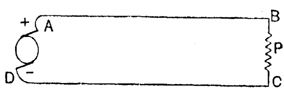


FIG. 152.

Transmission by Three Wires.—Let Fig. 152 represent a generator supplying direct current to a distant point P, and let us consider the conditions governing the flow of current in the conductor A B.

Current cannot flow along $A B$ unless this conductor forms part of a closed circuit connected to the generator. Hence the circuit must be closed at the end P, and there must be a return conductor from P capable of transmitting at any moment the same current from C to D as is flowing from A to B. The return current need not flow by a single conductor, but may flow partly through the earth, as in the case of a tramway installation, or may be unequally divided between a number of conductors. The only absolutely essential condition for the flow of a current from A to B is that the same amount of current shall flow at the same moment from C to D. This condition applies to any kind of current, whether direct or alternating.

Let $A\ A^1$, $B\ B^1$, $C\ C^1$ (Fig. 153) represent the six conductors carrying a 3-phase current, each of the dotted lines representing a conductor forming the return wire for the current carried by the conductor just above it.

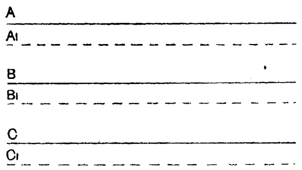


FIG. 153.

Then the current in A^1 is equal and oppositely directed to the current in A *at every instant*, and similarly for the remaining pairs.

It will be shown later that the special characteristic of a 3-phase system is that the sum of the currents in any two circuits at any instant is equal and oppositely directed to the current in the third circuit. Thus, the sum of the currents in B and C is always equal and opposite to the current in A. Consequently, if a suitable connection were made between the conductors A, B, and C, the conductors B and C might, without any change in the current flowing in them, provide the return path for the current in A. That is, the current supplied by the generator to the circuits B and C may be looked upon as being the return current of the circuit A. Thus the conductor A^1 might be dispensed with, without the current in A being affected.

Similarly, the conductors A and B carry an equal and opposite current to that in the conductor C, and the conductor C^1 may be dispensed with without altering the current in C, if A and B are so connected to C that they can form the return path for the current in it.

In this way no circuit requires a separate return conductor, and the 3-phase currents can be transmitted along three conductors instead of six.

The simplest way of showing that the sum of the currents in two circuits is equal and opposite to that in the third is to draw curves representing the simultaneous values of the currents. This has been done in Fig. 154. Taking at random any instant represented on the curve, it will be seen that the algebraic sum of the ordinates of two curves is equal to the height of the ordinate of the third, and is opposite to it in sign.

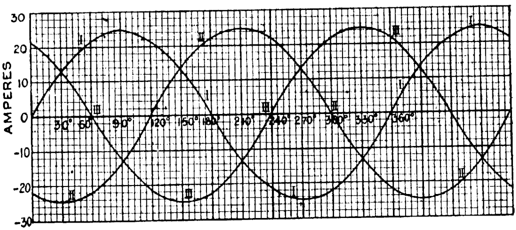


FIG. 154.—Curves of Current in a 3-phase Circuit.

The mathematical proof of the proposition is simple when the quantities are represented by vectors, as in Fig. 155 where $O\ I_1$, $O\ I_2$, $O\ I_3$ show the three currents.

The angles referred to are plainly marked. I is the common maximum value of the currents.

The instantaneous values of the currents are obtained by horizontal projection. It should be remembered that the cosine of an angle is equal to the sine of its *complement*.

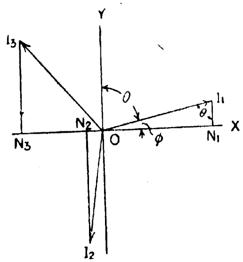


FIG. 155.—Three-phase Currents.

(1) $O\ N_1 = I \cos\phi = I \sin\theta.$

(2) $O\ N_2 = I \cos(120 - \phi).$

$= I (\cos 120 \cos\phi + \sin 120 \sin\phi).$

$= I \cos 120 \sin\theta + I \sin 120 \cos\theta$ (A)

(3) $O\,N_3 = I \cos (120 + \phi).$
$= I (\cos 120 \cos \phi - \sin 120 \sin \phi).$
$= I \cos 120 \sin \theta - I \sin 120 \cos \theta$ . . (B)

Therefore, from equations (A) and (B) above we get :—
$O\,N_2 + O\,N_3 = 2\,I \cos 120 \sin \theta.$
$= -2\,I\,\tfrac{1}{2} \sin \theta = -I \sin \theta = -O\,N_1.$

Evidently, also, this equality is independent of the value of θ and is true throughout the cycle.

Methods of Connection for 3-Phase System.—In the preceding paragraph it was assumed that each conductor was connected to the other two, so that they should form the return path for its current. The method of connection between the circuits must evidently fulfil two conditions : (1) It must be symmetrical ; (2) the current must flow through the resistances or machines forming the three load circuits.

There are two methods of connecting the load circuits fulfilling these conditions, both in general use. They are indicated in the diagrams, Figs. 156 and 157, and are called respectively the *star* and the *delta* or *mesh* connections.

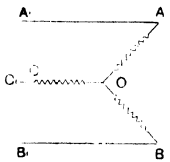


FIG. 156.—Star Connection.

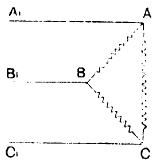


FIG. 157.—Mesh Connection.

In the star connection, Fig. 156, the three load circuits are connected with one end to a common or neutral point, the free ends being joined directly to the supply lines.

In a mesh-connected system, Fig. 157, the load circuits are all joined end to end in series, so as to form a closed circuit, and the supply conductors are joined to each junction point.

Connection must be made between the three windings or phases with which the alternator is wound. This can either be done in the machine itself, so that it has three terminals for connection to the three transmission lines, or the alternator may have six terminals, and these may be interconnected, so as to supply current of the right character to the line. In either case there are available only the two alternative methods just described, and the three windings of the armature must be connected in one of the ways shown in the diagrams, Figs. 156 and 157, in star or mesh connection. The three zig-zag lines in either diagram will then represent the phase windings of the alternator armature. The effect of this inter-connection of the phase windings upon the voltage of transmission must now be discussed.

Star-connected Alternator.—Let the three spirals, Fig. 158, represent the 3-phase windings of a star-connected alternator, while the vectors e_{oa}, e_{ob}, e_{oc} represent the voltages generated in these windings. Then the voltage between a pair of terminals c, a is due to the two windings $c\,o$, $o\,a$. These windings act in exactly the same way as two alternators giving equal voltages and coupled together at such an angle that they differ in phase by $\frac{1}{3}$ period or 120°. The resultant voltage due to these alternators might be found as in Experiment I., and would be represented in magnitude and phase by the diagonal of the parallelogram of which the separate electromotive forces are drawn to form the sides.

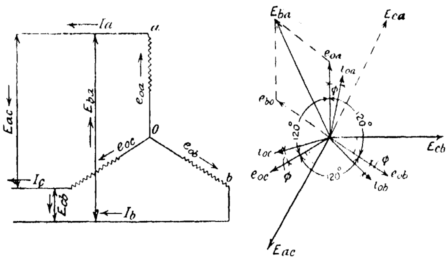


FIG. 158.—Star-connected Phases.

The voltage between the terminals c and a is the sum of the voltages generated in the windings c to O and O to a (see Fig. 158). It is most important to notice that this is *not* the same as the sum of the voltages O to c and O to a. Let the voltage induced in the phase from O to a be represented by e_{oa} in the vector diagram Fig. 158, and the voltage induced from O to c by e_{oc}. The voltage e_{oc} acting from O to c is equivalent to an exactly opposite voltage e_{co} acting from c to O. The voltage between c and a is, therefore, the sum of the two voltages e_{co} $(= - e_{oc})$ and e_{oa}. We may write this

$$E_{ca} = e_{co} + e_{oa} = - e_{oc} + e_{oa}$$

the signs indicating the vectorial addition or subtraction.

The voltage which is the resultant of e_{co} and e_{oa} is that marked E_{ca} in Fig. 158. This is the terminal voltage of the alternator. From the diagram its value is seen to be

$$E_{ca} = 2\,e_{oa} \cos 30^\circ = 2\,e_{oa}\,\frac{\sqrt{3}}{2} = \sqrt{3}\,e_{oa}.$$

The voltages between the other terminals are shown as vectors E_{ab}, E_{bc}. Each of these is the resultant of two phase voltages, and would be obtained by a similar construction to that just given.

Stated generally, in a star-connected alternator, the *terminal voltage = voltage of one phase* $\times \sqrt{3}$.

Evidently from the diagram the current supplied to any circuit is the same as the current in each winding of the armature.

Mesh - connected Alternator.—The phase connections and corresponding vector diagram for this case are shown in Fig. 159. As before, the phase-voltages are first drawn as three vectors making angles of 120° with each other. These phase-voltages are e_{ba}, e_{cb}, e_{ac}. The three phase-currents i_{ba}, i_{cb}, i_{ac} are three vectors each lagging behind the corresponding voltage vector by an angle ϕ. The current given out at the terminals is in each case the sum of the currents conveyed to any terminal along the two adjacent

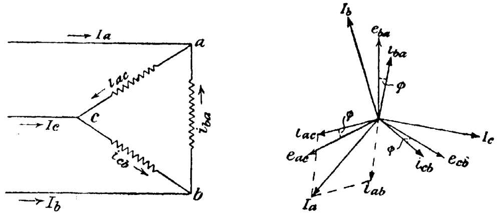


FIG. 159.—Mesh-connected Phases.

phase windings. Thus the current I_b is the sum of the currents flowing towards b along the paths $c\ b$ and $a\ b$. This may be stated briefly

$$I_b = i_{cb} + i_{ab} = i_{cb} - i_{ba}.$$

Hence the current I_b must be obtained vectorially by the addition of the vector i_{cb} and the vector i_{ab} ($= i_{ba}$ reversed). This addition is shown in the vector diagram (in Fig. 159). The vectors I_a, I_b, I_c obtained in this way show the magnitude and phase of the terminal currents given by the alternator. Thus we have in a mesh-connected circuit a terminal current equal to 2 cos 30° ($= \sqrt{3} = 1{\cdot}73$) times the phase-current.

The terminal voltage of a mesh-connected alternator is seen to be the same as the phase-voltage.

The results of the preceding sections may be summarised as follows :—

Let I, E be respectively the current in the line and the voltage between each pair of conductors, and
i, e the current and voltage of each phase winding of the generator armature.

For star-connected armatures

$$I = i,$$
$$E = \sqrt{3}\, e,$$

For mesh-connected armatures

$$I = \sqrt{3}\, i,$$
$$E = e.$$

The formulæ just given also apply to the case of motor armatures supplied from the line, i and e representing the current and pressure in each phase winding of the motor. Similarly, for any resistance or other form of load circuit supplied from the line, the above rule applies if i and e are taken as the current and voltage of each branch, when the branches are connected either in star or mesh connection, and I and E are the current and voltage in the main circuit.

Use of Neutral Wire.—In the case of circuits supplying a mixed load of motors and lamps, it is not unusual to employ a fourth (or neutral) wire connected to the star-point of the generator or transformer. Three-phase motors are connected to the 3-line conductor in the ordinary way, while lamps and other small loads requiring a single-phase current are connected to one line and the neutral wire. Evidently the voltage applied to the lamps is only $\frac{1}{\sqrt{3}}$ or 0·578, of the line voltage, which is usually an advantage, as the motors can be run advantageously at a higher voltage than the lamp. The lamps are arranged to make as equal a loading as possible and, in combination, they form a load star-connected to the circuit.

Power of 3-phase Circuit.—Still employing the symbols just given, it is easy to see what the power transmitted by the 3-phase circuit is.

First, take the case of non-inductive load

The power given to each branch load circuit is $i\, e$ watts. Since there are three such branch circuits, the total power transmitted by the line to the branch circuits is $3\, i\, e$ watts.

In the star connection $i = I$ and $e = \frac{E}{\sqrt{3}}$, hence in this case total power $= 3\, i\, e = 3\, I \frac{E}{\sqrt{3}} = \sqrt{3}\, I\, E.$

In the mesh-connected system $i = \frac{I}{\sqrt{3}}$ and $e = E$, consequently power of system $= 3\, i\, e = 3 \frac{I}{\sqrt{3}} E = \sqrt{3}\, I\, E.$

Thus in either case the power is the same, and is equal to $\sqrt{3}\, I.E$ watts.

The relations just given should be verified by means of the following experiment :—

EXPERIMENT XXXVI.—DETERMINATION OF THE POWER TRANSMITTED BY A 3-PHASE CIRCUIT (LOAD NON-INDUCTIVE).

DIAGRAM OF CONNECTIONS.

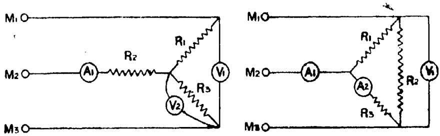


FIG. 160.

M_1, M_2, M_3 Terminals supplied with 3-phase alternating current.
R_1, R_2, R_3 Equal non-inductive resistances.*
A_1, A_2 Ammeters.
V_1, V_2 Voltmeters.

Instructions.—In order to verify the formulæ for both star and mesh connected systems, the connections should be made in turn according to both systems, as indicated in the diagrams shown above.

The diagrams sufficiently indicate the connections to be made. It is to be noted that in the case of the star connection a single ammeter measures both the current in the line and in the branch circuit. With the mesh connection a single voltmeter measures both line and branch voltage.

For each of the connections indicated in the diagram, take readings of the line current and voltage, and the current and voltage of one of the branch circuits. This should, if possible, be repeated for two or three values of the resistances. In each case the values of all these resistances should be equal. Enter the results in the form shown below, and note that the two values of the power entered in the last two columns should be the same.

COMPARISON OF POWER IN MAIN AND BRANCH CIRCUITS.

Line.		Branch.		Total Watts.	
Current = I.	Voltage = E.	Current i.	Voltage e.	$\sqrt{3}\, I\, E$.	$3\, i\, e$.

* Banks composed of equal number of similar incandescent lamps form a suitable non inductive load. A still more convenient form of 3-phase resistance is a liquid form with three symmetrical blades capable of being dipped into the solution simultaneously to any desired extent.

Power in Inductive Circuits.—If the load is partly inductive the phase-current and voltage will not be in phase with each other. In this case the power in each branch circuit will be $i\ e \cos \phi$ instead of $i\ e$ watts. If the angle of lag in all the branch circuits is the same ($= \phi$), the expression for the power in the line becomes

$$\text{Power} = 3\ i\ e \cos \phi \text{ or } \sqrt{3}\ I\ E \cos \phi,$$

which is the general expression for the power transmitted by a 3-phase line with equal loading on all phases.

It is most important to remember that ϕ is the phase angle between current and voltage in the *branch* circuits, and not between line current and line voltage. Also, the power-factor of a balanced 3-phase circuit is the power-factor of the three load circuits supplied. In general, the measurement of the power of a 3-phase system necessitates the employment of a wattmeter when the load is partly inductive, as the methods in which only ammeters and voltmeters are employed become very complicated.

There are several methods of connecting the wattmeter to the circuit, the most usual of which are now to be described.

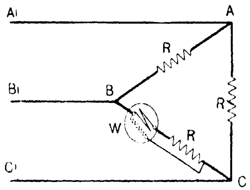


FIG. 161.—Connection of Wattmeter in Branch Circuit.

Wattmeter Measurements in Three-phase Circuit.—Method I.—One obvious method of measuring the power given to or taken from a balanced 3-phase circuit is to connect the wattmeter in one of the three branch circuits, as indicated in Fig. 161. If the power given to the circuit is to be measured, the wattmeter would be so connected as to indicate the output of one phase of the generator. If the power measured is that given to a motor, transformer, or other set of branch circuits, the wattmeter would measure the power in one branch circuit. As explained in the preceding section, the total power of the system $= 3\ i\ e \cos \phi$ where i and e are the current and voltage in one branch of the star or mesh. Thus the wattmeter will read one-third of the total power, and the reading must be multiplied by three, unless this use of it has been foreseen and allowed for in the calibration of the instrument.

Method II.—What would at first sight appear an equally simple method is to connect the wattmeter so as to read the product of current and voltage of the line, in the manner shown in Fig. 162 (page 256). A wattmeter connected in this way will not, however,

indicate the power of the circuit, because the line current I and line voltage E are not in phase with each other, even in a non-inductive circuit. Referring to Figs. 158, 159, it will be seen that for both star and mesh connections, E and I differ in phase by an angle equal to $30° \pm \phi$. Hence the reading of the wattmeter connected as shown in Fig. 162 will read the product $I E \cos 30° = \frac{\sqrt{3}}{2} I E$ when there is no lag or lead in the circuit, *i.e.*, when the power-factor is unity.

If the load is inductive, so that the branch current i is not in phase with the voltage e, but has an angle of phase difference ϕ (see Figs. 158, 159), then the phase difference between the current and voltage in the line will not be $30°$ but $30° \pm \phi$.

Imagine the series coil of a wattmeter to be inserted in one line, carrying the current I_a (Fig. 158), for instance. Suppose also that the volt coil of the wattmeter is connected so as to read the voltage between the same line and one of the others, say E_{ca}. The wattmeter reading will then be $i_{oa} E_{ca} \times$ cosine of angle between i_{oa} and E_{ca}, that is $I E \cos (30° + \phi)$. Now imagine the volt coil thrown over to read the voltage between the line in which the current flows and the other line, that is E_{ba}. The watts read will now be $i_{oa} E_{ba} \times$ cosine of angle between i_{oa} and E_{ba} or $I E \cos (30° - \phi)$. Thus the two readings of the wattmeter will be respectively $I E \cos (30° + \phi)$ and $I E \cos (30° - \phi)$,

$$i.e., \; I E (\cos 30° \cos \phi - \sin 30° \sin \phi)$$
$$\text{and } I E (\cos 30° \cos \phi + \sin 30° \sin \phi).$$

The sum of these is $2 I E \cos \phi \cos 30° = \sqrt{3}\, I E \cos \phi$, which is the value in watts of the power in the circuit.

It will not be difficult for the student to deduce a similar result for a mesh-connected circuit from the diagram Fig. 159.

We have, therefore, the following rule for determining the power in a balanced inductive 3-phase circuit. Connect the wattmeter with its current coil in one of the line wires, and join the volt terminal alternately to the other two line wires. The sum of the two readings thus obtained gives the total power on the circuit. The load and power-factor on the three lines are, of course, assumed equal.

Incidentally, it is worth noticing that the difference between the two wattmeter readings when divided by the line voltage gives the idle current directly.

The difference of the readings is $2 I E \sin 30° \sin \phi$.

$$= 2 I E \tfrac{1}{2} \sin \phi = E . \; I \sin \phi,$$

and $I \sin \phi$ is the wattless current in the line. (See page 72).

If the circuit is so highly inductive that ϕ is greater than $60°$, the value of $\cos (30° + \phi)$ becomes negative. The wattmeter is then deflected in a reverse direction, and in order to obtain readings it becomes necessary to reverse the connections to two of its terminals. The reading obtained under these conditions must be

looked upon as negative, and the total power of the circuit will be the *difference* between the readings of the 2 wattmeters.

Also the power-factor of the circuit may be arrived at from the relation

$$\tan\phi = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2}$$

Where W_1 and W_2 are the two readings of the wattmeter.

Instead of altering the connections of the volt coil of the wattmeter in order to obtain the total power by means of a single wattmeter, it is often preferable to throw the current coil alternately into 2 of the main circuits. The sum of the readings then gives the total power independently of equal loading of the circuits. This method is practically that described on page 258 when 2 wattmeters are employed.

The following experiment illustrates Method II. given above.

EXPERIMENT XXXVII.—DETERMINATION OF POWER AND POWER-FACTOR IN A BALANCED THREE-PHASE LINE

DIAGRAM OF CONNECTIONS.

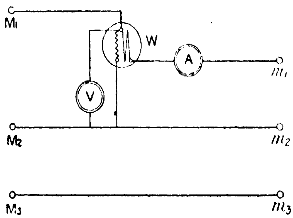


FIG. 162.

M_1, M_2, M_3 Source of 3-phase alternating current.
m_1, m_2, m_3 Points connected to a balanced partly-inductive load.
W Wattmeter.
A Ammeter for measuring line current.
V Voltmeter for measuring line voltage.

Instructions.—Connect three conductors from the source of 3-phase alternating current to a partially-loaded transformer, induction motor, or other inductive resistance.

Insert in one line wire the current coil of the wattmeter and an ammeter. Connect a voltmeter between one pair of wires, and connect the free end of the volt coil of the wattmeter alternately to the two wires not containing its current coil, *i.e.*, alternately to $M_2\,m_2$ and $M_3\,m_3$ in the diagram above.

Make sure that the voltage between each pair of line wires is the same, and then take readings on the ammeter, voltmeter, and two readings with alternative connections of the wattmeter for several values of the load on the circuit.

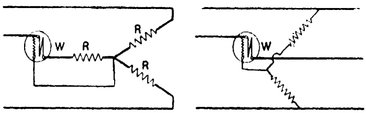
Enter the results of the readings thus :—

DETERMINATION OF POWER-FACTOR IN THREE-PHASE CIRCUIT.

Line.			Wattmeter Readings.			Power-Factor $\frac{W_1 + W_2}{\sqrt{3}\, I \times V}$	Idle Current $\frac{W_1 - W_2}{V}$
Current I.	Volts V.	Volt-amperes $I \times V$.	First W_1.	Second W_2.	Total Watts $W_1 + W_2$.		

The method just given, although simple in use for occasional measurements, or when arranged with a throw-over switch, has the disadvantage that two readings have to be taken and added together. It is therefore not a direct reading method suitable for switch-board use.

By the use of the next method of connection the power is recorded directly.



(a) Circuit with neutral point available. (b) Wattmeter connected to artificial neutral point.

FIG. 163.—Wattmeter Connections in 3-phase Circuit.

Method III.—Three non-inductive resistances are connected together to form a " star point," the potential of which will remain constant. The wattmeter is then connected so as to read the current in one line, and the voltage from this point to the same line. (See Fig. 163b). The voltage applied to the volt coil of the wattmeter is consequently the same as would be generated in one phase of a star-wound generator ($= e$). Also the current coil carries the same current as would be carried by the winding of the generator in the same case ($= i$). Hence (employing the previous notation) the reading of the wattmeter will be $i\ e \cos \phi$, or $\frac{1}{3}$ the total power of the circuit.

Usually a wattmeter intended for permanent connection to a circuit in this manner would have its scale so calibrated that the readings would be three times the power actually producing the deflection. In this case the power of the circuit would be registered directly.

If the neutral point of the generator winding, or the neutral point of the motors or branch circuits supplied, is available, the resistances shown in Fig. 163b are not necessary, and connections are then made as in Fig. 163a. The resistances are only required in order to create a neutral point when this is not otherwise available.

Measurement in Unbalanced Circuit with Two Wattmeters.—In cases where the load on the three wires is not balanced, the usual way to determine the power of the system is to insert a wattmeter in each branch circuit. The readings of the three wattmeters must then be added together to give the total power of the circuit.

Two wattmeters connected as shown in Fig. 164 suffice to measure the power in an unbalanced 3-phase system. The sum of the readings of the two instruments is equal to the total power transmitted. Sometimes the two volt coils are mounted on a single spindle, and the two current coils act on their respective volt coils in a single instrument. In this way the deflection of the needle is the sum of the deflections due to each pair of coils, and the instrument then reads the total watts of the circuit directly.

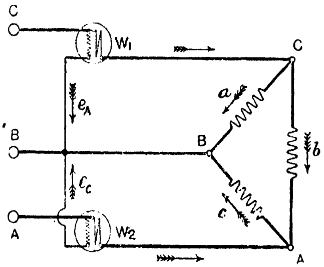


FIG. 164.—Measurement of Power in an Unbalanced Circuit.

The proof that the sum of the readings of wattmeters W_1 and W_2 in Fig. 164 gives the true watts of the circuit may be stated as follows.

Referring to Fig. 164, it is desired to measure the power delivered to the three branch circuits, *a*, *b*, and *c*.

Let the symbols represent the instantaneous values of the various quantities, as follows :—

e_a, e_b, e_c = the voltages of the branch circuits respectively.
i_a, i_b, i_c = the currents in the same circuits.
e_A, e_B, e_C = the voltages between the lines.
i_A, i_B, i_C = the currents in the line conductors.

Also consider the directions indicated by the arrows in the figure to be positive directions, in order to give definite meanings to the signs to be employed.

The power to be measured

$$= W = e_a\, i_a + e_b\, i_b + e_c\, i_c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

At the given instant it is evident from an inspection of the figure that

$$e_a = e_b + e_c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\text{also } i_c = i_a + i_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\text{and } i_x = i - i_b \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The reading of wattmeter W_1 at any moment

$$= W_1 = e_x\, i_c = e_a\,(i_a + i_b) = e_a\, i_a + e_a\, i_b$$

Substituting from equation (2)

$$W_1 = e_a\, i_a + i_b\,(e_b + e_c) = e_a\, i_a + e_b\, i_b + e_c\, i_b \quad . \quad (5)$$

Similarly the reading of wattmeter W_2

$$= W_2 = e_c\, i_x = e_c\,(i_c - i_b) = e_c\, i_c - e_c\, i_b \quad . \quad . \quad (6)$$

Adding equations (5) and (6)

$$W_1 + W_2 = e_a\, i_a + e_b\, i_b + e_c\, i_c = W \quad . \quad . \quad . \quad (7)$$

Since the wattmeters in each case read the mean of the product of current and volts, the sum of the readings will be the mean value of the watts W as given in equation (7).

The proof just given is based on the distribution of currents and voltages in a mesh-connected circuit. A similar proof may be obtained for a star-connected circuit. This is, however, hardly necessary, since from the method of connecting the wattmeters, it is evident that they will read the power transmitted by the line quite independently of the manner in which the branch circuits may be connected. Thus the circuits a, b c, in Fig. 164, might equally well be connected in star, instead of mesh, and the sum of the readings of the two wattmeters would still give the total power passing in the lines.

In the present case, as in the Method II. of measurement by one wattmeter given above, it is to be noticed that the common terminal of the wattmeters is connected to the source of supply. Under these conditions wattmeters of the usual construction will read in the correct direction. If it is found that the reading of the instrument is reversed, the shunt coil connections should be interchanged and the readings of the wattmeter considered negative and must be subtracted from the value recorded by the positively reading wattmeter.

CHAPTER X

The Rotary Converter.

The Rotary Converter.—The currents induced in the conductors of a direct-current generator are actually alternating currents which have given to them a single direction in the external circuit, owing to the action of the commutator. Thus, an alternator with rotating armature is essentially the same machine as a direct-current generator, except for the substitution of simple collecting rings directly connected to the armature winding, in the place of a commutator of which the segments are connected to a great number of separate conductors.

A closed circuit armature may be provided with both commutator and slip rings, and may then be used alternatively as a direct or alternating-current generator. It may also be used to give simultaneously direct and alternating currents. A more frequent application of a machine constructed in this way is to drive it as a motor by means of current of one kind, and to take current of the opposite kind from the armature winding. In this way the machine acts as a motor-generator, with the important difference from the true motor generator, that the same armature winding receives the driving current and gives out the generated current.

As in the case of direct-current motor generators, the fact that both generator and motor currents circulate round a single armature core renders the resultant armature magnetic reaction very small. This is due to the motor and generator currents being in opposite directions, and consequently exerting magnetising forces which approximately neutralise each other.

In the rotary converter a further advantage is secured by the fact that the individual conductors carry only the difference between the two currents which they receive as motor conductors and give out as generating conductors.

Ratio of Transformation.—The armature voltage of a multiple-circuit direct-current machine is given by the following formula:

$$E = \frac{N\,n\,F\,p}{60 \times 10^8}$$

E = Armature voltage.
N = Number of armature conductors.
n = Revolutions per minute.
p = Pairs of poles.
F = Flux per pole.

This voltage will be the *maximum* value obtained between two slip rings connected to the same armature. The virtual alternating volts will therefore be

$$E = \frac{N\,n\,F\,p}{60 \times 10^8} \times \frac{1}{\sqrt{2}}$$

assuming the wave form to be a sine curve.

Hence, if we neglect the losses in the armature conductors, the ratio of direct to virtual alternating single-phase voltage is

$$1 : \frac{1}{\sqrt{2}} \quad i.e., \ 1 : 0{\cdot}707.$$

Thus, if the rotary converter is supplied with 100 volts direct current, it will give out 70·7 volts alternating. If supplied with 100 volts alternating, it will give out 141·4 volts continuous current. In either case, the *maximum value* of the alternating voltage is the same as the voltage at the commutator, and the virtual value is the maximum value $\div \sqrt{2}$, if the wave form of the alternating current is a sine curve.

With a power-factor of unity on the alternating side, the ratio of direct to alternating current is the inverse of the ratio of the voltages. This follows from the fact that the input and output watts must be equal (except for the incidental losses in the machine), and hence, if we postpone the consideration of the effect of lagging or leading currents,

Volts (direct) $\times$ amperes (direct) $=$ volts (alternating) $\times$ amperes (alternating).

But it has just been shown that

Volts (direct) $=$ volts (alternating) $\times$ 1·414.

Hence,

Volts (alternating) $\times$ 1·414 $\times$ amperes (direct) $=$ volts (alternating) $\times$ amperes (alternating);

Or,

Amperes (alternating) $=$ 1·414 $\times$ amperes direct,

i.e., the (virtual) alternating current (with sine wave form) is nearly $1\frac{1}{2}$ times the value of the direct current supplied to, or taken from the commutator.

With a power-factor of $\cos\phi$ it is evident from the equality of input and output that the alternating current for a given output will be greater than the value just given. In this case,

$$\text{Amperes (alternating)} = \frac{1{\cdot}414}{\cos\phi} \times \text{amperes (direct)}.$$

Multipolar Rotary Converter.—A single-phase 2-pole converter will have two slip rings connected to conductors situated at opposite points on the armature. If the machine has a number of poles, the slip rings will each have as many connections to the conductors as there are pairs of poles. The angular pitch between two successive connectors to one ring will be equal to the angle between one N pole and the next N pole. The total number of

connections from the armature will thus be the same as the number of the poles.

Polyphase Converter.—Assuming the case of a 2-pole converter fed from the direct-current side to give a single-phase alternating current, the armature will have two connections to the slip rings which will twice in every revolution receive the full voltage given to the commutator. The value of the alternating voltage produced will consequently have a maximum value equal to the direct-current voltage, and a virtual value of $\frac{1}{\sqrt{2}}$ or 0·707 times this voltage.

In the case of a 2-pole 3-phase converter the conductors connected to slip rings will be situated 120° apart.

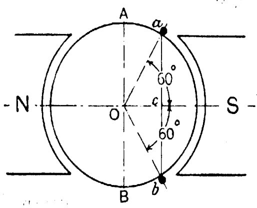


FIG. 165.—Voltage of Three-phase Converter.

The maximum voltage between two rings will occur when the conductors joined to the rings are in such a position as the conductors a b shown in Fig. 165, since the conductors between a and b will then be moving most nearly in a direction at right-angles to the lines of the field. In this position the flux being cut by the conductors will be less than the maximum flux, which would be cut by the conductors between A and B in the ratio of ab to AB, if we assume the armature to rotate in a uniform field.

Since the angle $aOb = 120°$, the length $a\ c = O\ a \sin 60° = O\ A \sin 60°$. Thus the line $a\ b = A\ B \sin 60°$.

Hence maximum alternating voltage $= V_D \sin 60°$, where V_D = direct-current voltage of the converter.

The virtual value of the alternating voltage

$$= \frac{1}{\sqrt{2}} \text{ maximum value} = \frac{1}{\sqrt{2}} \sin 60° \ V_D = \frac{\sqrt{3}}{2\sqrt{2}} V_D$$

$= \cdot 612 \ V_D$ volts.

It may be stated generally that in a rotary converter with m rings, if the electromotive force between the direct-current brushes

is V_D and v_A the virtual alternating pressure between two adjacent rings

$$v_A = \frac{1}{\sqrt{2}} V_D \sin \frac{\pi}{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The relation between the direct and alternating currents is best obtained by equating D C and A C powers.

Thus, if I_D and i_A are the direct and virtual alternating currents of a converter having m rings, *i.e.*, arranged for an m-phase alternating current,

$$\text{Direct current power} = V_D \; I_D$$

If i_A is taken to be virtual current in the armature conductors, and I_A the virtual current at the slip ring, the alternating current power per phase $= v_A \; i_A \cos \phi$, and the total A C power $= m \; v_A \; i$ $\cos \phi$.

$$\therefore \; V_D \; I_D = m \; v_A \; i_A \cos \phi,$$

or substituting the value already obtained for v_A,

$$V_D \; I_D = \frac{m \; i_A \; V_D}{\sqrt{2}} \sin \frac{\pi}{m} \cos \phi$$

$$\therefore \; i_A = \frac{\sqrt{2} \; I_D}{m \sin \frac{\pi}{m} \cos \phi} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It can be shown that

$$I_A = 2 \sin \frac{\pi}{m} \; i$$

$$\therefore I_A = \frac{2 \sqrt{2} \; I_D \sin \frac{\pi}{m}}{m \sin \frac{\pi}{m} \cos \phi}$$

$$= \frac{2 \sqrt{2} \; I_D}{m \cos \phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In a non-inductive 3-phase circuit

$$i_A = \frac{I_D \sqrt{2}}{3 \sin 60^\circ} = \frac{2 \sqrt{2}}{3 \sqrt{3}} \; I_D$$

and

$$I_A = I_D \; \frac{2 \sqrt{2}}{3} = 0{\cdot}943 \; I_D$$

The ratios of direct and alternating currents and voltages may be conveniently tabulated in the following manner, the power-factor of unity and a form-factor equivalent to that of a sine wave being assumed.

The influence of the pole-width in modifying these values is referred to on page 270.

	Direct Current.	Single Phase.	2-phase. 4-ring.	3-phase. 3-ring.	6-phase. 6-ring.	m-ring.
Volts between slip rings	1	·707	·707	·612	·354	$\frac{\sin \frac{\pi}{m}}{\sqrt{2}}$
Amperes at slip rings	1	1·414	·707	·943	·472	$\frac{2\sqrt{2}}{m}$

Heating of Converter Armature. — The rotary converter, when run from the alternating-current side, acts simultaneously as an alternating-current synchronous motor, and as a continuous-current generator. Since the machine has only a single armature winding, the currents flowing in the conductors of the armature are always the algebraic sum of the currents which the armature would carry if it were considered to be alternately (1) an alternating-current motor armature, and (2) a continuous-current generator armature. On the whole, the currents which would drive the

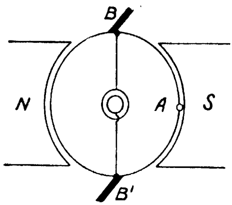


FIG. 166.—2-ring Converter.

armature as a motor are opposite in direction to the currents which would be generated in the armature of a generator. In general, therefore, the armature conductors carry currents which are less than those flowing in the armature of an alternating-current motor of the same output.

The magnitude of the direct current flowing in the armature conductors is constant in amount (for a given load), but changes in direction twice in each period. During each period, the alternating current passes through all its values, positive and negative, in succession. The exact relation which the alternating and direct currents bear to one another in phase is different for conductors situated at various points in the armature. In order to illustrate this, Figs. 167 and 168 have been drawn for conductors situated respectively midway between the slip-ring connections and adjacent to the slip rings of a single-phase converter.

Referring to Fig. 166, it will be evident that the conductor A, situated midway between the slip-ring connectors, will pass under the brushes B (and will in consequence have the direct current in it reversed), when the voltage between the slip rings is zero. Assuming a power-factor of unity, we see that the direct current in conductor A will undergo reversal at the moment when the

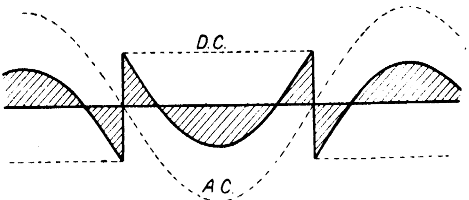


FIG. 167.—Current Variation in 2-ring Rotary Converter. Conductor Midway between Rings.

alternating current in it is zero. This condition is shown in Fig. 167, where the values of the direct and alternating current are shown by dotted lines, and the resulting current in the conductor, which is obtained by addition of the ordinates of the dotted curves, is shown by the full line. The shaded area shows the actual amount of current variation in this conductor

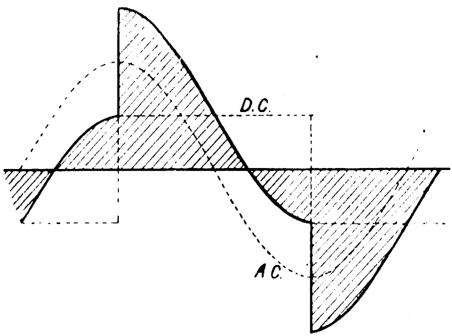


FIG. 168.—Current Variation in 2-ring Rotary Converter. Conductor Close to Slip Rings.

In a similar manner, Fig. 168 shows the current fluctuation in a conductor situated adjacent to the slip-ring connectors. This conductor experiences a reversal of the direct current at the moment when the armature is carrying its maximum of alternating current. The summation of the two curves of current, resulting in the curve

which bounds the shaded area, will be readily followed. It is evident that in this case the average current is far higher than in the conductor illustrated by Fig. 167. We have, in fact, chosen the two conductors in the armature which carry respectively the least and highest average value of the current, so that the conductors in intermediate positions will carry currents which are intermediate also in value.

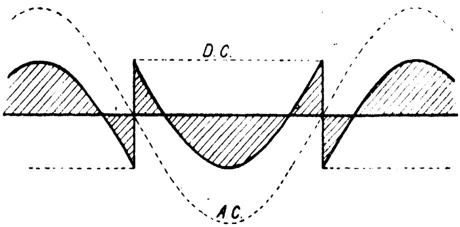


FIG. 169.—Current Variation in 3-ring Converter. Conductor Midway between Rings.

It will be seen that the maximum value of the alternating current is exactly double the value of the direct current, this being the condition for a single-phase converter. The maximum resultant current in Fig. 168 is three times the continuous current in value.

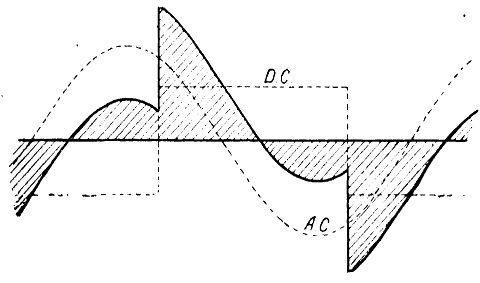


FIG. 170.—Current Variation in 3-ring Converter. Conductor Close to Slip Ring.

For purposes of comparison, Figs. 169 and 170 are given to show the current variation for the corresponding conductors in a 3-ring converter having the same direct-current output as the previous single-phase converter. The first curve (Fig. 169) resembles the corresponding curve of Fig. 167, except that the value of the alternating current is 77 per cent. of its former value. The second curve (Fig. 170) is, however, somewhat different from the previous

one. Referring to Fig. 171, it will be seen that the voltage between the connectors A and B has a maximum value at the instant represented on the diagram. A conductor situated between A and B immediately adjacent to A will pass under the brush B one-twelfth of a revolution after the position shown, *i.e.*, one-twelfth of a revolution, or 30°, after the maximum alternating current flows in it (again on the assumption of unity power-factor). The currents in the conductor adjacent to A will therefore have to be added in the manner shown in Fig. 170, so that the reversal of the continuous current occurs 30° from the point of maximum alternating current.

On comparing Figs. 167 and 168 for the single-phase converter with Figs. 169 and 170 for the 3-phase converter, it is at once evident that the average currents carried by the conductors in the latter are of less value, and that the heating of the armature for the same output must be considerably less. Also it is at once evident that in Figs. 167 and 169 the average actual current is much below the value of the direct current.

The heating of the conductors will be proportional to the average value of the square of the current, and will evidently be very unequally distributed in the armature of a converter. The greater the number of slip-rings employed, the more evenly will the heating be distributed, and the more nearly will the continuous and alternating currents neutralise one another. The following table is of interest as showing how an increase in the number of slip-rings with a given armature will increase its output when employed as a rotary converter as compared with a direct-current generator. The power-factor is assumed to be unity.

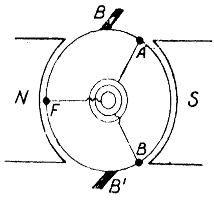


FIG. 171.—Three-ring Converter.

POWER RATING FOR SAME AVERAGE HEATING.

Continuous Current Generator.	Converter.			
	2 rings.	3 rings.	4 rings.	6 rings.
1·00	0·85	1·32	1·62	1·92

Magnetisation Curve or Open-circuit Characteristic of a Rotary Converter.—In order to ascertain the magnetic properties of the converter, and the most suitable excitation, this curve should be obtained, as in the case of a direct or alternating current generator.

If it is possible to drive the machine by external means, the test should be carried out exactly as described in Experiment XXIX. for an alternator. The machine is driven by a belt or coupling at a constant speed, and the excitation, which should be supplied from an external source of direct current, is gradually increased from zero to a value considerably above the normal. Voltmeters should be connected to both alternating and direct sides of the armature, and must be read for each value of the excitation. After a similar series of readings has been taken with decreasing values of the exciting current, the mean curve should be plotted with amperes or ampere-turns horizontal, and direct and alternating voltages vertical.

If it is not possible to drive the converter mechanically, it must be driven electrically from the direct-current side. The experiment is then carried out as follows :—

Connect the direct-current side of the machine to a source of direct current in series with a variable resistance. Supply the field windings with direct current, through a variable resistance and ammeter. Connect a voltmeter to the alternating side of the armature.

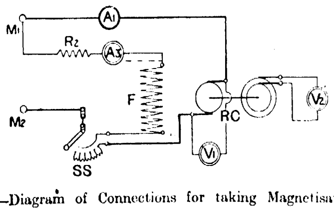


Fig. 172.—Diagram of Connections for taking Magnetisation Curve.

The connections will then be as shown in Fig. 172.

Begin by giving the magnets their full excitation, and start the machine, gradually cutting out the armature resistance until full speed is attained. Read the excitation and alternating and direct-current voltage. Slightly decrease the field, and increase the resistance in the armature circuit, so as to obtain the same speed as before. Repeat the readings and take a succession of such observations with decreasing values of the excitation.

When plotted to give the magnetisation curve, the terminal voltage on the direct-current side will be higher than the true voltage, which should be considered to be the voltage transformed into alternating volts on the A.C. side, by the amount lost in armature drop and armature reactions. If the armature drop

(= current × armature resistance) be subtracted from each D.C. ordinate of the curve, a curve which is practically the true magnetisation curve of the machine will be obtained.

Fig. 173 shows the magnetisation curve or no-load characteristic of a $2\frac{1}{2}$ kw. rotary converter obtained by driving the machine by a separate motor and varying the excitation. Curve I. shows the readings of a voltmeter connected to the direct-current side, and Curve II. shows similar readings obtained on the alternating side.

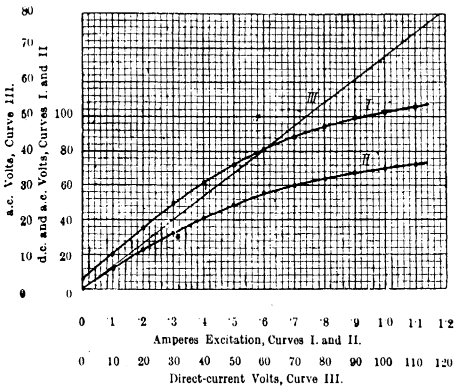


FIG. 173.—No-load Characteristic of Rotary Converter.

Curve III. shows the relation between the direct and alternating voltages. The ratio is seen to be constant over the whole range of readings taken, and to be $\frac{\text{alternating volts}}{\text{direct volts}} = \cdot 675$. This value is less than the theoretical value of ·707, which would have been found if the wave form of the alternating voltage generated had been sinusoidal. The wave form in a rotary converter is affected mainly by the shape of the poles, since the armature winding is a distributed one, and consequently does not influence the form of the wave. The width of the poles is consequently the chief factor in modifying the wave form produced, and its effect is usually stated in terms of the ratio of the width of pole to the pitch of the poles (*i.e.*, the distance from centre to centre of the poles).

This ratio consequently largely determines the *virtual* value of the alternating voltage (its *maximum* value is determined by the direct-current voltage) and the ratio of transformaton of a rotary converter. The following table shows the influence of the pole width upon the ratio of transformation of a single-phase rotary

converter. In a polyphase converter the ratio of transformation is altered in the same proportion.

$\frac{\text{Polar arc}}{\text{Pole pitch}}$	0·8	0·75	0·7	Sinusoidal field	0·65	0·6	0·55
$\frac{\text{Alternating volts}}{\text{Direct volts.}}$	0·67	0·69	0·71	0·71	0·73	0·75	0·77

In the case for which the curve, Fig. 173, was drawn, the polar arc was rather more than ·75 of the pole pitch, and the normal ratio of transformation, without armature drop, &c., was therefore ·675 instead of $\frac{1}{\sqrt{2}}$ or 0·707.

EXPERIMENT XXXVIII.—DETERMINATION OF CHARACTERISTIC AND EFFICIENCY OF A ROTARY CONVERTER. (1) WHEN RUN FROM THE DIRECT-CURRENT SIDE.

DIAGRAM OF CONNECTIONS.

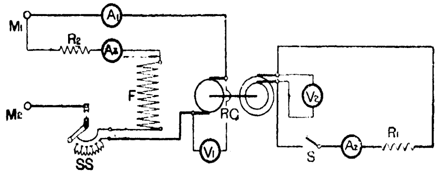


FIG. 174.

M_1, M_2 Source of direct current.
$R\,C$ Armature of rotary converter.
F Field windings.
A_1 Ammeter reading direct current supplied to armature.
A_2 Ammeter reading alternating current given out by armature.
A_3 Ammeter reading exciting current.
V_1 Voltmeter reading direct voltage applied.
V_2 Voltmeter reading alternating voltage given out.
$S\,S$ Starting switch.
R_1 Variable non-inductive resistance in load circuit.
R_2 Shunt regulating resistance.
S Switch for breaking load circuit.

Instructions.—Connect the brushes of the direct-current side of the converter to source of direct current. Connect also the field windings to the same supply in series with an ammeter and regulating resistance. Connect the terminals of the alternating

side to a variable non-inductive resistance in series with an ammeter and switch. Connect voltmeters to the terminals of both the direct and alternating sides.

Start the machine in the same manner as an ordinary direct-current motor, with the switch in the alternating circuit open. When running at normal speed and with normal excitation, read the voltage at both direct and alternating terminals, and also the direct current supplied to the armature. Keeping the direct-current voltage at the armature terminals and also the excitation constant, close the switch in the load circuit. For a series of increasing values of the load current take readings on both voltmeters and ammeters. The power in each circuit should be calculated and the results entered in tabular form as indicated below.

From the results the following curves should be plotted on a load base: (1) Direct current supplied, (2) ratio of transformation, (3) efficiency.

It must be remembered that in all rotary converter experiments the position of the brushes affects the voltage ratio very much. The brushes should be kept fixed at the neutral position.

LOAD CHARACTERISTIC OF ROTARY CONVERTER.

Rotary converter No. .... Type ....

Alternating voltage............................. Current...........

Excitation......................... Speed.........revs. per minute.

Direct Current			Alternating Current			Ratio Alternating to Direct		Efficiency
Volts	Amps	Watts	Volts	Amps	Watts	Volts	Current	

The efficiency is calculated by dividing the A.C. watts given out by the D.C. watts supplied. It must be remembered that the excitation losses should be added to the input watts to give the total efficiency. Since the load circuit is non-inductive, the output may be taken as the product of current and voltage on the A.C. side.

Figs. 175 and 176 show the results of such test, which was carried out on the same 2-pole machine for which the magnetisation curves in Fig. 173 were drawn. The curve in Fig. 175 shows the ratio of voltage transformation, the voltage on the direct-current side being constant at 100. The curve is seen to drop fairly rapidly at first, but afterwards to become practically a straight line. The loss of voltage indicated by the curve is due to the same causes as those producing the drop in a separately excited generator, although the armature reactions and loss due to armature resistance will both be smaller in the rotary converter.

Fig. 176, which shows the relation between the primary direct current and the output alternating current, is also a straight line, but does not pass through zero. The ratio of current transformation appears consequently not to be a constant.

The explanation of this apparent want of proportionality in the ratio of transformation is that the machine required 2·5 amps. to drive it when the alternating output was zero, as indicated by the point at which the curve cuts the vertical axis. As the power required to drive the converter when giving out current on the alternating side will be at least equal to the current taken to drive it at no-load, there will always be at least 2·5 amps. of the direct current supplied which are not converted into alternating current,

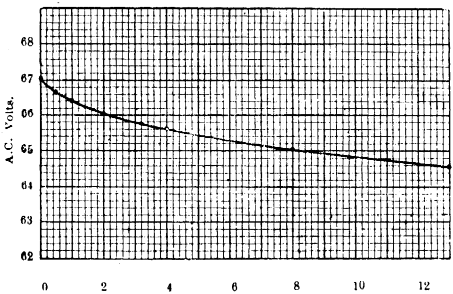


FIG. 175.—External Characteristic of Rotary Converter Driven from D.C. Side.

D.C. Volts = 100.

but which pass through the armature from brush to brush on the commutator, and are spent in overcoming the frictional losses, so as to maintain the revolution of the machine. Thus, if the power spent in driving the machine were a constant, the true ratio of current transformation would be obtained by subtracting the constant no-load current from each reading of the direct current supplied. This can most simply be done on the diagram by drawing a horizontal line through the point where the curve cuts the vertical axis, and taking this as the horizontal axis from which to measure that part of the direct current which is converted into alternating current. Proceeding in this way we find the ratio of transformation with a power-factor of unity to be

$$\frac{\text{alternating current}}{\text{direct current}} = \frac{18}{12{\cdot}4} = 1{\cdot}45.$$

We can calculate from the true voltage ratios obtained from the open-circuit characteristic (Fig. 173) what the ratio of current transformation should be, since

D.C. watts = A.C. watts, and $\frac{\text{A.C. volts}}{\text{D.C. volts}} = \cdot 675$ as already measured, hence

$$\frac{\text{A.C. amps.}}{\text{D.C. amps.}} \text{ should be } \frac{1}{0\cdot 675} = 1\cdot 48.$$

The difference between the ratio 1·45 actually observed and the true ratio of 1·48 is due to the slight increase in driving current required to overcome the increased iron losses when the machine is loaded. These losses are probably chiefly due to eddy currents in the pole faces and to increased hysteresis losses in the armature core In Fig. 176 the dotted line drawn just below the curve

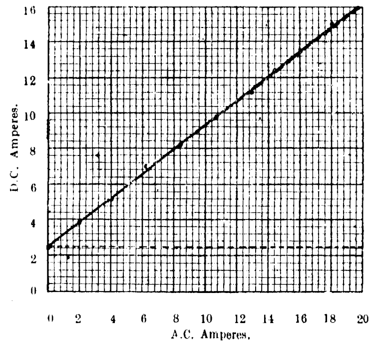


FIG. 176.—Current Characteristic of Rotary Convertor Driven from D.C. Side.

indicates the theoretical ratio of current transformation. The vertical distance between this line and the curve actually obtained is therefore a measure of the increased current taken in driving the machine when loaded on the alternating-current side.

Referring again to Fig. 175, showing the ratio of voltages, we can now separate to some extent the loss of voltage into its constituents. There will be a certain loss of voltage due to the direct current driving the converter, equal to (no-load current × armature resistance). In the present instance this was 2·5 amps. × ·145 ohm = ·36 volts. At no load this will be the only source of loss. As the armature begins to supply alternating current, there will be an additional armature ohmic drop due to this current. The value of this drop will not be simply the product of either the alternating or direct current by the armature resistance, since the alternating current will not flow through the whole of the armature, but at certain positions of the armature will largely flow from the direct-current brushes to the slip-rings. The value of the drop may be calculated from the following figures, where k is the constant by which the product (direct current × armature resistance) must

be multiplied to obtain the true loss of voltage due to that part of the current which is transformed into alternating current.

LOSS OF VOLTAGE IN ROTARY CONVERTER.

	Two-ring.	Three-ring.	Four-ring.	Six-ring.
k	1·175	0·75	0·615	0·515

To find the drop in the armature of a rotary converter due to any load, we must use the following formula, the value of k being taken from the table above :—

$$e = k\, I_c\, R$$

Where e drop in volts,

I_c = direct current taken or supplied,

R = armature resistance between brushes.

The loss of voltage due to armature resistance at any load may be determined in the way illustrated by the following example.

Take the point corresponding to 12 amps. A.C.

From Fig. 176 it is seen that the direct current taken by the converter when giving this output is 10·7 amps. Of this, 2·5 amps. are spent in driving the machine, while 8·2 amps. are converted into alternating current.

The value of the total ohmic drop will therefore be calculated as follows, using the constant for a 2-ring converter given above, and employing the value 0·145 ohm. for the armature resistance.

Drop due to

driving current $2{\cdot}5 \times 0{\cdot}145 = 0{\cdot}36$ volts.

converted current $8{\cdot}2 \times 0{\cdot}145 \times 1{\cdot}175 = 1{\cdot}4$ volts.

Total drop . . . . 1·76 volts.

It is to be remembered, however, that this calculation is based on the current and voltage measured on the input (direct current) side of the converter.

In order to obtain the volts lost in armature resistance in terms of the output (alternating) voltage, the result of our calculation must be multiplied by the ratio of conversion of the converter.

From Fig. 173, page 269, this ratio is seen to be 0·675 for the 2-ring converter used for the test. Hence we get the value of the alternating voltage lost through armature resistance

Total A.C. drop $= 1{\cdot}76 \times 0{\cdot}675 = 1{\cdot}19$ volts.

It is to be noted that the machine for which the curves are drawn was an experimental machine of small size. The losses due to resistance of the armature and friction of the machine appear unduly high in consequence. In a machine of commercial size when working under normal load, the losses due to the no-load currents would be entirely negligible, and the total loss in volts would be obtained directly by employing the formula at the top of the page.

From Fig. 176, which shows the ratio of current transformation, it is seen that the ratio is a constant one, since the curve is a straight

line. It cuts the vertical axis at 2·5 amps., showing that this was the current required to drive the converter when running light and supplied at the normal voltage on the direct-current side. The power taken under these conditions ($= 2{\cdot}5 \times 100 = 250$ watts in the present case) represents the power required to overcome friction, windage and no-load iron losses. The iron losses will increase somewhat under load, but approximately the no-load losses + the copper losses, which can be calculated for any load, will give the total losses at any load. The approximate efficiency of the converter when driven from the direct-current side could therefore be calculated from the no-load observation.

Effect of Varying Excitation of Rotary Converter.—The effect of a variation in exciting current is different according to the side from which the converter is driven.

When driven from the direct-current side, a lessening of the exciting current will cause the machine to run more rapidly, since the driving side behaves similarly to any direct-current motor. The increase of speed will not, however, produce a corresponding increase in voltage on the alternating side, since the increase of speed is not more than enough to make up for the decrease in the strength of the field in which the armature rotates. The conductors will therefore rotate more rapidly, but in a weakened field, and, if the losses remained the same as before, it would appear that the voltage generated should be unchanged by change in excitation. It will be found in the experiment which follows that the voltage is affected to some extent by changes in excitation.

EXPERIMENT XXXIX.—DETERMINATION OF EFFECT OF VARIATION OF EXCITATION UPON RATIO OF TRANSFORMATION OF A ROTARY CONVERTER. (1) CONVERTER DRIVEN FROM DIRECT-CURRENT SIDE

DIAGRAM OF CONNECTIONS.

Same as for Experiment XXXVIII., Fig. 174.

Instructions.—Make connections as described in the case of Experiment XXXVIII., above.

With the alternating-current circuit open, vary the excitation, and for each value of the exciting current take readings of the direct and alternating voltages, the current spent in driving the machine, and the speed. The applied voltage should be kept constant during the test.

The alternating circuit should then be closed and a similar series of readings taken with one or two values of the load current. For each set of readings the resistance in the load circuit should be varied so as to maintain the current at a constant value.

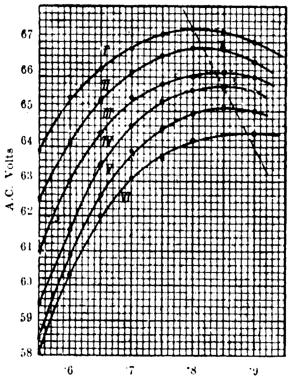
The readings should be entered in columns as shown for Experiment XXXVIII.

Curves showing the variation in secondary voltage should be plotted on a base of exciting current, each curve corresponding to a definite value of the alternating-current output.

Fig. 177 shows a set of six curves obtained on the same machine as that employed for the results shown in the previous Figs. 173-176

The direct voltage applied was maintained constant at 100, so that the alternating voltage gives directly its percentage value in terms of the direct voltage.

It will be seen that there is one value of the excitation which gives the maximum ratio of voltage-conversion, and further, that this value varies for different loads. The inclined dotted line passes through the maximum point of each curve, and shows the



D.C. Volts = 100
Curve I. = 0 Amperes load.
II. = 1 " "
III. = 2 " "
IV. = 4 " "
V. = 8 " "
VI. = 14 " "

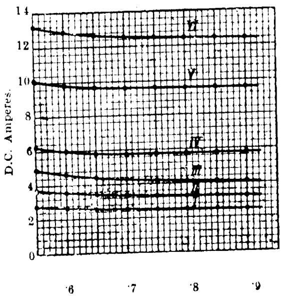
FIG. 177.—Rotary Converter Driven from D.C. Side. Relation between A.C. Volts and Excitation.

variation of exciting current which would be necessary in order that the converter should give the maximum voltage ratio for varying loads.

Fig. 178 shows the values of the direct current observed during the same experiment, at the same values of the alternating current. It is seen, as might have been expected, that within the limits taken the currents decreased as the strength of field was increased and as the speed of the converter became less. Less power was consequently required to drive the converter with higher excitations. Within the limits taken in the experiment, the efficiency consequently increased with the excitation.

If the convertor supplies an inductive circuit, the effect upon its field will be similar to that in the case of an alternating-current generator. A lagging current will tend to weaken the field, while a leading armature current will strengthen it. A change of power-factor will tend to produce a change in speed, and a consequent change in the frequency of the A.C. output.

On short-circuit, or on a highly-inductive load, the field may be so much weakened that the speed may become excessive. On account of this danger, it is usual in the case of "inverted" rotaries (*i.e.*, converters driven from the D.C. side) to excite the field from a special exciting machine coupled to the rotary converter. If this is done, any tendency for the speed to increase, due to the



Curve I.	= 0	Amperes	load.
II.	= 1	,,	,,
III.	= 2	,,	,,
IV.	= 4	,,	,,
V.	= 8	,,	,,
VI.	= 14	,,	,,

FIG. 178.—Rotary Converter Driven from D.C. Side. Relation between Current and Excitation.

demagnetising effect of the armature currents, is checked by the increased excitation given by the exciter, whose speed varies with that of the rotary.

Rotary Converter Driven from Alternating-current Side.—When used to convert alternating into direct currents, the converter is driven from the alternating side as a synchronous motor, and generates direct currents which are supplied to the load circuit from the commutator. This is the most usual application of rotary converters

The field windings must be supplied with direct currents, which may conveniently be obtained from the D C. side of the converter itself.

EXPERIMENT XL.—DETERMINATION OF CHARACTERISTIC AND EFFICIENCY OF A ROTARY CONVERTER. (2) WHEN RUN FROM THE ALTERNATING-CURRENT SIDE.

DIAGRAM OF CONNECTIONS.

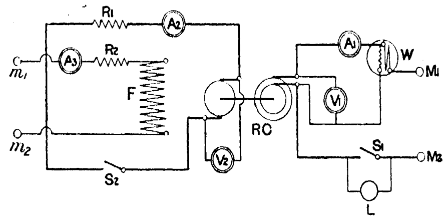


FIG. 179.

M_1 M_2 Source of alternating current.
m_1 m_2 Source of direct current.
R C Armature of rotary converter
F Field windings
A_1 Ammeter in alternating supply circuit.
A_2 Ammeter in direct-current load circuit.
A_3 Ammeter in field circuit.
V_1 Voltmeter reading alternating supply voltage.
V_2 Voltmeter reading direct-current voltage.
W Wattmeter reading power supplied.
S_1 S_2 Switches.
L Synchroniser.
R_1 Variable resistance in load circuit.
R_2 Field-regulating resistance.

Instructions.—Connect the alternating side of the machine to the source of alternating current in series with an ammeter, series winding of a wattmeter and switch. A synchroniser of some kind must also be provided. Connect a voltmeter and the volt coil of the wattmeter to the alternating terminals of the converter. Connect the field winding to a supply of direct current through an ammeter and regulating resistance. Connect the direct-current machine terminals to a variable load resistance in series with an ammeter and switch.

First excite the field windings, then run up to speed and synchronise the rotary converter with the alternating supply, after adjusting the excitation, so that the voltage is the same as that of the supply. Usually the most convenient method of running the machine up to speed will be to use the D.C. side as a motor and to drive from a source of direct current. In order to do this, break the load circuit, shown on the left of the diagram Fig. 179, and join the two free ends of the circuit thus obtained to the direct-current supply. On closing the switch S_2, the variable resistance

R_1 may be made to serve the purpose of a starting resistance, and the speed of the machine may be regulated by varying this and the field-regulating resistance. If the direct and alternating supply voltages are not of suitable values, difficulty may be experienced in getting both speed and voltage on the alternating side to the correct value for synchronising. In such cases it is often best to run the motor up to a speed which is considerably above synchronism, then switch off the direct-current supply and switch on to the alternating circuit when the motor has slowed down to the correct speed. In this case exact voltage on switching in will usually not be of importance.

After synchronising, cut out the connection to the D.C. supply, then adjust the excitation until the motor takes the minimum current, and complete the direct-current load circuit.

First for no load, and then for a succession of increasing loads, take readings of the alternating and continuous currents and voltages, and also the power supplied as registered by the wattmeter. The speed should also be observed; it should, of course, be constant.

The results should be entered in tabular form, the power-factor being calculated for each set of readings by dividing the watts supplied by the volt-amperes.

Several sets of readings should be taken: first, one or two sets each with constant excitation, and then a set with the excitation varied to give the minimum current on the A.C. side.

TEST OF ROTARY CONVERTER DRIVEN FROM A.C. SIDE.

Rotary converter No. .... Type....................

A.C. supply .... volts.................... amps.

D.C. output .... volts.................... amps.

Speed	Excitation	Alternating			Direct			Primary Power Factor	Efficiency
		Volts	Amps	Watts	Volts	Amps	Watts		

Fig. 180 shows some curves taken on the same machine from which the previous curves were obtained.

The curves C_1 and V_2, drawn as continuous lines, show the variations of alternating current supplied, and direct voltage given out, for various outputs of the converter, the field being adjusted throughout so as to give a mimum current. Under these conditions the power-factor was found to be unity, except when the converter was working at no load, when the power-factor was ·99.

The curve marked "excitation" shows the excitation supplied in order to maintain these conditions.

The dotted curves marked C_1 and V_2 show the primary current

and secondary voltage for the same loads as before, but with a constant excitation of ·35 amps. A comparison of these curves shows a most important property of rotary converters, which will be further alluded to in connection with the compounding of converters. The upper curve V_2 shows that if the power-factor of the supply circuit remains constant there is considerable drop of voltage due to armature reactions as the load increases. Also, in order to maintain the power-factor constant, a decrease of excitation is required.* Further, contrary to what might have been expected, with a constant exciting current, although the current supplied to the armature is much larger, the variation in voltage

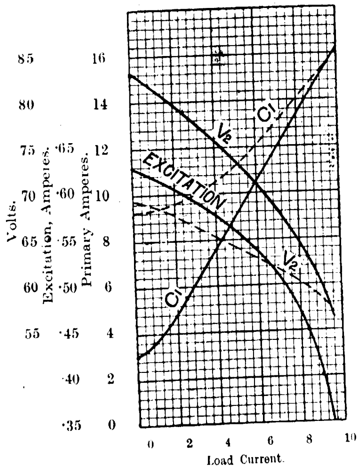


FIG. 180.—Rotary Converter Driven from Alternating Current Side.
Primary Volts 58.
Full-line curves—Excitation for maximum power-factor.
Dotted curves—Constant excitation of ·35 amp.

is much smaller. It is thus found that although the efficiency is greater with the excitation adjusted so as to maintain constant voltage, and the voltage generated is higher, the effect of thus decreasing the excitation is to increase the *variation* of voltage.

Since a diminution in excitation with increase of load produces an *increased* drop in voltage on the direct-current side, it appears at once to be likely that if the excitation were increased with the load, a *decreased* variation in voltage might result. This reasoning

* The percentage variation in voltage and excitation shown in the curves is greater than would be obtained in practice in a large commercial machine.

is supported by facts, and the idea is usually carried out by adding a series winding to the field so as to increase the ampere-turns on the field automatically as the load increases. This matter is further discussed on page 285.

Effect of Variation of Excitation.—In the case of a rotary converter driven from the alternating-current side, it is evident that no change in speed can follow from an alteration in exciting current, since the machine must continue to run synchronously at a speed depending only on the frequency of the supply. It would at first appear as if the direct voltage generated in the armature should vary approximately in the same proportion as the excitation, since the conductors always rotate at a constant speed in the field. This will be found from the experiment which follows not to be the case, for reasons given later in the discussion of the curve obtained from the results.

When driven from the alternating-current side, a rotary converter behaves in most respects similarly to a synchronous motor. Thus, variation in the excitation will produce changes in the phase relations between the current and terminal voltage of the machine.

EXPERIMENT XLI.—DETERMINATION OF EFFECT OF VARIATION OF EXCITATION UPON RATIO OF TRANSFORMATION OF A ROTARY CONVERTER. (2) CONVERTER DRIVEN FROM ALTERNATING SIDE.

DIAGRAM OF CONNECTIONS.

Same as for Experiment XL., Fig. 179.

Instructions.—Connect up and start the machine as described in the case of Experiment XL., page 278.

With the direct-current circuit open, vary the excitation, and for each value of the exciting current take readings of both direct and alternating voltages and the current supplied to the armature. The voltage and frequency of the alternating supply should be kept constant.

The direct-current load circuit should then be closed, and a similar series of readings taken for one or two values of the load current, which should be kept constant during each complete series of readings.

The results should be entered in tabular form, as shown in the case of Experiment XXXVIII., page 271.

Curves should be plotted showing excitation measured horizontally, and primary voltage, primary current, primary watts, and power-factor vertically.

Figs. 181, 182 and 183 give some curves obtained in the manner just described on the same converter as that already experimented upon.

In Fig. 181 are shown the curves of alternating current supplied, and power-factor for no load and for a load of 4 ampeves, equal to about a quarter full load. The smaller the load on the converter,

the greater is the current variation produced by a change in excitation, and the more steep and pointed at the bottom does the curve become. At heavy loads the current curve is comparatively flat and rounded, showing that the permissible variation of excitation is then much greater.

The power-factor curves show somewhat similar features, although the curves are inverted, since an increase of primary current corresponds to a decrease of power-factor when the load is maintained constant. The no-load curve is very steep and pointed, whereas with heavy loads the curves become flat or well

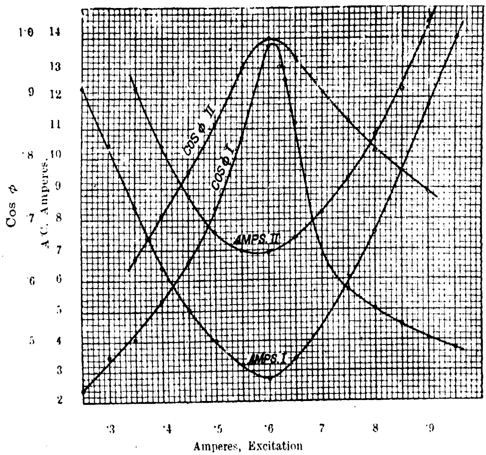


FIG. 181.—Rotary Converter Driven from A.C. Side.
Curves of Current and Power-Factor.
Curves I. no load.
Curves II. 4 amperes load.
Primary volts = 58.

rounded at the top. When the converter is loaded, the power-factor curves will generally rise to unity, as shown in Curve II. At no load, on the other hand, it is not generally possible to obtain a point when the machine works non-inductively. This is due to variations in the wave form of the current which occur under these conditions.

The similarity of the curves in Fig. 181 to those already obtained with a synchronous motor is at once apparent. As explained in connection with the motor curves, the conditions to the left of the point of maximum power-factor correspond to a lagging current, while points to the right indicate leading currents in the supply

circuit. In the present case it is apparent that with less than about ·6 amperes excitation the converter works with a lagging current, while with more than this excitation the current begins to lead. The importance of these V-curves in connection with compounding and automatic regulation of converters will be mentioned later.

In Fig. 182 is shown the ratio of voltage transformation at various excitations, at no-load and with a load of 4 amperes.

Curve I. shows this ratio at no load, and Curve II. at about quarter load consisting of 4 amperes.

The first thing that will be noticed in connection with these curves is the wide divergence they show from the true ratio of

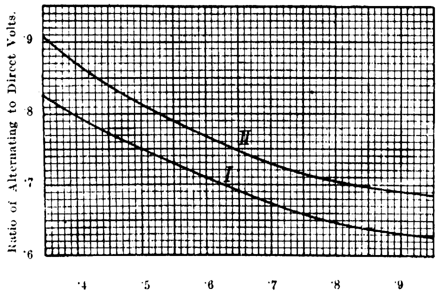


FIG. 182.—Rotary Converter Driven from A.C. Side. Relation between Ratio of Transformation and Excitation.

Curve II. load 4 amperes.
Curve I. no load.
Primary volts = 58.

transformation, which was seen in Fig. 173 to be ·675 for this particular machine.

It must, however, be remembered that the volts lost due to the driving current are not simply the product of current and armature resistance, as was the case when the converter was driven from the D.C. side. The volts lost are now the product of current and armature impedance. Also the phase of the volts thus lost will be more nearly opposed to the applied voltage when the power-factor of the circuit is low and the current lagging. There is thus a very great armature drop at low excitations, both on account of the comparatively high armature current and also on account of the low power-factor and lag in current.

Referring to Fig. 181 the power-factor is at its maximum value at an excitation of ·61 amperes, the power-factor then being ·99. Under these conditions the current taken by the machine is 2·8 amperes. The armature drop is then almost entirely due to armature resistance, and will be nearly in phase with the applied volts.

Thus the drop will be $2 \cdot 8 \times \cdot 145$ volts $= \cdot 41$ volts, the armature resistance being $\cdot 145$ ohms. The A.C. volts actually converted are therefore $58 - \cdot 41 = 57 \cdot 59$, and the ratio of conversion $\frac{57 \cdot 59}{82 \cdot 3} = \cdot 7$ (approximate), $82 \cdot 3$ being the actual voltage on the D.C. side.

In calculating this ratio we have not taken account of the fact that $\cos \phi$ is not exactly unity. The difference between $\cdot 7$ and the ratio of transformation $\cdot 675$ previously obtained is to be attributed to this, to armature magnetic reactions, and the fact that the wave form of the alternating current supplied was considerably more "peaky" than the form of wave given by the converter when driven as a generator.

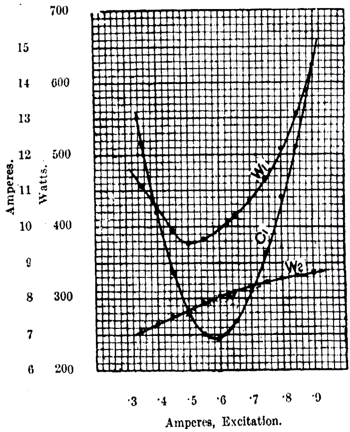


FIG. 183.—Rotary Converter Driven from A.C. Side.
Primary and secondary watts and primary current for constant load of 4 amps.

It is important to notice that the direct voltage continues to rise with increase of excitation after the point of maximum power-factor is reached. This is due to the impedance voltage in the armature now becoming more nearly in phase with the impressed voltage, instead of the generator back voltage, as the current now leads the voltage in the supply circuit.

In Fig. 183 are shown the primary and secondary watts and primary current observed while the converter was supplying a constant direct current of 4 amperes.

The very great variation of primary current and considerable variation in primary power taken are shown very strikingly in

contrast to the nearly constant power output. The secondary watts increase slightly on account of the rise in the ratio of voltage transformation.

Compounding Rotary Converters.—It has been shown (see Fig. 180) that the *most constant* ratio of transformation is not obtained by maintaining the power-factor of the circuit constant, as the fall in pressure on the secondary side is considerable in that case. If a *constant* and not a *maximum* ratio of voltages is required, it would be possible by altering the power-factor of the circuit to produce a lagging current at light loads, and thus reduce the secondary voltage, and to make the power-factor unity at full load, so as to get a maximum voltage in this case which might be equal to, or even higher than, the voltage at no load.

It has been shown by the results of the experiment just described how the power-factor of the supply circuit is altered by change of excitation—an increase of excitation produces an increase of lead in the current.

It would consequently be possible, by increasing the excitation of the converter in the correct proportion to the load, to obtain an approximately uniform ratio of conversion from no load to full load.

The principles governing the compounding of a rotary converter depend, in the first instance, on the behaviour of a synchronous motor supplied with varying excitation, as discussed on pages 238 and 242.

In any synchronous motor (or rotary converter) supplied from a constant voltage there are two sources of excitation, viz., ampere-turns of the field winding and reaction ampere-turns of the armature. To balance a constant applied voltage the sum of these two excitations must remain constant.* If the ampere-turns in the field winding are less than necessary, a current circulates in the armature in such a manner as to assist the field in magnetising the armature. On the other hand, if the ampere-turns of the field winding are more than sufficient to produce the constant voltage required to balance the applied voltage, a current circulates in the armature in such a manner as to oppose the magnetising action of the field winding and to reduce the magnetic induction.

The actual current in the armature may thus be regarded as being composed of (*a*) the useful current driving the machine and enabling it to give the required output at the commutator; this current depends only on the load; and (*b*) a magnetising current, which will be a lagging or leading component, determined by the amount of excitation supplied to the field magnets.

The resulting field thus remains constant and unaffected by changes in exciting current supplied to the field. The only effect of varying the excitation is to change the phase relations of current and voltage in the armature, but to maintain the generated voltage

* The point of view here adopted is that the armature *reactance* may be neglected and that armature currents produce *reaction* only. See p. 242.

at a practically constant value. It is for this reason that it is not possible to vary the commutator voltage directly by regulation of the field current.

The effect of change of excitation may be shown in diagram form, as in Figs. 184 and 185, where the vectors might be taken to represent either voltages or ampere-turns.

In Fig. 184 the voltages are shown for the case of a converter in which the excitation is adjusted so that the terminal voltage and current are in phase, *i.e.*, when $\cos \phi = 1$, $a\ c$ is the voltage spent in overcoming armature impedance, $a\ d$ is the terminal applied volts, $c\ d$ is the generated voltage which determines the direct-current voltage at the commutator.

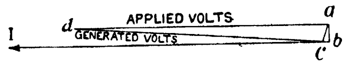


FIG. 184.—Cos $\phi = 1$.

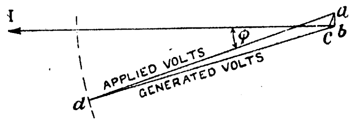


FIG. 185.—Current Lagging.

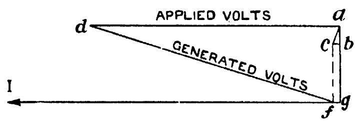


FIG. 186.—Reactance Added. Cos $\phi = 1$.

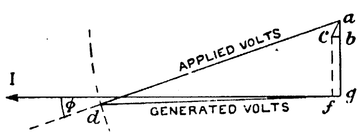


FIG. 187.—Current Lagging.

Phase Diagrams Illustrating Compounding of Rotary Converter.

If the excitation is diminished, the condition becomes that shown in Fig. 185, where the current vector is seen to lag behind the applied voltage vector $a\ d$.

In Fig. 185 the voltage $c\ d$ is seen to be slightly less than in Fig. 184, chiefly because of the reactance voltage $a\ b$ of the armature. The effect on the voltage in an actual machine, if changing from the conditions of Diagram 1 to those of Diagram 2, would be very small, since the sides of triangle $a\ b\ c$ would be very small compared with $a\ d$, or $c\ d$.

By increasing the reactance of the circuit (*e.g.*, by the insertion of a choking coil), we have the means of producing a considerable change in the drop in the reactance and converter, as illustrated in the next two diagrams, Figs. 186 and 187.

In these diagrams a reactance has been introduced of such magnitude as to require a voltage *b g* to send the current through it. The applied voltage (measured outside the converter and inductance) is *a d*, as before, while the generated voltage in Fig. 186 is *f d*, which lags behind the current in phase by the angle *d f l*, and has a greater value than in Fig. 184. The excitation of the motor in Fig. 186 is, in fact, greater than in Fig. 184. In Fig. 186 the leading current due to the over-excitation of the field is just sufficient to compensate for the lagging current due to the reactance introduced into the circuit.

In Fig. 187 is shown the effect of reducing the field excitation and thus producing a lag of the current. It is there seen that the generated voltage *f d* is considerably decreased, as compared with that in Fig. 186. If the excitation were still further increased beyond the value it has in Fig. 186, a further rise in the generated voltage *f d* would occur.

It is thus seen that by inserting a reactance in series with the converter, and maintaining a constant voltage at the terminals of the combination, it is possible to vary the continuous-current voltage by alteration of the field current. This voltage variation is not primarily due to change of field, but is due to the change of power-factor which accompanies the change of excitation, and which produces a variation in the effect due to the voltage lost in the reactance.

This action is made automatic, so as to compensate for drop in the feeders and armature due to load, in the following manner :—

The fields are provided with a series winding, through which the direct-current load passes, in addition to the usual shunt winding.

At no-load, when the shunt windings act alone, the fields are under-excited, so that a lagging current is produced, as shown in Fig. 187. The voltage at the commutator has then its least value. As the load on the converter increases, the excitation increases and gradually raises the power-factor to unity at, say, three-quarters of full load, when the conditions are those shown in Fig. 186. At full load, the current is made to lead the terminal volts, and a further increase in the generated volts is the result. The reactance in series with the converter and the degre of compounding adopted for the fields may be adjusted to maintain the commutator voltage constant at all loads, so that the drop in the armature and alternating-current circuit only are compensated for. It may also be arranged, by choosing a higher value of reactance and greater number of series turns, that a predetermined rise in voltage at the commutator shall occur at full load.

The necessary reactance is usually provided by adjustment of the magnetic leakage of the transformers supplying the converter, so that no extra expense is incurred to produce it.

CHAPTER XI.

THE POLYPHASE INDUCTION MOTOR.

Production of a Rotating Field.—Let the diagrams in Fig. 188 represent a ring-shaped iron core with two separate windings upon it in positions at right angles to each other. The two windings are each wound in halves on opposite sides of the ring. For briefness we will call the winding composed of the top and bottom coils the "vertical" winding, and the other the "horizontal" winding. A current sent through either winding will have the effect of

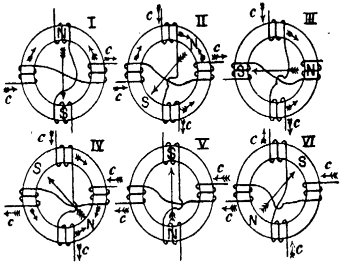


FIG. 188.—Production of a Rotating Field by 2-phase Currents.

magnetising the ring, producing north and south poles at points in the ring half-way between the two coils composing the winding. Imagine a magnet needle to be pivoted in the centre of the iron ring so as to be free to rotate in the plane of the ring. This is represented by the long arrow in the centre of the ring.

If a current is sent through the horizontal winding only, as indicated in Diagrams I. and V., the ring will be magnetised with a north pole at the top or bottom, according to the direction of the current, and the needle will set itself at right angles to the lines of this page, as shown. If the same current is sent through both vertical and horizontal coils (Diagrams II. and VI.) the needle will take up a position 45° from its previous direction. It will occupy this position whatever the strength of the current may be, as long as the same current passes through both coils; consequently, if the current in the two coils is an alternating one, the field will always be formed along this axis, but the sense and strength of the magnetic field will undergo the same changes as the current.

The magnet would in this case tend to set itself in the direction of the field, with its axis along a fixed line, but with the relative position of north and south poles rapidly reversed. The magnet would consequently not rotate (unless given an initial rotation by hand), but would receive rapid impulses in the nature of alternate pulls and pushes in the direction of its axis at every reversal of the current.

If the currents in the two coils do not vary simultaneously, but alternate one after the other, the result will be that the magnet will tend to rotate in the manner illustrated by the six diagrams in Fig. 188.

Thus, suppose (Diagram I.) the current flows from left to right, through the horizontal coil, the needle will point downwards. On a current being started downwards in the vertical coil (Diagram II.) the needle is deflected to the left. If, now, the current in the horizontal coil ceases (Diagram III.), the needle will point horizontally to the left. As a current is started in a reverse direction, *i.e.,* from right to left, the needle inclines upwards (Dia. IV.). And so the changes may be followed further.

A step-by-step motion of rotation would thus be maintained by such a sequence of changes in the currents. The cycle of changes just suggested would be experienced if a 2-*phase* current were supplied to the windings, one phase being connected to the vertical coil, and the other phase to the horizontal coil, except that in this case the changes would occur gradually, instead of step by step. It will be shown that the resultant field produced by a 2-phase current flowing in two windings situated perpendicular to each other is *constant in strength,* and that its direction changes gradually and uniformly from the axis of one coil to the axis of the other. A 2-phase current thus produces a *rotating magnetic field* of constant strength, which would make one complete revolution of the field formed within the two coils in the time of one period of alternation of the current.

A similar effect may be produced by means of three coils, situated at an angle of 120° apart, and supplied each with current from one phase of a 3-phase system. In this case, also, the current in the several coils will attain its maximum value successively, and the magnet would be attracted into a position normal to each coil in turn. The magnetic field in this case also makes one revolution in the time of one complete cycle of the current.

The rotating field is employed in the case of all polyphase induction motors, and the principle of its production is the same as that just described. The method of winding the wire upon the iron magnetic circuit of a motor is necessarily somewhat different from that shown in Fig. 188.

In an actual induction motor a laminated ring-shaped core is employed as typified by the ring in Fig. 188. In place of the pivoted magnet, however, a cylindrical iron core is used with conductors embedded in slots in its circumference. The nature of the magnetic field produced in the ring and inner core is illustrated

by Fig. 189, where a single winding only is shown carrying current. Exactly the same magnetic field would be produced by applying the winding to the ring in any other position, so long as the winding embraces the lines forming the magnetic flux. Thus, in Fig. 190 the winding is shown as embracing the magnetic flux at the point where it comes to the air gap and passes from the ring to the inner core.

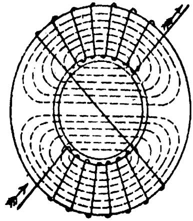


FIG. 189.—Field in Induction Motor.

The advantage of putting the winding in the position shown in Fig. 190 is that the coil can be wound complete, and then applied to the inside of the ring, the projecting end connections of the winding being bent over, as indicated in the diagram.

Proof of the Constancy of the Rotating Field.—The simplest method of proving generally that a constant rotating field is

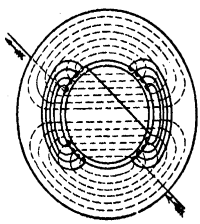


FIG. 190.—Field in Induction Motor.

produced by a polyphase current flowing in equally spaced coils is to resolve the field due to each phase into two components in two fixed directions at right angles to each other. The component fields in each of the two directions are then added together, and the resultant of the two fields thus found is calculated as the square root of the sum of the squares of these two mutually perpendicular fields

The method is generally applicable to currents having any number of phases; it assumes that the distribution of the field due to each winding is sinusoidal. The fact that this condition is only imperfectly fulfilled in an actual motor accounts for a certain amount of fluctuation which occurs in the strength of the field in practice.

The following calculation applies to a 3-phase circuit.

In this case there are three coils, situated at 120° from each other, and carrying varying currents, which are assumed to produce fields proportional at any moment to the strength of the current in the coil. Also the strength of the field due to each coil is taken as varying sinusoidally round the air-gap.

Let f_1, f_2, f_3 = the number of lines induced in the three coils I., II., and III., at the instant under consideration.

F = max. value of the number of lines induced in each coil.

F_r = number of lines forming the resultant field.

The values of the fields due to the several coils may be written in the form $F \sin \theta$, since they vary harmonically in the same manner as the current, and attain a maximum value F.

Hence, at the instant under consideration, they will have values as follows, where θ can be chosen to have any value from 0 to 360°.

$f_1 = \mathrm{F} \sin \theta$ in the direction $0\,F_1$ in Fig. 191.

$f_2 = \mathrm{F} \sin (\theta - 120)$ in a direction $0\,F_2$.

$f_3 = \mathrm{F} \sin (\theta + 120)$ in a direction $0\,F_3$.

Resolve each of these fields in two directions respectively parallel and perpendicular to the direction of the field of coil I.

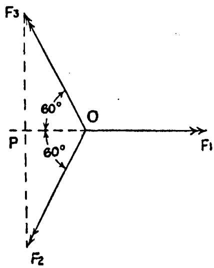


FIG. 191.—Three Magnetic Fluxes due to 3-phase Current.

From Fig. 191 it is clear that the components are

	Horizontal.		Vertical.
for f_1, . . .	$0\,F_1$	and	*zero*
„ f_2, . . .	$0\,P$	„	$P\,F_2$
„ f_3, . . .	$0\,P$	„	$P\,F_3$

Let us first evaluate the **horizontal** components :—

for f_1, $\quad 0F_1 \qquad = F \sin \theta.$

" f_2, $\quad 0P = 0F_2\,(-\cos 60°)$
$= F \sin(\theta - 120°)\,(-\cos 60°) = -\tfrac{1}{2} F \sin(\theta - 120)$

" f_3, $\quad 0P = 0F_3\,(-\cos 60°)$
$= F \sin(\theta \times 120)\,(-\tfrac{1}{2}) \quad = -\tfrac{1}{2} F \sin(\theta + 120).$

By addition we get : $0F_1 + 0P + 0P$

$$= F \ \sin \theta - \tfrac{1}{2}\sin(\theta - 120) - \tfrac{1}{2}\sin(\theta + 120).$$
$$= F\{\sin \theta - \tfrac{1}{2}[\sin(\theta + 120) + \sin(\theta - 120)]\}$$
$$= F\{\sin \theta - \tfrac{1}{2}\, 2 \sin \theta \cos 120\}$$
$$= F\{\sin \theta + \tfrac{1}{2} \sin \theta\}$$
$$= \frac{3}{2} F \sin \theta$$

On evaluating the **vertical** components we get :— $\qquad$ *i.e.*, 0.

for f_1, $\qquad$ *zero*

" f_2, $\quad PF_2 = 0F_2\,(-\sin 60°) = -F \sin(\theta - 120)\dfrac{\sqrt{3}}{2}.$

" f_3, $\quad PF_3 = 0F_3 \quad \sin 60° = +F \sin(\theta + 120)\dfrac{\sqrt{3}}{2}$

Adding,

$$0 + PF_2 + PF_3 = F\left\{\sin(\theta + 120) - \sin(\theta - 120)\right\}\frac{\sqrt{3}}{2}$$
$$= F\frac{\sqrt{3}}{2}\, 2 \cos \theta \sin 120$$
$$= F\sqrt{3} \cos \theta \frac{\sqrt{3}}{2}$$
$$= \frac{3}{2} F \cos \theta.$$

The three fields are thus equivalent to two fields at right angles to each other, whose values are :

$$\frac{3}{2} F \sin \theta \text{ and } \frac{3}{2} F \cos \theta.$$

The resultant field formed of these two will be :

$$F_1 = \frac{3}{2} F \sqrt{\sin^2 \theta + \cos^2 \theta} = \frac{3}{2} F.$$

Hence the resultant field is constant in value and equal to $\frac{3}{2}$ the maximum value of each field taken singly.

Further, if a is the angle which this resultant field makes with the vertical

$$\tan a = \frac{\text{sum of horizontal components}}{\text{sum of vertical components}} = \frac{\frac{3}{2} F \sin \theta}{\frac{3}{2} F \cos \theta} = \tan \theta$$

Hence $a = \theta$, and the angle of the resultant field will vary at the same rate as the phase of the separate fields, and the resultant field will complete one revolution in the time of one period of the alternating current, rotating with constant angular velocity.

It may be shown that if the system is that of an m-phase current, the magnitude of the resultant field would be $\frac{m}{2} \times$ maximum strength of each individual field.

With regard to the direction of rotation of the field, it is evident that it will revolve in the direction in which the current in the successive coils attains its maximum value. For instance, if after the coil I. has its maximum current, the current in coil II. increases to its maximum value, the field will rotate in the direction from I. to II., and then to III., &c. If, on the other hand, the current in coil III. reached its maximum value next after coil I., the direction of rotation would be I., III., II., &c. Thus by interchanging the connections of coils II. and III. the direction of the rotation of the field is reversed. Consequently in a 3-phase motor supplied with current at three terminals, the direction of rotation will be reversed by interchanging the connections of any two terminals to the supply circuit.

Rotation of Rotor of Induction Motor.—It has been explained in the previous paragraph that the effect of polyphase currents passing through a polyphase winding on a circular magnetic field system is to produce a rapidly rotating pole, so that the result is similar to the rotation of a permanently magnetised ring with poles produced at fixed and equal intervals along the circumference.

In an induction motor, the field system surrounds an armature consisting of an iron core carrying a number of conductors round its circumference. These conductors are usually all connected together, so as to form a number of complete electrical circuits. The lines of force produced by the windings on the ring pass into the armature or rotor, and complete their path to the nearest magnet pole of opposite polarity, as indicated for the case of a single pair of poles in Fig. 190. In passing from ring to armature, and again from armature to ring, the lines of force cross the layer of armature conductors. If the armature remains stationary, the revolving poles will generate a rapidly-alternating electromotive force in the armature conductors in the same way as the revolving field of an alternator induces currents in the conductors of its armature.

If an alternator armature carries no current there is no torque between the armature and field. When the armature circuit is closed, so that the conductors have a current formed in them, a force is at once set up tending to retard the revolving field and opposing the relative motion of armature and field. This force increases in direct proportion to the current carried by the armature conductors.

In the induction motor exactly the same action occurs, and, if the electromotive force generated in the armature conductors is

allowed to produce a current, this current will produce a force acting on the rotating field opposing the relative motion of the field and armature, and tending both to retard the rotating field and to cause the armature to revolve with it.

Since the field always rotates at a fixed speed, depending only on the frequency of the current supplied to it, the force cannot retard the field, but the field will drag the armature round with it at a speed depending upon the ease with which the shaft can be turned. The force producing the motion of the armature of an induction motor is, consequently, essentially the same as that which is sustained by the stationary armature of a revolving-field alternator, and its value is calculated in a similar manner.

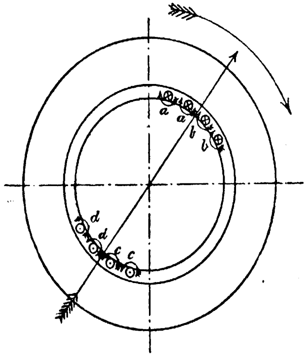


FIG. 192.—Induction of Currents in Rotor.

Thus the torque of the motor is proportional to the product (field strength) × (armature current).*

In an induction motor the armature is usually called the *rotor*, since it is the rotating member, while the circular field system is called the *stator*, since it forms the stationary part. These terms will in future be used.

Production of Current in Rotor Winding.—The diagram (Fig. 192) indicates the outline of the stator and rotor of an induction motor, a few conductors being shown to indicate the rotor winding. The diagonal arrow indicates the position of the rotating field at the moment under consideration, and the curved arrow shows the direction of its movement.

Applying Fleming's rule for the production of a current (see Fig. 1, page 14), it will be seen that the direction of the current in the conductors in the upper part of Fig. 192 will be away from

* Phase difference between these quantities is neglected in this statement.

the reader (as indicated by the crosses), while in the lower conductors the current will flow towards him. The direction of the lines of force set up in the immediate neighbourhood of the conductors by these currents is shown by the small circular arrows. The direction of these arrows shows that immediately in front of the rotating lines of the main field the induced field in the rotor is in the same direction as, and consequently repelled by, the main field, while behind the main field the lines are oppositely directed, and are attractive.

The strength of the current in the rotor conductors is given by the usual rule :—

$$\text{Current} = \frac{\text{E.M.F.}}{\text{Impedance}}$$

$$\text{E.M.F.} = \frac{\text{rate of cutting lines of rotating field}}{10^8}$$

The impedance depends upon the resistance of the conductors, the method of connecting them together, and the self-induction of the rotor winding.

If the rotor is stationary, the lines of each pole of the rotating field will cut each of the conductors f times per second (where f = frequency of current supplied to the motor). If, however, the rotor rotates in the same direction as the field n times per second, the flux under each pole will only be cut $f - n$ times per second by each conductor.

Consequently, with a constant rotating field the electromotive force induced in the rotor conductors is proportional to the *difference in speed* of rotating field and rotor.

The difference between the revolutions per second of the rotor and of the rotating field of a 2-pole motor is called the *slip* of the motor.

Thus in a 2-pole motor : slip* $= f - n$ cycles per second.

where f = periodicity of current
= revolutions per second of rotating field.
n = revolutions per second of rotor.

The slip of a multipolar motor is $f - n\,p$ cycles per second where p = number of pairs of poles, and the synchronous speed (*i.e.*, the speed of the rotating field) of such a motor is $\frac{f}{p}$ revs. per second.

It is important to notice that, according to the definition of slip just given, *the slip is equal to the periodicity of the alternating currents induced in the rotor.*

The electromotive force induced in the rotor conductors by a constant rotating field is a maximum when the rotor is stationary, *i.e.*, with a slip equal to f, and is zero with the rotor revolving

* Sometimes the slip is defined as being equal to $\frac{f - n}{f}$, *i.e.*, the ratio of the difference of speeds to the frequency. This would be better defined as the "fractional slip," or, when multiplied by 100, as the "percentage slip."

synchronously, *i.e.*, at the same rate as the field, when slip $= O$. The curve representing the dependence of induced rotor voltage upon slip is a straight line, as in the case of an alternator or direct-current dynamo.

The wave-form of the voltage induced in the conductors of the rotor must depend on the distribution of the magnetic field in the air-gap between stator and rotor in which the conductors rotate. Usually, the field is so distributed that no serious error is introduced by assuming that the electromotive force produced is sinusoidal, *i.e.*, the curve representing its variation is a sine curve.

If the effect of magnetic leakage could be neglected, we might say that this electromotive force would give rise to a current given by the fraction

$$\frac{\text{E.M.F.}}{\text{resistance of rotor winding}}$$

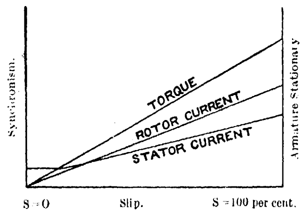


FIG. 193.—Relation between Slip and Torque and Currents in Motor without Leakage.

The current would consequently increase in direct proportion to the slip, producing a turning moment also proportional to the slip. We shall find later that this simple relation is modified by the rotor magnetic leakage or self-induction.

The relations which would exist between slip, torque, and current in the ideal motor without magnetic leakage are indicated in Fig. 193. The stator current would be proportional to the rotor current except for the no-load current, which is indicated by the height at which this curve cuts the zero ordinate.

Effect of Magnetic Leakage.—The leakage occurring in an induction motor is of two kinds, viz., that in the stator and that in the rotor.

The stator leakage field is formed of magnetic lines which do not enter the rotor, but pass across from tooth to tooth of the stator core without crossing the air-gap between stator and rotor, and so do not affect the rotor conductors.

Similarly, the rotor leakage lines are formed in the rotor core, and do not pass into the stator.

The leakage field thus consists of all the magnetic lines which do not form part of the rotating field. The leakage in either stator

or rotor is increased by making the slots nearly or entirely closed, since it then becomes relatively easier for the lines to pass from tooth to tooth in the same core rather than to pass twice across the air gap into the opposite core. The path of the leakage lines is illustrated in Fig. 194.

The effect of the stator leakage is that a certain number of the lines formed by the stator current have no effect upon the rotor, and that a portion of the applied voltage is spent in overcoming the back electromotive force set up by this portion of the field. In other words, the leakage field of the stator forms the self-induction of the stator winding. It does not produce any waste of power, but diminishes the output of the motor, since it lowers the amount of useful voltage applied to the machine.

The effect of the magnetic leakage in the rotor is somewhat more complicated. The leakage lines do not pass into the stator, and are consequently not neutralised by the stator field; but

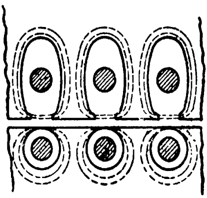


FIG. 194.—Leakage Lines in Stator and Rotor.

they give rise to a back electromotive force of self-induction in the rotor winding, and produce an increased apparent resistance in this circuit, and a lag in phase of the rotor current behind the induced voltage.

The voltage induced in the rotor conductors will consequently be spent in overcoming the reactance (due to this self-induction), as well as the resistance of the conductors. The voltage relations in the rotor may be represented by a triangle, as in Fig. 195.

Here $O\ A$ is the voltage induced in the rotor conductors, proportional to the slip of the rotor, $O\ B$ is the component of this voltage overcoming resistance and proportional to the rotor current. $B\ A$ is the idle voltage overcoming reactance. The value of the reactance is not constant, but varies with the frequency of the currents in the rotor conductors. This frequency has already been stated to be equal to the slip. The reactance is consequently expressed as $2\ \pi\ s\ L_2$, where s is the slip of the motor in cycles per second and L_2 is the coefficient of self-induction of the rotor winding.

As the slip of the motor increases, the total induced rotor voltage will increase in the same proportion. The rotor current, therefore, also rises, but in a less ratio, because of the increased

reactance. Also its phase will show an increasing angle of lag behind the voltage, due to the same cause.

The angle ϕ in the diagram is the angle of phase difference between the current in a rotor conductor and the air-gap flux in which the conductor is situated, because the induced electromotive force $O\,A$ is proportional to the flux cutting the conductors at any instant, so that $O\,A$ shows the phase of the flux cut by the conductor.

The current, therefore, lags behind the flux in phase. This is of importance, because the torque of the motor is due to the action of the rotor currents on the flux in the air-gap, and its magnitude is reduced by this phase difference between the two quantities. The triangle shown in Fig. 195 forms the basis of further discussion on page 312.

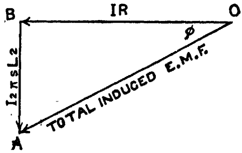


FIG. 195.—Voltages in Rotor Circuit.

Measurement of Slip.—Since the slip of a motor is only a small percentage of its speed, it cannot be accurately measured by a tachometer of the usual type, as the readings of such an instrument cannot usually be read with a closer approximation than about 1 per cent, on account of the large range of the scale. Even this degree of accuracy can hardly be relied upon when two belt-driven tachometers are employed—one on the generator and one on the motor. As the slip of an induction motor is only about 2 to 5 per cent of its speed, it is evident that more exact methods of measurement must be employed.

The simplest method is to apply simultaneously a speed counter to the shaft of both motor and generator for one minute. If both machines are in the same room, two observers can easily signal to each other, so as to obtain exact coincidence in the times of reading. If the generator is at a distance, it is often possible to run a temporary pair of wires for a signal bell or lamp from one to the other. In such a case a code of signals for "ready," "on," "ready," "off," must be arranged, the warning "ready" before either applying or removing the counter being essential for reliable working.

If n_1 and p_1 are the revolutions per second and number of pole pairs of the generator, and n_2 and p_2 are the revolutions per second and number of pole pairs of the motor, the slip in cycles per second is given by

$$s = n_1\,p_1 - n_2\,p_2,$$

or, when stated as a percentage of the supply frequency,

$$s \text{ per cent} = 100 \frac{(n_1 p_1 - n_2 p_2)}{n_1 p_1}$$

In some cases it may be possible to arrange a gong on the shaft of the generator and a hammer on the shaft of the motor in such a way that the hammer strikes the gong each time the generator shaft makes one revolution more than the motor. If the two machines have the same number of poles, the number of blows per minute divided by $60 \times p$ equals the slip of the motor in cycles. If the number of poles is not equal, the method is not applicable, since the sounds would be too rapid to be distinguishable.

Instead of a bell and hammer, discs may be fixed to the shafts of the machines with sectors cut out of them, bearing a definite ratio to the number of poles. By viewing the discs in line with one another, the number of times one disc makes one more revolution than the other can be seen.

Another simple method is to insert an ammeter in one of the phases of the rotor circuit. The needle of the ammeter will be deflected by the slowly varying currents in the rotor. The number of deflections may be counted, and will equal the number of half-cycles of slip of the rotor. By employing a moving-coil direct-current ammeter, the number of vibrations will be equal to the rotor slip, since the needle will only respond to currents in one direction. It is consequently easier to count the vibrations than when an alternating-current instrument is used, which gives a deflection for the currents induced in both directions. It is hardly possible to count much more than 150 vibrations per minute, which, with a frequency of 50, corresponds to a maximum slip of 5 per cent. if a C.C. ammeter is used.

If the rotor is of the squirrel-cage type, an ammeter cannot be inserted in its circuit. In this case a revolving contact maker may be employed. Any form of contact maker described in Chapter IV. might be employed, but a small stud on the rim of the motor coupling will serve the purpose. In this case one of the mains is connected to the shaft of the motor, and another to a light spring (which is touched by the stud as it revolves) in series with a dead-beat voltmeter. The voltmeter will then show a full deflection each time the moment of contact coincides with a maximum voltage. If there are as many contact points as there are pairs of poles, the deflections of the voltmeter in one direction will equal the slip. If only one contact is employed, the deflections must be multiplied by the number of pairs of poles.

Another method of determining the slip when this is not great is to employ an alternating arc lamp or metal-filament incandescent lamp. This lamp is supplied with current from the same source as the motor, and the light given out by it will flicker with the velocity of the alternation of this current. If alternate white and black sectors are painted on the motor pulley, or on a piece of card attached to the motor shaft, these sectors will rotate with a speed slightly less than that of the flickering of the arc. At each period

of maximum illumination the sectors will consequently be slightly behind the position which they occupied at the last time of illumination. The sectors will thus appear to rotate slowly backwards, and, if the number of white segments is equal to the number of N-poles on the motor, they will make the same number of apparent revolutions as the revolutions lost by the motor.

The time taken by one dark segment to take the place of the next dark segment, corresponds to the duration of one cycle of slip.

A very simple device which may be employed to measure the slip in an induction motor provided with external starter is a small pivoted compass needle.

This needle is placed immediately above, or below, one of the connections to the rotor starting resistance, this wire being placed in such a direction that the needle normally points along the wire, so that the alternating currents induced in the rotor will deflect the needle. Thus, as in the case of an ammeter in the rotor circuit, the number of deflections in either direction gives the value of the slip.

Another method, often adopted in works, is to connect a sensitive galvanometer to the terminals of a coil of many turns (*e.g.*, the field winding taken off a shunt motor), and to place the coil near the motor. After moving the coil by trial into the best position for the purpose, it will be found that the galvanometer will give a kick in one direction for each period of slip of the motor. This method has the advantage that it is applicable to any kind of polyphase induction motor, and may be used even with a totally enclosed motor having a squirrel-cage rotor.

Calculation of Slip from Rotor Resistance.—The slip at any load can be approximately predetermined without actual measurement from a knowledge of the rotor current and resistance, as given in the following formula for a motor having a 3-phase rotor :

$$\frac{s}{f} = \frac{3\, i_2^2\, r_2}{W + 3\, i_2^2\, r_2}$$

the symbols having the following significations :—

s = Slip in cycles per second.
f = Frequency of supply.
p = Pairs of poles.
i_2 = Current in rotor winding per phase.
r_2 = Resistance of rotor winding per phase.
W = Output of motor in watts, including frictional losses.

The above formula may be arrived at from the following considerations (see page 327) :—

$$\text{The torque on a generator armature} = \frac{\text{watts output}}{\text{speed relative to field}}$$

The watts generated in the rotor armature are $3\, i_2^2\, r_2$, and hence

$$\text{torque on motor shaft } \frac{3\, i_2^2\, r_2\, p}{2\pi\, s}$$

Also torque exerted by the rotor is equal to

$$\frac{\text{output of motor}}{\text{speed of rotation of shaft}} = \frac{W}{2\pi n} \text{ when } n = \text{revs. per sec. of rotor.}$$

Hence torque $= \dfrac{3\, i_2^2\, r_2 p}{2\pi\, s} = \dfrac{W}{2\pi\, n}$

Adding the two ratios together we obtain

$$\frac{W + 3\, i_2^2\, r_2}{2\,\pi \left(n + \frac{s}{p}\right)} = \frac{3\, i_2^2\, r_2\, p}{2\pi\, s}$$

or since $n + \dfrac{s}{p} = \dfrac{f}{p}$ we obtain

$$\frac{s}{f} = \frac{3\, i_2^2\, r_2}{W + 3\, i_2^2\, r_2}$$

as in the equation above.

Measurement of Resistance of Windings.—The measurement of the resistance is usually carried out by sending a measured direct current through the windings, and noting the drop in voltage. In 3-phase motors the resistance per phase will not be obtained by this method. In a star-connected stator there will be two windings between each pair of terminals, and the resistance measured will be twice the resistance per phase. In the case of a mesh-connected winding, there will be two paths in parallel between the terminals, one having the resistance of one phase and the other of two phases in series. The measured resistance will in this case be two-thirds of the resistance of one phase.

If the system of connection is not known, the copper loss may be obtained as follows :—

The resistance between each pair of terminals is measured, the three resistances so obtained being then added together ; the result when divided by two is sometimes called the "equivalent"* resistance of the winding. It is evident that for a star-connected stator the equivalent resistance

$$= R = \frac{3 \times 2r}{2} = 3r$$

where r = resistance of each phase winding.

In the case of a mesh-connected winding

$$R = \frac{3}{2} \cdot \frac{1}{\frac{1}{r} + \frac{1}{2r}} = \frac{3}{2} \times \frac{2}{3} r = r$$

The special convenience of this equivalent resistance is that when multiplied by the square of the current supplied to the motor terminals it gives the copper losses in watts directly.

Thus in the case of the star winding, losses $= 3\, i^2\, r$, and since line current = current per phase and $3r = R$,

$$\text{losses} = I^2\, R.$$

* The term "equivalent" as here employed must not be confused with its more general use in connection with transformers, &c. (see p. 1.0).

Similarly, with mesh connection, phase current $\frac{1}{\sqrt{3}}$ line current,

and losses $= 3\left(\frac{I}{\sqrt{3}}\right)^2 r = I^2 r$

And since in this case $r = R$,
We have watts lost in resistance $= I^2 R$.

In the case of large motors, eddy currents will have the effect of increasing the resistance of the conductors, as mentioned on page 193, in connection with alternator armature windings.

No-load Curves of an Induction Motor.—No-load curves may be obtained on an induction motor under either of two conditions, viz.: (1) with the armature shaft free to rotate, so that the motor runs (except at very low voltage) at almost the speed of synchronism, and (2) with the rotor rigidly clamped, so that it remains stationary and the slip is 100 per cent.

Experiment XLII.—No-load Test of an Induction Motor at Varying Voltages.

Diagram of Connections.

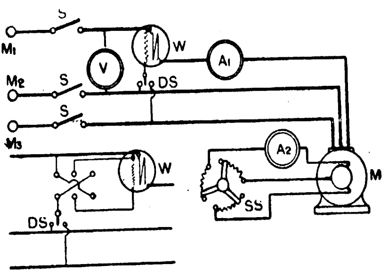


Fig. 196.

$M_1 M_2 M_3$ Source of 3-phase alternating current.
M Induction motor.
A_1 Ammeter in supply circuit.
A_2 Ammeter in rotor circuit.
V Voltmeter reading supply voltage.
W Wattmeter.
$S\ S\ S$ 3-pole switch.
$D\ S$ 2-way voltmeter switch.
$S\ S$ Motor starting switch.

Connections.—Connect the stator windings of the motor to the source of alternating-current, inserting an ammeter for reading the current supplied, and a voltmeter to read the voltage. Connect

the rotor windings to the usual starting switch after inserting an ammeter of low resistance in one of the phases. The starting switch and resistance may be dispensed with, and the rotor may be short-circuited directly through the ammeter if the motor is always started at a reduced voltage. If the motor is of the squirrel-cage type, of course no starter or ammeter can be employed.

Means must be provided for varying the primary voltage through a considerable range. This may be done either by inserting a variable 3-phase resistance in the primary circuit, or preferably by varying the voltage of the alternator supplying the circuit by means of its shunt regulator. In either case the periodicity of the circuit is to be kept constant.

A wattmeter must be inserted in the supply circuit in order to read the input watts. This may be a single-phase wattmeter connected in one phase, and so reading $\frac{1}{3}$ of the total power.* It may be a 3-phase wattmeter reading directly the total power, or it may be a single-phase wattmeter arranged for the volt coil to be thrown over from one main to the other. This is indicated in the diagram in Fig. 196.

It will probably be found necessary to add a reversing switch for the shunt coil of the wattmeter, as indicated in the left-hand lower corner of Fig. 196. This necessity is owing to the fact that in throwing over the shunt coil from one main to the other the reading of the wattmeter will be found to be reversed in some cases, although not in all, since the relative direction of the readings will depend upon the value of the power-factor. (See page 255.) If the reversing switch has to be thrown over between the readings on the two mains, one value observed must be considered *negative*, and must be subtracted from the other to give the total watts supplied. In other cases the *sum* of the readings gives the total watts. If a 3-phase wattmeter is employed, it reads the power of the whole circuit directly without any throw-over switches.

In the case of an unloaded motor, it is not usually necessary to take special precautions to guard against want of symmetry in the three circuits of the motor, as the current is small, and the power depends on the iron circuit, rather than on the resistance of the windings.

Instructions.—After starting the motor in the usual way, light and with belt off, adjust the primary voltage to the full working pressure of the motor. Take readings of the stator and rotor currents, watts supplied (by readings on the wattmeter with the shunt coil connected first across one pair of mains and then across the other). Observe also the primary voltage and the slip (by any of the methods described on page 298, *e.g.*, by counting the swings of A_2.)

Repeat the readings for gradually decreasing values of the

* See Method III. for reading power, p. 257.

primary voltage, the periodicity of the supply being maintained constant.

Readings should be continued in this manner until a point is reached when the motor speed falls rapidly.

From the readings of the wattmeter and primary ammeter and voltmeter, the value of the power-factor $\cos\phi$ should be calculated. The power in a 3-phase circuit $= \sqrt{3}\ I\ E\ \cos\phi$ where I and E are the line current and voltage as measured on A_1 and V. Consequently the power-factor is calculated from the formula

$$\cos\phi = \frac{\text{true watts.}}{\sqrt{3}.\ I \times E}$$

The following table shows a few sample readings from a test on a 5 h.p. Electrical Company's motor, and illustrates the manner of entering up the results. The readings are plotted on the curves shown in Fig, 197.

NO-LOAD TEST OF 3-PHASE INDUCTION MOTOR.

Motor No.......... Type : Electrical Company's 4-pole.
Output 5 h.p., at 1,460 revs. per minute.
Voltage 200. Frequency 50 cycles per second.

Primary Volts $= E$	Primary Current $= I$.	Speed Revs. per min.	Secondary Current.	Primary Watts.			Apparent Watts $\sqrt{3} EI$	Power Factor $\cos\phi$
				Volt Coil Phases I. & II.	Volt Coil Phases I. & III.	Total.		
Volts decreasing from Maximum Value.								
234	8·1	1450	1·2	1000	− 496	·504	3280	·153
124	3·8	1450	2·3	300	− 7	293	817	·359
25	6·0	1260	6·0	144	+ 84	228	260	·878
Volts increasing from Minimum Value.								
20	11·5	20	16·7	210	− 20	190	398	·478
30	17·4	107	25·1	500	− 70	430	895	·482
36	21·0	600	30·3	780	− 100	680	1310	·519

In Fig. 197 it will be noticed that there are two completely separate curves for each of the quantities plotted—one portion to the extreme left of the diagram and another more extended portion to the right. We meet here for the first time with an illustration of the two conditions under which an induction motor will run. It is found in many experiments that under certain conditions the motor will run at either of two speeds one of these speeds being more stable than the other In the present case, for instance, at 30 volts (see Fig. 197), the motor would run either

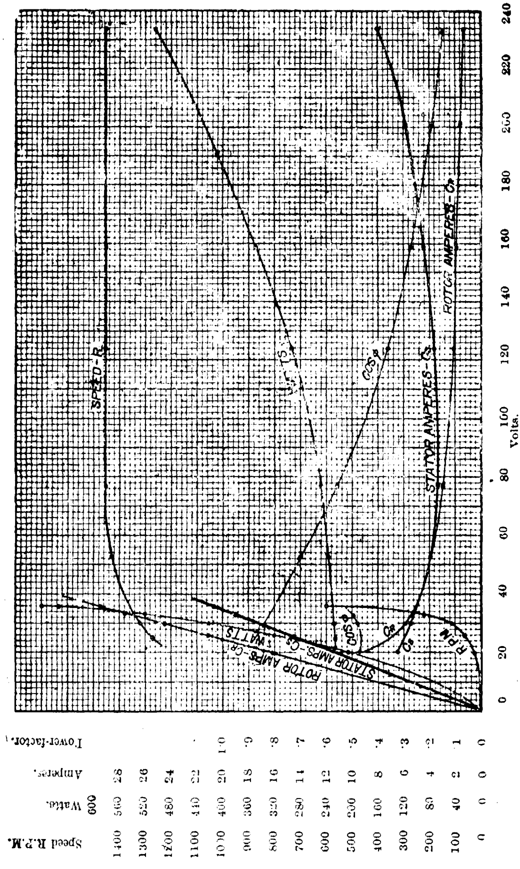


FIG. 197.—No-Load Test of Induction Motor.

at 1,320 revs. per minute, as indicated on the upper curve, or at 110 revs. per minute, shown on the lower curve. Corresponding to either speed is found a set of internal conditions of currents, watts, &c., quite distinct from those existing at the other speed. The external condition, *i.e.*, the nature of the voltage applied, was identical in the two cases. At points where the two sets of curves overlap, the curves corresponding to the higher speed in all cases represented the stable conditions—*i.e.*, the motor after running a short time at the lower speed would generally speed up until the conditions of the right-hand curves were reached. The unstable curves are not of practical value, since they do not represent the usual running conditions. To obtain them, the voltage was raised very gradually from zero, readings on the instruments being taken as rapidly as possible at points where the motor tended to increase its speed to the values of the upper curve. In order to get the overlapping part on the higher speed curves, the voltage was gradually reduced, care being taken to avoid sudden changes of any kind, which would result in the speed of the motor falling to its lower value when nearing the lower end of the curve.

The most important characteristics of the curves taken on the motor on no-load may be briefly summarised :—

(1) *Speed* remains practically constant until very low voltages are reached. Unless heavily loaded, the speed of an induction motor is affected very litttle by fluctuations of voltage. The slip depends under normal conditions only on the load, and will be found in the later experiments to vary almost in a constant ratio with the load within the working range.

(2) *Rotor current* is small, and falls almost uniformly over the greater range of voltages. The torque being nearly constant, the rotor current varies in an inverse ratio to the strength of the rotating field (*i.e.*, to the stator voltage), in order that the product of rotor current × flux may give the required torque.

(3) *Stator current* rises gradually on account of the increase in magnetising and iron-loss current required to produce the stator flux, which, in turn, bears a constant ratio to the applied voltage. The component of the stator current which provides the ampere-turns balancing the rotor ampere-turns will steadily diminish as the rotor currents decrease. The increase in the magnetising component is, however, more than sufficient to balance this decrease. At very low voltages the induction is so low that almost the whole of the stator current is employed in balancing the rotor currents. At normal voltage the rotor currents require only a small proportion of the stator currents to balance them, and the higher saturation of the magnetic circuit requires a much stronger magnetising current to maintain the air-gap flux. This gradual change is well indicated by the next curve.

(4) *Power-factor.* As just explained, the magnetising component of the stator current becomes larger as the voltage increases. This accounts for the continuous fall in the curve of $\cos\phi$. It

is to be remembered that the friction losses of the motor are practically constant, since the speed variation is so small. At low voltages there is very little magnetising current, but almost the normal amount of energy current to overcome friction losses. With increased voltage both magnetising and iron-loss currents will increase, but since the iron losses will at first be only a small fraction of the total no-load loss, the proportion of magnetising to energy current will increase, producing a decrease in power-factor.

(5) *Watts.* The curve of watts is a curve of total no-load loss, and includes watts spent in overcoming both iron and friction losses. As already stated, the friction losses are nearly constant at all voltages, or until the motor speed falls rapidly, while the iron losses continue to increase with the induction in the iron circuit. By producing the watt-curve to the left, until it cuts the ordinate of zero voltage, when there can be no iron loss, we may make a rough estimate of the power spent in friction and windage, by measuring the height of the point of intersection. In the curve of Fig. 197 the power lost in friction thus appears to be about 220 watts. Assuming this value to be correct, we should take the total no-load losses of the motor at the normal voltage of 200 to be 428 watts, of which 220 are due to friction and windage, and 208 due to iron losses in hysteresis and eddy currents. Obviously, the decrease in the speed of the motor at low voltages makes this method of determining the friction losses not a very accurate one.

(6). *Ratio of Transformation.* This should be measured by applying normal voltage to the stator, and measuring the voltage at the slip rings when open-circuited and with the rotor in the position giving a maximum voltmeter reading between the rings. This ratio is of importance in calculating the value of the rotor currents from the known value of the stator current at any load; also in calculating the magnitude of the starting resistance necessary to reduce the currents to any desired value at starting.

It is important to remember that the ratio of stator to rotor *currents* in an induction motor does not give the ratio of transformation, because of the large magnetising currents taken by the stator.

EXPERIMENT XLIII.—TEST OF LOCKED INDUCTION MOTOR.

DIAGRAM OF CONNECTIONS.

As for Experiment XLII., Fig. 196, page 302.

Connections.—These will be the same as for the previous experiment (see page 302), except that the ammeter and wattmeter must be suitable for the larger currents to be employed in the present test.

Instructions.—Tie or clamp the rotor shaft in such a way that it cannot rotate. Apply first a low voltage, and then gradually increasing voltages to the stator windings with the rotor short-circuited, until a current about 50 per cent. above the normal

full-load current of the motor is reached. Take the same readings as in the previous test, entering them in a table similar to the one given on page 304.

Since the currents taken by the motor will depend chiefly on the resistances of the windings, a slight want of symmetry may produce a considerable out-of-balance current, *i.e.*, the currents taken by the three phases may not be equal. It will, therefore, be desirable to employ an ammeter and wattmeter in each line, or to employ throw-over and short-circuiting switches, so that the instruments may be introduced into each of the phases in turn.

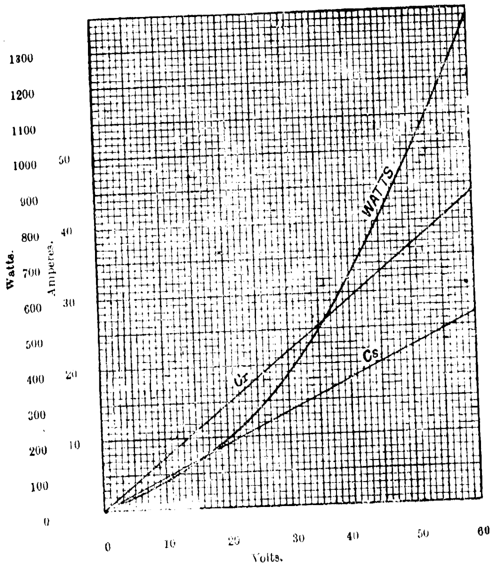


FIG. 198.—Curves of Stator and Rotor Currents and Watts in Stationary Motor.

The curves given in Fig. 198 were taken on the same 5 h.p. motor as was employed for the previous test.

The power taken by the motor when locked is almost entirely due to copper losses, increasing as the square of the current. The iron losses are only small, even at the maximum voltage shown in Fig. 198, since this corresponds to an iron saturation much below the normal working value.

In the conditions under which the curves shown in Fig. 198

were taken, viz., with stationary rotor, the motor is practically a static transformer, the stator and rotor forming the primary and secondary windings. The test is consequently similar to the curve taken on a short-circuited transformer in order to separate the copper losses (see Fig. 72, page 135).

It will be noticed on comparing the two curves that the shape of the watt curve, showing the copper losses, is similar in the two cases.

The current curves are straight lines. In the case of the rotor this is easy to understand, since the current will naturally increase in proportion to the increased voltage induced in it by the transformer action. The primary current increases in proportion to the secondary current, as in a transformer, in order to counteract the demagnetising action due to the rotor currents. Practically the only field which the stator windings can maintain (with stationary short-circuited rotor) is a leakage field between the teeth

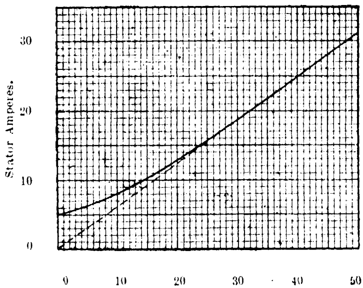


FIG. 199.—Ratio of Currents.

Full line = Full voltage, motor running under load.
Dotted line = Motor clamped, voltage varied.

of the stator core, since any flux entering the rotor is at once neutralised by the rotor currents which it induces. The experiment represents, consequently, the condition of maximum magnetic leakage.

In Fig. 199 the values of the currents shown in Fig. 198 are plotted in another way, viz., as the ratio of stator to rotor current. The partly-dotted straight line passing through zero is the curve referred to, and shows that the ratio of the currents is almost constant when the motor is stationary. This curve is similar to the curve of ratio of primary to secondary current in a transformer, and gives the ratio of transformation of the induction motor when considered as an alternating-current transformer.

It is interesting to note that the ratio of the two currents approximates to the dotted straight line, even when the motor is

running, no matter what may be the load, primary voltage, or resistance in the rotor circuit, since these do not affect the ratio of transformation of the two windings.

As a result of this experiment it would always be possible to calculate the value of the rotor current from a curve of stator currents obtained in a load test. This might be useful as it is not usual to measure the rotor current when carrying out a test. The full-line curve in Fig. 199 shows the ratio of primary to secondary currents when the motor works at full voltage (as in the next experiment), and shows that when fully loaded the ratio of the currents approximates to the theoretical ratio of transformation. This ratio would be maintained at all loads if it were not for the considerable no-load current of the motor when working at normal voltage.

The special practical application of the two measurements just described will be further alluded to later.

Circle-law of Stator Current.—The following test, although not of a commercial nature, is introduced in order to enable the student to prove for himself the important fact that the current taken by an induction motor may be represented approximately by vectors drawn from a fixed point to the circumference of a circle.

EXPERIMENT XLIV.—DETERMINATION OF VARIATION IN CURRENT AND POWER-FACTOR OF AN INDUCTION MOTOR UNDER LOAD.

DIAGRAM OF CONNECTIONS.

Same as for Experiment XLII., Fig. 196, page 302, except that no ammeter is required in the rotor circuit.

Connections.—These should be made as described on page 302, except that the ammeter in the rotor circuit may be omitted. A brake should be arranged to act on the motor pulley, or the motor may be made to drive a generator or other load. It is not necessary that the power thus exerted should be measured.

Instructions.—In order to obtain as wide a range readings as possible, it will be well to employ a voltage considerably below normal—say, one half the usual working voltage of the motor. Start the motor unloaded, and take readings of the volts, amperes, and watts supplied to it. Keeping the same voltage and frequency of supply, vary the load on the motor through a wide range, taking readings on the ammeter, voltmeter, and wattmeter in the stator circuits for each load. The power exerted need not be observed. By keeping on the load for short intervals, just sufficiently long to enable readings to be taken, it will be possible to obtain values of the current much beyond the normal working range without overheating.

Readings should be entered in tabular form as shown below, and curves of stator current and cos ϕ should be plotted, as in Fig. 200. If the motor output is not observed, the curves may be plotted on a base of watts input.

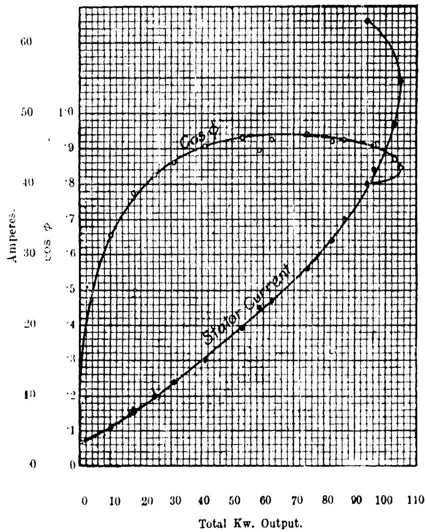


FIG. 200.—Variation of Current and Power-factor with Load.

The values of the current per phase taken by the motor should then be drawn as vectors from a common point O, as shown in Fig. 201, which shows the method of drawing them from the values obtained in a test on a 4-pole 6 h.p. British Thomson-Houston motor, as shown in the curves Fig. 200 and the table given below. The angles of the current vectors may be measured off from the vertical line $O\ E$ by the construction given on page 81. The centre of the circle $F\ B\ D$ must be found by bisecting chords, such as $F\ B$ by a line drawn at right angles to the chord to cut the horizontal line $O\ D$ at the point f, which is the centre of a circle passing through F and B. Several trials should be made in order to find the circle passing most nearly through all the points.

VARIATION OF STATOR CURRENT IN 3-PHASE INDUCTION MOTOR.

Output—6 h.p at 1,500 r.p.m. Voltage 200 50
Stator star connected.

Voltage maintained constant at 200.

Stator Current.	Total Watts Input.	Total Watts Output.	Cos ϕ
5·5	1250	1040	·656
10	2860	2480	·825
15	4700	4110	·904
28	9100	7440	·94
63	17500	9400	·8

In order to explain the significance of the results of this experiment, we must consider in greater detail the influences producing the change in power-factor of the rotor currents.

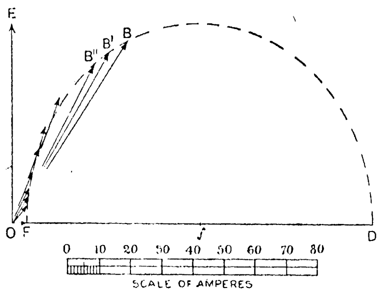


FIG. 201.—Vectors of Stator Current.

Variation of Rotor Current with Slip.—We have seen that the electromotive force induced in the rotor conductors varies in the same ratio as the slip. The frequency of this voltage is equal to the slip, and, therefore, the reactance of the rotor conductors ($= 2\ \pi\ L_2\ s$) bears a constant ratio to the slip. It follows that as the slip increases, a greater current will be sent through the rotor circuit, but will lag behind the phase of the induced voltage by an increasing angle on account of the increasing reactance of the circuit.

Let $O\ B\ A$ (Fig. 202) be the triangle of rotor voltages for a particular speed of the motor, being the same triangle as $C\ B\ A$ (Fig.

195, p. (298). The effect of a decrease in speed, *i.e.*, an increased slip, will be to increase the induced voltage e, representd by $O\ A$ to $O\ A^1$. The current will be increased as a result, so that $O\ B$ will have the value $O\ B^1$. The length of $B^1\ A^1$ will, however, increase in a greater ratio, since both current and reactance have become greater. The triangle of voltages for the greater slip will therefore resemble $O\ B^1\ A^1$, the angle of lag ϕ having increased to ϕ^1. The vector $O\ B$ may be taken to represent the rotor current in magnitude (since r_2 is a constant resistance), and also in phase relatively to the vector of induced voltage $O\ A$. It is evidently important to ascertain the law governing the value and phase of the current vector $O\ B$, as the slip varies. We shall find that the path of B is the circle $O\ B\ D$, having $A\ O$ as a tangent and the vertical line $O\ D$ as diameter, the length of this diameter being the maximum value of e, *i.e.*, its value at standstill. The proof of this statement is given in a separate paragraph.

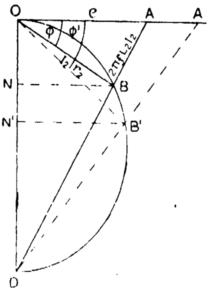


FIG. 202.—Variation of Rotor Current.

Proof of Circle Law.—We have said before that the rotor may be regarded as the armature of an alternating-current generator, being acted upon by the rotating field. The speed with which the field cuts the armature conductors depends on the *slip* of the motor. The motor may therefore be regarded as a variable-speed alternator having constant resistance and inductance in the armature circuit, and constant field strength. The frequency of the induced armature currents will then be the same as the slip of the motor.

The alternator induced voltage e will be directly proportional to the speed.

Let X, R be the total reactance and resistance of the load circuit (in this case the rotor circuit), and I the current produced by the voltage e.

$$\text{Then } e = I\sqrt{R^2 + X^2}$$

Since both e and X are directly proportional to the speed, we may write $e = k\,X$ where k is some constant

$$\therefore I = \frac{k\,X}{\sqrt{R^2 + X^2}}$$

X being proportional to e for any given excitation within the limits of saturation.

Draw $O\,D$, $O\,B$ at right angles (see Fig. 203), and make $O\,D$ equal to X (= reactance at full speed), $O\,B$ equal to R

As the speed falls, the reactance X will assume a smaller value x represented by $O\,D^1$

Joining $D^1\,B$, the line $D^1\,B = \sqrt{R^2 \times x^2}$.

The current in the circuit has been shown to be proportional to

$$\frac{x}{\sqrt{R^2 + x^2}}\ \textit{i.e.}, \text{ to } \frac{O\,D^1}{D^1\,B} \text{ or } \sin\phi.$$

Now a line drawn from O perpendicular to $B\,D^1$ will mark off a triangle $O\,A\,B$ similar to the triangle $D^1\,O\,B$. Also, as D^1 changes

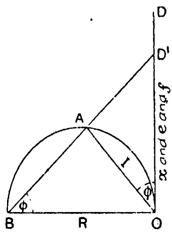


FIG. 203.—Proof of Circle Law.

in position the triangle $O\,A\,B$ will always be a right-angle triangle described on the constant hypotenuse $O\,B$, so that the locus of A will be the semicircle described on $O\,B$.

Also $\sin\phi = \frac{O\,A}{O\,B}$ or, since $O\,B$ is constant, we have

$$I \propto \sin\phi \propto O\,A,$$

whence $O\,A$ can be taken to represent the current in the circuit in magnitude. It will also represent it in phase relative to the voltage (shown by $O\,D^1$), since the angle $A\,O\,D^1$ = angle $D^1\,B\,O = \phi$.

That ϕ is the angle of lag in the circuit is seen from the fact that

$$\tan D^1\,B\,O = \frac{x}{R}.$$

As already stated, the height of $O\,D^1$ is proportional to the total generated volts, and may thus be taken as representing the voltage induced by the rotating field in the rotor circuit.

We have thus proved the statement made on p. 313 that the locus of the point B in Fig. 202 is a circle. Note that $O\ D$ is equal to the maximum value of e, *i.e.*, $O\ D$ represents the rotor voltage at standstill.

Relation between Stator and Rotor Currents.—Having considered the variation of the rotor current, we must now find how the stator current varies with change of speed.

The connection between stator and rotor currents may best be studied by regarding the motor as a transformer in which the stator winding forms the primary and the rotor the secondary. In order that the rotating field may have a constant value, the stator will take a certain no-load current which is just sufficient to produce this field and to overcome the iron losses, as in the transformer *

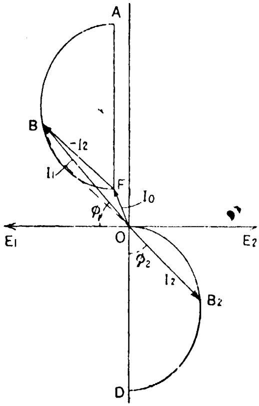


FIG. 204.—Relation between Stator and Rotor Currents.

When the motor is loaded, the rotor currents which are formed are balanced by an equal and opposite increase in the ampere-turns in the stator, and the vectorial difference between the total stator and rotor ampere-turns gives the same magnetising ampere-turns as on no-load.

The stator current is thus always equal to the

(rotor current ÷ ratio of transformation) + no-load current.

This is the same law as for the static transformer.

If we now show the connection between the currents as in the transformer diagrams (see page 160), we may draw the rotor current

* The no-load current of the induction motor has also a component overcoming the frictional losses due to the rotation of the motor. This current is however balanced by a corresponding current in the rotor, and is best regarded as part of the load current.

$O B_2$ as moving in the semi-circle $O B_2 D$. The stator current will then be obtained by drawing the no-load current $O F$ (see Fig. 204), and adding to it vectors equal and opposite in phase to $O B_2$. The vectors $F B$, representing the variable current in the stator, will evidently be chords of the semicircle $F B A$ described on a diameter $F A$ equal to $O D$. The vectors of stator current will therefore be lines drawn from O to points on the circumference of this semicircle for all possible values of the slip or load.

The verification of this important law for the variation of the current of an induction motor forms the subject of Experiment XLIV., previously given, the circle $F B A$ being the same as that obtained in Fig. 201. It is usual to draw this semicircle with its diameter horizontal, as shown in Fig. 205 and subsequent diagrams.

Dispersion Coefficient.—The circle diagram gives at once the coefficient of magnetic dispersion of the motor. We have already stated that magnetic leakage exists both in the stator and rotor. The total leakage of the motor is the sum of both leakages.

In the diagram Fig. 205, $O A$ represents the motor current when the rotor is stationary, while $O F$ represents the current when the motor runs at synchronous speed. This diagram is drawn on the assumption of no losses, so that $O F A$ is a horizontal line.

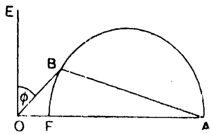


FIG. 205.—Simple Circle Diagram.

So long as the same voltage is maintained at the stator terminals, the flux linked with the stator winding is the same in both cases. With locked rotor there is no resultant flux through the rotor, and the flux is entirely leakage flux. At synchronism there is no rotor current, and the stator flux is free to enter the rotor. Hence $O A$ represents the ampere turns necessary to drive this flux across the leakage paths, while $O F$ is the ampere-turns required to send the same flux through the useful path and leakage path in parallel.

Thus $O A$ is proportional to the reluctance of leakage paths, and $O F$ is proportional on the same scale to the joint reluctance of the useful and leakage paths.

Putting these statements in the form of equations, and employing the term permeance to represent the inverse of reluctance,

$$\frac{O A}{O F} = \frac{\text{reluctance of leakage path}}{\text{joint reluctance of useful and leakage paths}}$$

$$= \frac{\text{permeance of leakage path} + \text{permeance of useful path}}{\text{permeance of leakage path}}$$

$$= 1 + \frac{\text{permeance of useful path}}{\text{permeance of leakage path}}$$

Now the ratio $\frac{\text{permeance of useful path}}{\text{permeance of leakage path}}$ will be the same as the ratio $\frac{\text{useful flux}}{\text{stray flux}}$ if the same ampere-turns act upon both magnetic paths (as at no load).

Hence $$\frac{O\,A}{O\,F} = 1 + \frac{\text{useful flux}}{\text{stray flux}}$$

Also $$\frac{O\,A}{O\,F} = \frac{O\,F + F\,A}{O\,F} = 1 + \frac{F\,A}{O\,F}$$

$$\therefore \frac{F\,A}{O\,F} = \frac{\text{useful flux}}{\text{stray flux}}$$

The leakage factor of any magnetic circuit

$$= \frac{\text{total flux}}{\text{useful flux}} = \frac{\text{useful flux} + \text{stray flux}}{\text{useful flux}} = 1 + \frac{\text{stray flux}}{\text{useful flux}}$$

$$= 1 + \frac{O\,F}{F\,A}$$

Among writers on induction motors the letter σ has been adopted to represent another leakage ratio, usually termed the "dispersion coefficient."

Behn-Eschenberg uses for the dispersion coefficient

$$\sigma = \frac{\text{leakage flux}}{\text{total flux}} = \frac{O\,F}{O\,A}$$

whilst Hobart and others take another coefficient,

$$\frac{\text{leakage flux}}{\text{useful flux}} = \frac{O\,F}{F\,A}$$

Adopting Behn-Eschenberg's coefficient, we have

$$\text{dispersion coefficient} = \sigma = \frac{O\,F}{O\,A}$$

Employing another usual symbol for the quantity employed by Hobart,

$$v = \frac{O\,F}{F\,A}$$

we obtain the leakage factor as first defined

$$= 1 + \frac{O\,F}{F\,A} = 1 + v$$

$$= 1 + \frac{O\,F}{O\,A - O\,F} = 1 + \frac{\sigma}{1 - \sigma}$$

which gives the relation between the forms usually adopted for the leakage factor and dispersion coefficient.*

* A full discussion of the various coefficients and their relation to the reactance and self-induction of a motor is given in Cramp and Smith's Vectors and Vector Diagrams, Longmans and Co.

The actual values for v or σ may be obtained by various formulæ from the shape of the core teeth and width of air gap, or more directly by actual measurement of the open-circuit and short-circuit tests on the completed motor in the manner already described.

Experiment XLV.—Load-test of an Induction Motor.

Diagram of Connections.

(Same as for Experiment XLII., Fig. 196, page 302.)

Instructions.—The instruments and connections required will be the same as those given in the connections for Experiment XLII. In this case, however, both voltage and periodicity of the supply are to be maintained constant. The motor must be provided with a brake on its pulley, or must be coupled to a generator which has been previously calibrated, so that it may serve as a brake or absorption dynamometer. The power given to a generator employed as load in this way will be the measured output of the generator divided by the efficiency of the generator at this output.

The test is carried out by observing the primary amperes taken by the induction motor, the watts supplied, and the slip, first at no-load, and then for a series of increasing values of the load up to about 25 per cent. overload for an ordinary commercial test, or up to the point of stoppage of the motor in the case of a more complete experiment.

The results should be entered under headings similar to those shown below, and curves plotted with either watts or horse-power output horizontal, and primary current, slip, efficiency, and power-factor and watts absorbed vertical.

It is not the general practice to measure the current in the rotor circuit, as its exact value is not of first importance in the behaviour of the motor, and its approximate value can be deduced from the other quantities observed, as already explained on page 309.

Load Test of 3-phase Induction Motor.

Motor No. .... Type..........................
Output..............h.p., at..............revolutions per minute.
Voltage............................. Frequency.......................

Supply					Slip per cent.	Rotor Current	Apparent Watts Supplied	Power-Factor	Efficiency
Current	Volts	Watts							
		a.	b.	Total					

The three columns for watts are to be employed when readings are taken in two phases, so that their sum gives the total power. Where a 3-phase wattmeter is employed only one column will be required.

The nature of the curves to be obtained from this experiment may be judged from Fig. 206 taken from tests of a 3½ h.p. 3-phase Oerlikon motor running at a frequency of 50, and having a synchronous speed of 1,500 revs. per minute.

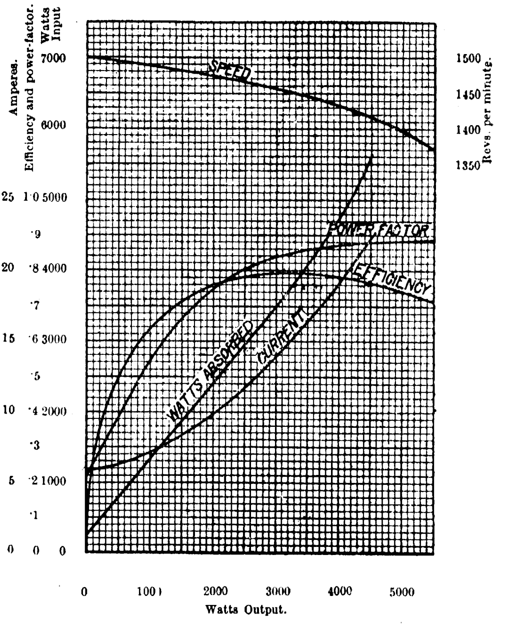


Fig. 206.—Load Curves of Induction Motor.

As a further illustration, some additional curves obtained in another experiment in a rather different manner are shown in Fig. 207. The motor employed in this case was a 5 h.p. Electrical Company's motor with slip rings, having a synchronous speed of 1,500 revs. per minute on a 200-volt 50 cycle circuit. These curves are given as an illustration of a simple method of obtaining load curves of a motor by coupling it to a direct-current generator.

The test was carried out with a voltage supply of 120 volts instead of 200, in order that the motor might be loaded beyond the

pull-out point without overheating. The load on the motor shaft was a direct-current generator driven by a belt, and excited with a constant current. The currents plotted horizontally in Fig. 207 are the currents generated by the direct-current machine + the no-load current required to drive both machines at each speed. In this way the horizontal scale is a scale of torque, but measured in terms of the armature current of the direct-current generator. Assuming the generator field to remain constant for all currents, during the experiment, the armature current would be proportional to the amperes thus plotted. In order to make this as nearly true as possible, the excitation of the generator was carefully adjusted to a constant value, and the brushes were fixed in the neutral position to avoid weakening of the field due to armature reactions.

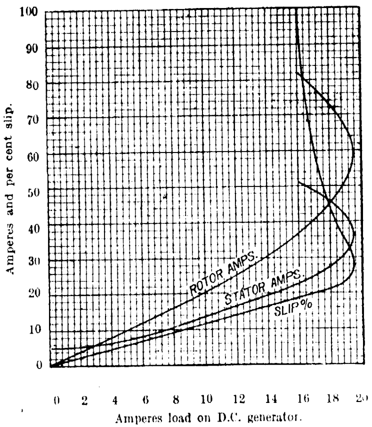


FIG. 207.—Load Test of Induction Motor Coupled to Direct-current Generator.

Some weakening of the field due to magnetic distortion must have taken place; but as the machine was not worked up to its maximum output this was probably not serious. This assumption was made after testing the proportionality between speed and voltage of the generator, when on open circuit and when fully loaded, and finding that the observed loss of voltage determined with the heaviest armature current at the reduced speed was nearly all accounted for by the resistance of armature and brushes. In any experiment carried out in the same manner this point should be similarly tested.

The method of obtaining the curves in Fig. 207 was briefly as follows:—

The motor was connected to the supply of 3-phase current as

described in the instructions given above. A direct-current generator was coupled to the motor by a belt, and its armature was connected to a variable resistance in series with an ammeter; a voltmeter was connected to read the terminal voltage. The generator was separately excited, the excitation being kept constant by means of a regulating resistance. The motor was supplied at a constant voltage of 120 throughout the test, and the generator was allowed to send gradually increased currents, so as to increase the load on the motor. For each value of the load on the generator, watts supplied, rotor and stator currents, speed and generator voltage were read.

The final readings, at and near the point of stoppage of the motor, were taken by supplying the direct-current machine armature with current from an external source in series with a resistance,

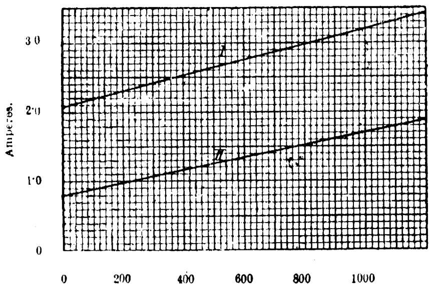


FIG. 208.—No-Load Current taken by D.C. Machine.
I. Driving Induction Motor by Belt.
II. Belt thrown off.

and reversing the fields, so as to make it tend to drive the alternating-current motor in a reverse direction. By this means the armature current corresponding to the torque at stand-still was obtained.

A test on the direct-current generator was then made to ascertain the current equivalent to the frictional and other losses at various speeds. The machine was run as a motor at the same excitation as before, and the current taken by the armature at various speeds was noted, both with the belt coupling it to the motor and with the belt thrown off. The readings taken with the belt on included the friction of the induction motor and belt as well as of the direct-current generator.

The results of this test are shown in Fig. 208. In plotting the curves of Fig. 207 the no-load current required to overcome the total losses of both machines (as shown on the upper curve) were added to the current actually given out by the generator. The

frictional losses of the induction motor itself are thus counted as torque exerted by the rotor.

In order to determine the relation between the horizontal scale of amperes in Fig. 207 and torque, measured in lb.-ft. or kg. cm., it is only necessary to observe the voltage generated in the direct-current machine when running unloaded, and its speed. In the present case the no-load voltage at 1,320 revs. per minute was 210.

Thus, at an output of 10 amperes, and the speed observed of 1,160, the power generated including armature copper losses was $\frac{1{,}160 \times 210}{1{,}360} \times 10$ watts. The no-load driving current at this speed is (see Fig. 208) 3·4 amperes. Thus the power developed by the motor was

$$\frac{1{,}160 \times 210}{1{,}320} \times 13{\cdot}4 \text{ watts} = 2{,}480 \text{ watts.}$$

This power is equivalent to a torque of 15 lb.-ft.

This may be arrived at as follows :—

$$\text{H.P. developed} = \frac{\text{watts}}{746} = \frac{2{,}480}{746} = 3{\cdot}32$$

$$\text{Also, H.P.} = \frac{2\,\pi\,n\,T}{33{,}000}$$

$$\text{where } T = \text{torque in lb.-ft.}$$

$$\text{and } n = \text{revolutions per minute}$$

$$\text{Hence torque} = T = \frac{\text{H.P.} \times 33{,}000}{2\,\pi\,n} = \frac{3{\cdot}32 \times 33{,}000}{2\,\pi \times 1{,}160}$$

= 15 lb.-ft. approximately

We may briefly point out the characteristics of the experimental curves shown in Figs. 206, 207.

Primary Current.—The current supplied to the stator is determined by two factors, as in the case of a loaded transformer. It has, firstly, to supply the almost constant magnetising current required to maintain the rotating field, corresponding to the no-load current of a transformer, which we may call the no-load component. Secondly, the current has a component which produces a field equal and opposite to that formed in the stator by the rotor currents, corresponding to the component of the primary transformer current which overcomes the demagnetising action of the secondary circuit. This we may distinguish as the load component.

The primary current of an induction motor is consequently in most ways similar to that in a transformer. There are two respects in which it differs. On account of the air-gap in the magnetic circuit of the induction motor the magnetising current is much larger than in a transformer, and the no-load current is in consequence usually from one-quarter to one-third of the full-load current. This is clearly shown by the curve in Fig. 206.

The relation between primary flux and primary voltage is given by the same formula as that on page 116, for the transformer

$$e_1 = 4{\cdot}44\, f\, F\, T_1\, 10^{-8}$$

When F = maximum flux produced
f = periodicity.
T_1 = turns in winding.
e_1 = primary voltage.

The flux depends only on the number of stator turns and the voltage applied, and is independent of the load.*

In a 3-phase motor it has been shown on page 292 that current in each phase winding must produce a flux equal to two-thirds of the total strength of the rotating field. The value of the no-load current, or magnetising current, will be given by applying the same formula as that given for the magnetising current in a transformer, account being taken of the fact that the current in each phase is that required to produce two-thirds of the rotating flux.

The load component differs from that in a transformer because its angle of lag relative to the applied voltage varies in a definite manner with the load, obeying the law explained on page 312.

At heavy loads the primary current increases in a higher ratio than the load on account of the rapidly-falling power-factor. The current curve is thus made to bend upwards at heavy loads, instead of approximating to a straight line drawn through the origin, as with the transformer.

Power Absorbed and Efficiency.—The losses occurring in the motor are practically of three kinds : (*a*) The loss in the stator and rotor windings ; (*b*) iron losses in stator and rotor cores ; (*c*) frictional losses.

(*a*) In a 3-phase motor the watts lost in the stator winding are equal to $3\, i_1^2\, r_1$, where i_1 is the primary current per phase, and r_1 is the resistance of the winding per phase. If the rotor winding is a short-circuited 3-phase winding, the watts lost in the rotor are $3\, i_2^2\, r_2$, where the symbols i_2, r_2 denotes the current and resistance per phase of the rotor winding.

(*b*) The iron losses in the stator will depend on the saturation of the iron and the frequency of the magnetic changes. It has already been shown that the strength of the primary field is constant at all loads, depending only on the supply voltage. The rotating field makes $\frac{f}{p}$ revs. per second when f = frequency of supply and p = number of pairs of poles. Thus the iron of the stator passes through a complete magnetic cycle f times per second. This source of loss is consequently independent of the load.

The iron losses in the rotor will be proportional to the slip, since the polarity of the rotor core will change p times for each rotation of the rotating field relative to the rotor. The slip under ordinary working conditions is so small that the maximum rate of magnetic reversal is only about 5 per cent. of the speed of the rotating

* This is neglecting the loss of voltage in the resistance of the primary winding $= i_1 r_1$.

field. The iron losses in the rotor may consequently be safely neglected in comparison with the other losses.

(c) The friction losses consist of bearing friction and windage Since these depend almost entirely on the speed, which is practically constant, and not appreciably on the load, they may be taken as being constant for any motor.

From the consideration just given, we see that the friction and iron iosses are practically constant at all loads, while the no-load copper losses are very small (not more than 1-2 per cent.), but increase in proportion to the square of the current taken by the motor.

If we group together the useful output of the motor and the power spent in overcoming friction and iron losses and denote their sum by W_t, calling the power supplied to the motor W_s, we may write

$$W_s = W_t + 3\, i_1^2\, r_1 + 3\, i_2^2\, r_2.$$

If there were no copper losses, we should have

$$W_s = W_t,$$

and the curve showing the relation between total watts output and watts supplied would be a straight line passing through zero and inclined at an angle of 45° to the horizontal, if the same horizontal and vertical scales are chosen. The effect of the copper losses is to make the watts supplied increase more rapidly at higher loads, the increased power being proportional to the square of the load, so that the line will bend upwards in a curve

Referring to Fig. 206, we see that the curve of watts agrees with the statements just made. The line does not appear to pass through zero, because the horizontal scale is *useful* output only. If the curve were prolonged backwards to meet the horizontal axis, the distance to the left of the vertical axis would measure the power spent in overcoming friction and iron losses at no-load. This distance is the same as the height of the point where the line cuts the vertical axis, since the line is inclined at 45°, as may be seen. In the present case, therefore, the iron and friction losses may be taken to be 270 watts.*

By drawing a tangent to the watt curve inclined at 45°, we might measure the power spent in copper losses at any load by determining the vertical distance between this tangent and the curve

The curve of efficiency is of the usual shape If the copper losses are neglected, the equation to the curve would be

$$\text{efficiency} = \eta = \frac{\text{watts output}}{\text{watts output} + \text{constant losses.}}$$

This would represent a hyperbola approaching its asymptote $\eta = 1$ more nearly as the load increases. Owing to the copper losses the curve fails to reach this value and begins to bend downwards at higher loads, thus departing from the shape of the hyperbola. The curve should reach its maximum value at normal load, and

* These losses are not *accurately* constant at all loads, but sufficiently nearly so for most practical purposes.

being very flat near this point, the efficiency does not vary much between three-fourths and five-fourths of full load.

The Power-factor.—The power-factor is very low at no-load, on account of the large magnetising current taken to maintain the flux in the air-gap of the motor.

The shape of the power-factor curve is similar to that of the efficiency curve, but does not pass through zero. The general shape of the curve follows from the curves of power and stator current, already discussed.

It is one of the characteristic disadvantages of induction motors that they always work with a power-factor less than unity. In order to increase the power-factor of a motor, there are two ways which it is possible to adopt, both of which diminish the no-load currents. These are, either to increase the number of windings of the primary, or to reduce the reluctance of the magnetic path by reducing the air gap. The first method has the effect of increasing the resistance of the winding, and, consequently, diminishing the efficiency of the motor. This disadvantage is not possessed by the other method of improving the power-factor.

It is therefore usual to find the air gap of induction motors smaller than in any other type of electrical machine. In practice the air gap is only limited by the clearance necessary for mechanical safety

Relation between Slip and Torque.—One of the most interesting curves in connection with an induction motor is the slip-torque curve. The characteristic shape of such a curve is shown in Figs. 212 and 213.

With low values of the slip, the rotor behaves as if it were practically non-inductive ; slip, torque, and rotor current consequently all increase in the same ratio. The slip-torque curve is, therefore, a straight line for speeds near synchronism (c.f. Fig. 193, p. 296). As the slip and the frequency of the currents in the rotor, increase, the rotor reactance ($= 2\ \pi\ s\ L_2$) increases, and causes an increasing lag in the rotor currents. Finally, at low speeds, the reactance preponderates over the resistance, and the current lags nearly 90° behind the flux in phase.

These changes may be followed out in Fig. 202, which indicates the circle voltage diagram of the rotor. The torque is proportional to $I_2 \cos \phi$, *i.e.*, to the line $B\ N$.

Evidently this line has a maximum value when B is on the circle exactly half-way between O and D, *i.e.*, when $B\ D = O\ B$, or when $2\ \pi\ s\ L_2\ I_2 = I_2\ R_2$, or resistance and reactance are equal. This is the condition for maximum torque in an induction motor. The angle of lag ϕ in the rotor circuit is seen to be 45° under these conditions.

A further important result follows, viz. : With a constant rotating field (a constant length of $O\ D$ in Fig. 202) the maximum torque is independent of the rotor resistance, but will occur at a different speed of the motor (a different value of s) for each value

of this resistance. So long as the value of the rotor resistance does not exceed the maximum possible value of the reactance $2 \pi s L_2$ ($= 2 \pi f L_2$), the maximum torque which the motor can exert will be independent of the resistance.

The ratio of maximum torque to the full-load torque is called the *overload capacity* of the motor. This overload capacity refers, of course, to torque capacity alone, and not to output capacity, which is sometimes referred to by the same term.

We may regard the slip-torque curve as consisting of two portions—a straight line in which resistance of the rotor is greater than the reactance, and a hyperbolic portion in which reactance overpowers the resistance. These two parts merge into one another when reactance and resistance become equal at the point of maximum torque.

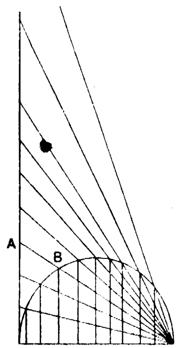


Fig. 209.—Construction for Finding Relation between Slip and Torque of Induction Motor without Losses.

In order to show how the special shape of the slip-torque curve of an induction motor arises, the diagrams in Figs. 209, 210, have been drawn. In Fig. 209 the triangle of voltage for the rotor (similar to Fig. 202, page 313) has been drawn for a number of speeds. This triangle consists of all such triangles as $D\ B\ O$. Now, $O\ A$ is proportional to the slip of the motor (see page 313), and $B\ M$ is proportional to the torque (see page 325); hence, by plotting the relation between lengths such as $O\ A$ and the corresponding length of $B\ M$, we obtain a curve similar to the slip-torque curve of the motor. This has been done (with a change of scale) in Fig. 210. The student will find it an interesting example to plot a curve of this kind for one or two values of rotor resistance and inductance.

Torque, Output, and Rotor Efficiency.—The torque of an induction motor may be expressed in terms of power and speed in three different ways, and the three expressions so obtained are very useful in showing the relation between other important factors.

We shall express the three values of the torque briefly thus :—

(*a*) Torque* = output of rotor ÷ speed of shaft.

(*b*) Torque = losses in rotor ÷ speed of slip.

(*c*) Torque = input to rotor ÷ speed of synchronism.

(*a*) The torque is to be taken as the total mechanical torque exerted by the rotor, including that overcoming friction. The watts output of the rotor are equal to the watts supplied to it, less

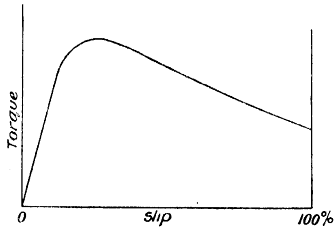


FIG. 210.—Relation between Slip and Torque of Induction Motor.

the losses in resistance. Calling w_0 the watts supplied to the rotor (c.f. p. 324), the watts usefully exerted will be $w_0 - m\, i_2^2\, r_2$,

When m = number of rotor phases.

i_2 = rotor current per phase.

r_2 = rotor resistance per phase.

Hence our first value for the torque may be written

$$T = \frac{w_0 - m\, i_2^2\, r_2}{n} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

n being the revolutions per second of the shaft. This relation follows directly from the ordinary laws of mechanics.

(*b*) We may regard the induction motor as an alternating-current generator in which the rotor forms the armature, and the poles rotate relatively to this armature with a speed of $\frac{s}{p}$ revolutions per second.

The torque on the rotating member of an alternator rotating at a speed of n revs. per second, when giving an output of W watts,

$$= T = \frac{W\, 10^7}{2\, \pi\, n} \text{dyne cm. or absolute units.}$$

* The torque as here given is not reduced to the ordinary system of units. When given in watts and revs. per second, the various products on the right of the equations give the torque in units equal to those of the absolute C.G.S. system $\times \frac{10^7}{2\,\pi}$. When the power is given in watts and the speed is given as a fraction of the maximum or synchronous speed, the torque becomes numerically equal to the "watts at synchronous speed" which this torque would produce (see also page 339).

or $$T = \frac{W}{n}$$

in terms of the special units used above.

The same expressions may be employed for the torque of an induction motor if we substitute for W the power developed electrically in the armature, *i.e.*, the watts spent in heating the rotor conductors, and in place of $n, \frac{s}{p}$ the revs. per second of the field relatively to the rotor conductors.

In a motor with m-phase rotor,

$$T = \frac{m\, i_2^2\, r_2\, p}{s} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This is the mathematical form of our previous equation (b).

(c) The third value of the torque follows directly from the previous two, from the application of the rule in proportion that if $\frac{a}{b} = \frac{c}{d}$, then $\frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$

Applying this rule to equations (1) and (2),

$$T = \frac{w_0 - m\, i_2^2\, r_2}{n} = \frac{m\, i_2\, r_2}{\frac{s}{p}} = \frac{w_0}{n + \frac{s}{p}}$$

$$= \frac{w_0}{\frac{f}{p}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The torque expressed in these terms is independent of the rotor losses. The rotor losses determine the slip of the motor, and the motor would run synchronously if the rotor losses were nil.

From our three definitions of the torque, we can see the following relations :—

Power given to rotor : power exerted at shaft : rotor losses : : speed of synchronism : actual speed of motor : speed of slip ; or, expressed in symbols,

$$w_0 : w : m\, i_2^2\, r_2 : : f : p\, n : s$$

w being the output of the motor (including work done against friction).

Further, the rotor efficiency

$$= \frac{w}{w_0} = \frac{p\, n}{f} = 1 - \frac{s}{f}$$

Whence it is evident that for high efficiency the value of s must be small, *i.e.*, the motor must work near synchronous speed

The relation between slip and rotor losses has been given on page 300. It follows immediately from the consideration just given.

Effect of Increase of Resistance in Rotor Circuit.—The next point to be considered is the effect of increasing the resistance of the rotor circuit. This is best studied by using a motor with

wound rotor and slip rings, so that equal variable resistances can be put in series with the phases of the winding. For the purpose of the experiment, the usual 3-phase starting resistance may be conveniently employed, provided that its carrying capacity is sufficient to prevent its being over-heated by the heavy currents induced in the rotor when the motor is overloaded.

Experiment XLVI.—Determination of Starting Torque of an Induction Motor with Varying Rotor Resistance.

Connections.

(These will be the same as for the locked test, Experiment XLIII., page 307.

Instructions.—The motor pulley should have a lever and spring balance attached to it, or a rope may be attached to the pulley, and after being wound round it, may be attached to a spring balance, so that the torque exerted by the shaft can be measured. The test is then carried out by applying a constant voltage to the motor terminals, and taking observations of the currents and watts supplied to the stator, and the torque exerted by the shaft for a series of different values of resistance in the rotor circuit. It will usually be found advisable to choose the voltage considerably lower than the normal working voltage of the motor, so as to reduce the currents to values which will not overheat the motor.

If T_1 is the torque observed at a voltage V_1, then the torque T at normal voltage V will be practically

$$T = T_1 \times \left(\frac{V}{V_1}\right)^2.$$

A practical difficulty may be met with in carrying out this test, owing to the torque varying with the relative position of the rotor and stator windings. In order to avoid variations in the readings of torque due to this cause, the rotor should be kept in the same position throughout the test, the point of suspension of the spring balance being adjusted in order to correct for the alteration in the length of the spring balance which occurs as its deflection alters.

Values of stator current, power-factor, and torque should be observed, and plotted in the form of curves with rotor resistance as a base.

Some results of such a test carried out on a 3 h.p. Oerlikon motor are shown in Fig. 211. It is seen that there is a definite value of the resistance, for which the starting torque is a maximum. With greater or less values of the rotor resistance the starting torque will be less. The explanation of this follows from the form of the slip-torque curves obtained in the next experiment.

Attention may be called to one further point in connection with the starting curves, Fig. 211. For a given value of the starting torque there are evidently two possible values of the rotor resistance. In designing the starting resistance, the greater value of the resistance should be chosen, since this corresponds to a smaller starting current. Consequently the rotor will *start* upon the *flatter* right-hand limb of the torque curve (Fig. 211). It will run when at full speed on the *steeper* limb of the running curves shown in Fig. 213.

Usually the value of the starting resistance to be used is determined by the permissible rotor current, and not by the desired torque. If e is the voltage between the slip rings when the motor

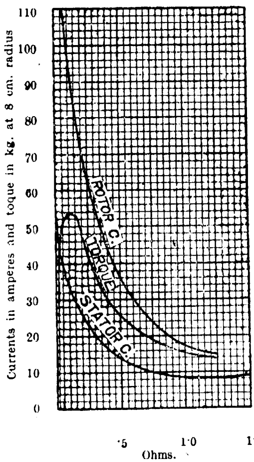


FIG. 211.—Relation between Starting Current and Torque and Rotor Resistance.

is stationery, and full voltage is applied to the stator, and $I_{max.}$ is the greatest starting current allowable, the value of a star connected starter may be taken to be $\frac{e}{\sqrt{3}\, I_{max.}}$ per leg, since the voltage acting on each leg of the resistance will be $\frac{e}{\sqrt{3}}$. This calculation obviously neglects the effect of the rotor impedance, which is usually small compared with the resistance of the starter but which may be allowed for if necessary.

Experiment XLVII.—Test of an Induction Motor with Variable Resistance in the Rotor Circuit.

Diagram of Connections.

As for Experiment XLII., Fig. 196, page 302.

Instructions.—The experiment may be carried out exactly as in Experiment XLV., but with a series of different values of the resistance in the rotor circuit. The series of curves are then plotted, each for a different value of the rotor resistance. From these curves a fresh curve may then be obtained, showing the variation of the primary current, torque, efficiency, and power-factor with the slip.

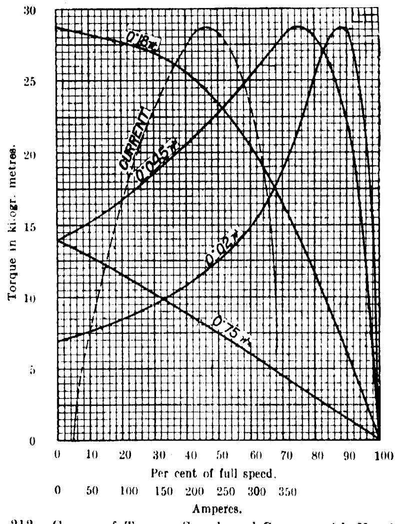


Fig. 212.—Curves of Torque, Speed, and Current with Varying Rotor Resistance.

For the particular purpose of determining the effect of the rotor resistance on the behaviour of the motor, it is, however, sufficient to take measurements only of the stator and rotor currents, slip, and torque, for each of a series of values of the rotor resistance. Complete sets of readings of each of these quantities should be taken and entered up as shown for the load test, curves being then plotted on a torque or load base.

Fig. 212 shows several curves of torque and slip obtained with different resistances in the rotor circuit of the same motor, to

which the curves in Fig. 211 apply. It will be seen that the effect of increasing the resistance is to make the curve slope upwards less steeply on the right, so that for small values of the torque the slip is increased. All the curves reach ultimately the same maximum value of the torque, so that the maximum torque exerted by the motor is not affected by the rotor resistance. The maximum torque is, however, reached with a different value of the slip in each case.

The point at which the curves cut the vertical line, corresponding to a slip of 100 per cent., gives the turning effort of a motor when started from rest. Evidently the greatest starting torque will be obtained by so choosing the rotor resistance that the curve has its maximum value on this vertical line. In the case of the motor for which Fig. 212 is drawn, this resistance of rotor winding and starter per phase is seen to be about ·18 ohm. A resistance greater than this would give a maximum torque at a slip of over 100 per cent.—that is, with the motor rotating in a reversed direction. Resistances less than ·18 ohm enable the motor to exert its greatest turning effort at speeds intermediate between stand-still and synchronism. By suitably choosing the successive steps of the starting resistance, and moving the starter handle over at the correct rate, as the motor increases its speed, it is possible to ensure that the motor is started and got up to speed under the most favourable conditions.

When the starting torque is plotted as a function of the rotor resistance, as in the last experiment (see Fig. 211), the value of the resistance giving a maximum starting torque is seen at once, being the resistance corresponding to the peak of the curve. This would correspond to the value of the resistance giving the curve of ·18 ohm. in Fig. 212. The points to the right of this are lower, because the maximum turning effort is only reached with a slip, greater than 100 per cent. Points on the curve to the left of the peak indicate values of the resistance, for which maximum torque is only reached after the motor has attained some speed.

The lower the resistance of the rotor circuit the nearer to synchronous speed is the point at which the motor can exert its greatest torque. Since the greatest slip usually occurring in practical working is about 5 per cent. it is an advantage to have an extremely low resistance of the secondary winding. In cases where a separate resistance is not employed for starting—that is, in the case of squirrel-cage motors—it is necessary to sacrifice some of the torque which might have been attained when running at full speed, in order to get a sufficient torque at starting and when running at the initial low speeds.

The general effect of moderate increase in resistance of the rotor circuit is seen from the curves to be that the inclination of the straight portion of the curve is diminished, and at the same time the hyperbolic portion is raised so as to cut the axis at a greater height from the horizontal axis.

It is of great importance to remember that the maximum torque of the motor is constant and independent of the rotor resistance.

The value of the slip at which it occurs is determined by the rotor resistance.

The starting torque is thus proportional to the rotor resistance, until this resistance exceeds the value corresponding to a maximum starting torque, as is also the slip for a given torque when running.

In Fig. 212 will be seen a dotted curve of current. This curve shows the value of the stator current (measured on the horizontal scale) corresponding to each value of the torque (measured vertically). It is possible thus to draw a single curve representing the relation between current and torque independently of the rotor resistance. This is because, for a given torque, the rotor will always take the same current, the slip automatically adjusting itself to enable this rotor current to be induced. Since the stator current depends only on the rotor current and the constant no-load current, there will be a definite value of the stator current for each torque independent of the rotor resistance. As seen from the current curve (Fig. 212), there are actually two values of the

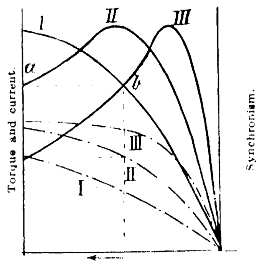


FIG. 213.—Rotor Current and Torque Compared with Slip.

current instead of a single one, as just stated. This is because there are two values of the slip for each value of the torque, as seen from the torque-slip curves. The running conditions always correspond to the lower slip, and, consequently, lower current.

The variation of rotor current with torque may be easily followed from Fig. 213, where rotor current and torque are plotted for three different values of the rotor resistance.

If two points on corresponding arms of two of the curves be taken, and each corresponding to the same torque, *e.g.*, *a* and *b* and the corresponding currents be noted on the current curves. the currents are seen to have the same value.

The student should read the section on the graphical method of obtaining the slip torque curves given on page 326, and compare the curves of Fig. 210 with those of Fig. 212.

Determination of Losses and Efficiency.—From the tests already given, it will be apparent that the losses in an induction motor may be determined in a manner very similar to that employed in the case of a transformer.

Corresponding to the determination of the copper losses in a transformer, we have the test on the motor while held stationary, described on page 307. The power supplied to the motor when locked is nearly all $I^2 R$ loss, since the iron losses are small at the low saturation employed. The losses due to the resitance of the windings are thus easily determined for any value of the stator current.

The core losses and friction losses are practically constant at all loads. Their value is determined from the no-load running test (see page 302). The power taken by the motor when running light is nearly all spent in iron losses, and running friction. A small part will be due to copper losses in the windings, and this should be subtracted from the wattmeter readings to give the true iron and friction losses. The value of the copper losses for any value of the no-load current may be derived from the results of the determination of the total copper losses.

From the results of these tests the efficiency of the motor at any load can be approximately calculated, since the iron and friction losses remain practically constant at all loads, and the losses due to resistance can be obtained from the curve obtained in the locked test (see Fig. 198, page 308).

The method of separating the mechanical losses of the motor from the other losses has already been described on page 307.

A paper dealing with the experimental determination of the losses in Induction Motors, by the author, will be found in the Proc. Inst. El. Engrs., Vol xxxix., page 437

The graphical representation of the various losses at all loads is discussed in connection with the circle diagram, page 335 and seq.

Braking and Return of Power to Line.—A 3-phase motor is reversed by interchanging any pair of supply connections, which has the effect of reversing the direction of rotation of the rotating field.

A reversing switch is usually made in the form of a 3-pole throw-over switch, the inter-connections being so arranged that with the switch thrown over to one side the motor receives current and runs in one way, and with the switch in the opposite position the motor runs in the opposite direction.

If, while the motor is running at full speed, the reversing switch be suddenly thrown over, so as to reverse the direction of the rotating field, the rotor will, at the moment after reversal, rotate in the opposite direction to the rotating field, and the slip will be practically 200 per cent. and large currents will be induced in the

rotor, since this will be running with the starting resistance cut out. Even with the large currents induced, the retarding torque is not very high. It is, consequently, not permissible to brake any except small motors with squirrel-cage rotors by reversing the field, although in case of serious danger it might be resorted to for larger motors.

It is not possible to brake an induction motor by disconnecting it from the line and short-circuiting the terminals through a resistance, as is frequently done with direct-current motors. As soon as the stator of an induction motor is disconnected from the supply, the rotating field ceases to exist, and no force exists to retard the motor, which will continue to rotate until gradually stopped by the friction of the shaft or gearing.

It is important to notice what happens if, while the rotor is driven by external means, the motor remains connected to the supply, and the rotating field consequently continues to rotate.

If the speed of the rotor is below that of synchronism, currents will be induced in the rotor in the usual manner, and will flow in such a direction that the motor tends to rotate with the field, and the external driving force will be less than that required to drive the motor without current. As the speed of the rotor is increased, the currents induced in it will fall until at the synchronous speed no currents will be formed, and the external force required to maintain the rotation of the rotor will exactly balance the friction and other forces opposing the rotation.

If the speed is now still further increased, the rotor conductors will begin to cut the lines of the rotating field in the opposite direction. This will cause rotor currents to flow in the reverse direction, and their action will be to *oppose* instead of to *assist* the rotation. Under these conditions the machine becomes a generator and supplies current to the line through the primary winding. It may thus be made to act as a brake, preventing the speed of the rotor rising much above synchronism.

Graphic Representation of Performance.

Circle Diagrams.—In practice, it is most convenient to represent the conditions in an induction motor by means of a diagram. By making a number of approximations it is possible to give such a diagram a very simple form, while retaining sufficient accuracy for most commercial purposes. The circle diagram affords a convenient practical method of obtaining approximate values for the output of the motor, which is fairly reliable for normal motors under commercial conditions.

Simple Circle Diagrams.—It will be convenient to summarise briefly the steps which led up to the construction of the simple circle diagram shown on page 316. A voltage is produced in the rotor winding proportional to the slip, due to the cutting of the rotor conductors by the constant rotating field.

This voltage e produces a rotor current having the value

$$i = \frac{e}{\text{rotor impedance}}$$

the voltage e will have two components, one in phase with the rotor current equal to $i \times R$, and one perpendicular to this in phase, equal to $i \times X$,

where i = rotor current per phase.
R = rotor resistance per phase
X = rotor reactance per phase.

Now, $X = 2\,\pi\,s\,L$

where s = frequency of rotor current = slip in cycles per second.
L = coefficient of self-induction of rotor winding.

$$L \text{ is equal to} \frac{\text{leakage flux} \times (\text{turns of winding per phase})^2}{i \times 10^9}$$

The inductance may also be calculated from the length and number of conductors and character of the slots.

Thus the reactance is not constant, but varies in proportion to the slip.

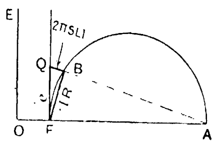


FIG. 214.—Diagram of Voltages in Rotor Circuit.

If the generated rotor voltage per phase be represented by vertical distances $F\,Q$ (Fig. 214), and $F\,B$, $B\,Q$ are drawn mutually perpendicular to represent the ohmic and reactive components of e, it has been shown (see page 314) that the point B will move on the semicircle $F\,B\,A$ as the value of e changes Since the resistance of the circuit is constant, the length $F\,B$ will be proportional to the rotor current. Thus, lines drawn from F to cut the semicircle will represent values of the rotor current.

The stator current per phase is equal to the sum of the stator magnetising (or no-load) current and of a current which produces the same number of ampere-turns as the rotor current. This portion of the stator current is, therefore, equal to

$$\text{the rotor current} \times \frac{\text{number of rotor turns}}{\text{number of stator turns}}$$

We can thus obtain the values of the stator current by redrawing our diagram as shown in Fig 215, where the magnetising current $O\,F$ (drawn perpendicular to the voltage e in Fig. 214) is shown combined with the current $F\,B$ to give the total stator current per phase $O\,B$ It must be remembered that since Fig. 215

is drawn to a scale of stator amperes, the line $F\,B$ is not now equal to the rotor current on the same scale, but is the rotor current divided by the ratio of transformation of the motor.

Fig. 215 represents the circle diagram for an induction motor without losses. The various lines of the diagram show the following quantities :—

$O\,F$ represents the no-load magnetising current per phase of the motor.

$O\,A$ represents the short-circuit current at the same voltage.

$O\,B$ represents the stator current under working conditions.

$E\,O\,B$ is the angle of lag at the stator terminals, so that $\cos E\,O\,B$ is the power-factor of the motor.

$O\,E$ is proportional to the rotor induced electromotive force, and may consequently be taken to represent the slip.

$B\,F$ is proportional to the rotor current, and represents it to the scale of stator current, *i.e.*, its actual value is obtained by dividing the length of $B\,F$ measured on the scale of amperes by the fraction

$$\frac{\text{rotor conductors}}{\text{stator conductors}}$$

$B\,G = I \cos \phi$; hence $B\,G$ is proportional to the watts supplied to the motor.

$\frac{O\,F}{O\,A} = \sigma$ is the "dispersion coefficient" of the motor, explained on page 316.

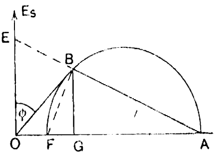


FIG. 215.—Diagram of Stator Currents.

When B moves round the circle to such a point that $O\,B$ becomes a tangent to the circle, we have the conditions corresponding to a minimum value of ϕ, *i.e.*, a maximum power-factor. The values of this power-factor, and of the slip and current corresponding to it, are easily obtained in this way.

The diagram shown in Fig. 215 does not take account of the various losses occuring in an actual motor.

We must next consider a more complete form of diagram, in which the effect of these losses is shown.

NOTE.—There are two distinct points of view from which the rotor circuit may be regarded for the purpose of the diagrammatic representation of the conditions which exist in it.

Firstly, the rotor may be regarded as a variable-speed alternator, having a voltage induced in it by the passage of its conductors

across the rotating field. From this point of view, both magnitude and frequency of the rotor voltage must be regarded as varying in direct proportion to the slip of the motor—as, indeed, is actually the case (see pages 295, 312).

On the other hand, the whole motor may be regarded as being equivalent to a transformer having its primary (stator) winding supplied at constant voltage and a secondary (rotor) winding, in which a constant voltage is also induced, and which carries a current which varies with the load on the motor.

The frequency of both voltage and current in the rotor must be regarded as constant and independent of the slip when observed from their effect on the stator * Consequently, when a diagram is constructed to show the behaviour of the rotor currents in terms of their influence on the stator circuit, we must treat the rotor currents as being of constant frequency and the rotor reactance as being constant.

This point of view is illustrated in the construction of the next diagram.

More Complete Circle Diagram.—This only differs from the simple diagram already given by the addition of two circles, which make an approximate correction for the losses in the stator and rotor windings.

Although originally drawn as a triangle of currents, the triangle $A\ B\ F$ in Fig. 215 may be looked upon as a voltage triangle for the motor, $A\ F$ representing the constant applied voltage, $F\ B$ the voltage overcoming reactance, and $A\ B$ the energy voltage in phase with the current, which may be supposed for this purpose to be represented by $A\ B$ in phase. The motor has a constant reactance, and consequently the line $F\ B$ will be proportional to the current in *magnitude,* since this line represents the product of the current by the constant reactance.

The line $A\ B$ represents the total energy voltage supplied to the motor, which consists of three parts, viz., voltage spent in overcoming stator and rotor resistances, and the induced back voltage. The line $A\ B$ can be subdivided into these components by two circles passing through A and F, and having their centres on the same vertical centre line as the circle $F\ B\ A$.†

These circles are shown in Fig. 216, where the length $B\ H$ represents the voltage per phase lost in stator resistance, $B\ H$ representing $i r_1$ volts to the same scale as the length $F\ A$ represents the constant applied voltage of the motor.‡ $H\ N$ is the voltage per phase spent in rotor resistance, $A\ N$ is the energy voltage due to rotation of the rotor. It has been shown that $B\ G$ is propor-

* Let the rotor conductor rotate past the stator windings with a frequency n. The frequency of the currents in the rotor conductors themselves is equal to the slip s. Hence the variations in the rotor current, as observed from a point on the stator, have a frequency $n + s = f$, which is the frequency of the stator supply.

† The point H lies in a circle because the triangle $F\ H\ B$ is constant in shape for all positions of B, since both $H\ B$ and $B\ F$ are proportional to the rotor current, and enclose a constant angle. Hence the angle at H is constant and lies in the arc of a circle.

‡ See footnote, page 341.

tional to the watts supplied to the stator ; $H J$ is similarly proportional to the watts given to the rotor (the difference representing watts lost in the stator winding) ; the ordinate of N is the watts output of the rotor, *i.e.*, the mechanical output of the motor, the difference in the heights of H and N being the watts lost in rotor resistance.

Now the watts supplied to the rotor are proportional to the torque,* hence the ordinates of the three points B, H, and N give respectively lengths proportional to input, torque, and output of the motor. The circles on the diagram are accordingly called the circles of input, torque, and output.

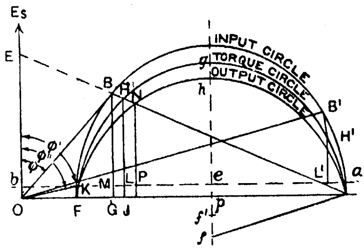


FIG. 216.—Complete Circle Diagram.

The slip of the motor would be proportional to $O E$ in Fig. 216, except for the stator losses. For speeds near synchronism this construction for the slip may be used. A more accurate value is obtained as follows, the construction being shown on Fig. 218. If $O B^1$ is the line of stator current with stationary rotor, the *slip line* is the line $B^1 T$ in Fig. 218, drawn from B^1 perpendicular to the radius $A f^1$ of the torque circle. The point T is where this line cuts the base $A O$. For any position of B the length $V T$ cut off by $A B$ on the slip line is proportional to the slip. The whole length $B^1 T$ represents 100 per cent. slip, so that the slip of the motor will be $\frac{V T}{B^1 T} \times 100$ per cent. slip.

"**Synchronous Watts.**"—It was shown in the footnote on page 327 that the numerical value of the watts supplied to the rotor of an induction motor is proportional to the value of the torque

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* $2 \pi n T$ = mechanical output of rotor (see pages 327 and 328).

$2 \pi \frac{s}{p} T$ = watts lost in rotor.

$\therefore \frac{2 \pi}{p} (p n + s) T$ = rotor output + losses = input to rotor.

or since $p n + s = f$ = constant.
$T \propto$ input to rotor.

exerted by the rotor. The ordinates of the second circle of the circle diagram may thus either be measured to a scale of units of torque, in order to give values for the torque exerted by the motor, or they may be taken as representing the power given to the rotor, measured to the same scale of watts as that employed for the two other circles. The watts thus indicated represent the power which would be given out by the motor if it were to run at synchronous speed and to exert the same torque as under the existing conditions. For this reason, the power given electrically to the rotor is sometimes called the *Synchronous Watts* of the motor. Since this quantity is numerically proportional to the torque exerted, a curve of "synchronous watts" is frequently plotted instead of a curve of torque for showing the behaviour of the motor. When employed in connection with the circle diagram, the advantage is obtained that ordinates of all three circles are measured to a common scale, viz., that of watts.

Experiment XLVIII.—Experimental Derivation of Circle Diagram for an Induction Motor.

Instructions.—(*a*) Run the motor without load at normal voltage and frequency. Measure the current taken by the stator $= I_0$, and the power by means of a wattmeter, as described in Experiment XLII.

(*b*) Then make a fresh experiment with the shaft clamped so that it cannot rotate, and the rotor conductors short-circuited. Again apply the normal voltage to the stator, and measure the current and watts supplied. If the current taken by the motor under these conditions is too high for safety or convenient measurement, reduce the primary voltage, and multiply the current observed by the ratio $\dfrac{\text{normal volts}}{\text{volts applied}}$ in order to obtain the true static current, and multiply the watts observed by the square of this fraction in order to get the static watts. This is done because the no-load watts will be nearly all copper losses, and proportional to I^2.

(*c*) After these tests, measure the resistance of the stator winding while still hot by passing a measured direct current through the windings, and observe the drop of potential in them.

(*d*) If the motor is provided with slip-rings, measure the ratio of transformation by applying an alternating voltage to the stator and measuring the voltage between one pair of slip-rings when open-circuited. When taking the slip-ring voltage, move the rotor round until it is in the position of maximum induced voltage. This maximum value is the voltage to be observed. The ratio of transformation is thus the ratio of the observed stator and rotor voltages.

These measurements are sufficient for the construction of the complete diagram, from which the performance of the motor at all loads can be predetermined.

The method of procedure is then, in outline, as follows :—

Let $\cos \phi''$ = power-factor at stator terminals when full voltage is applied and motor runs light.

i'' = current per phase under these conditions.

$\cos \phi'$ = power-factor at stator terminals when rotor is locked and sufficient voltage is applied to produce full-load current.

i' = current per phase which is observed under these conditions multiplied by the factor $\dfrac{\text{normal working voltage}}{\text{actual voltage during test.}}$

E = normal voltage per phase.

Referring to Fig. 216,

Draw vertical and horizontal lines $O\ E$, $O\ A$.

Set off $O\ K$ to represent to scale i'' amperes, making the angle $E\ O\ K = \phi''$.

Draw $K\ F$ vertical to cut the line $O\ A$ at F.

Draw a dotted horizontal line $a\ b$ through K. The height of this line represents the power and torque taken up in overcoming the friction and iron losses of the motor.

Set off $O\ B^1$ equal to i' on the scale of amperes, making the angle $B^1\ O\ E = \phi'$.

Draw the semicircle $A\ B\ F$ through points $B^1\ K$, its centre p lying on the horizontal line $O\ A$, thus fixing the point A.

Join $B^1\ A$, and draw $A\ f$ perpendicular to $B^1\ A$ to cut a vertical line through p at f, which is the centre of the output circle $A\ N\ F$, which is to be drawn through the points A and F.

From B^1 mark off* a length $B^1\ H^1$ on $B^1\ A$ such that $B^1\ H^1 = \dfrac{I_1\ R_1}{E} \times O\ A$, *i.e.*, make the ratio of the lengths $\dfrac{B^1\ H^1}{O\ A}$ the same as the ratio of the resistance drop per phase in the stator (= stator current × stator resistance per phase) to the stator applied volts per phase.†

Draw the torque circle $A\ H\ F$ through A, H^1, and F, its centre f^1 lying on the vertical line through p.

The diagram is now complete, the following being the quantities represented :—

Phase of terminal volts by $O\ E$

No-load current by $O\ K$.

No-load magnetising current by $O\ F$.

Stator current at any load by $O\ B$.

Power-factor $\cos \phi = \cos B\ O\ E$, which is a maximum when $O\ B$ is a tangent to the circle $A\ B\ F$.

No-load power-factor $\cos \phi'' = \cos K\ O\ E$.

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* It will usually be advisable to mark off the voltage $I_1\ R_1$ for some other value of the current $O\ B$ because $B_1\ A$ is so steep that it is not easy to draw a circle to cut it exactly at any desired point.

† It will be noticed that $O\ A$ is taken to represent the phase voltage, and not $F\ A$ as stated on page 338. This is an arbitrary rule made to introduce an approximate correction for losses in voltage not otherwise accounted for. In the completed diagram $O\ A$ is always total volts per phase.

Energy component of no-load current by $F\,K$.
Rotor current by $F\,B$ (multiplied by ratio of transformation).
Input to motor ($I_s \times E \times 1{\cdot}73 \cos\phi$) proportional to $B\,G$.
Torque of motor by line $H\,L$ (maximum torque $= g\,e$).
Output by $N\,P$ (maximum output by $h\,e$).
Slip of motor by $O\,E$ (or by $V\,T$ in Fig. 218).
Starting torque by $H^1\,L^1$ (if without added resistance).
Starting current by $O\,B^1$ (if without added resistance).
Power-factor at starting $\cos\phi' = \cos B^1\,O\,E$ (if without added resistance).

Stator ohmic volts ($= I_1\,R_1$) by $B\,H$ } to same scale as $O\,A$ represents stator volts per phase.
Rotor ohmic volts ($= I_2\,R_2$) by $H\,N$ }

Stator copper loss $B\,G - H\,J$ watts.
Rotor copper loss $H\,L - N\,P$ watts.

The scale of watts is easily obtained for one value of the input. This gives the scale for both input $B\,G$ and output $N\,P$.

The torque is best calculated from the speed and output at full-load. Thus

$$T = \frac{W \times 33{,}000}{2\,\pi\,n}$$

where $W =$ output in horse-power, obtained from output circle.
$n =$ revs. per minute.
$T =$ torque in pound-feet.

The synchronous watts may be read off directly as ordinates of the torque circle to the same scale as the input or output circle ordinates.

The slip can also be determined as a rule from its value at full-load or from construction shown in Fig. 218.

In the above description it has been assumed that the diagram was to be drawn from test results. In designing a motor the diagram has to be drawn from the calculated value of the leakage coefficient from which the input circle is derived : the other portions of the diagram may be put in so as to fulfil the specified working conditions of the motor.

The figures employed in the following example are taken from Mr. Eborall's paper, read before the Society of Arts in 1901.

The motor was rated to give 80 b.h.p. at 600 revs. per minute at 40 cycles and 350 volts.

Fig. 217 gives the results of the locked and no-load tests from which the diagram Fig. 218 was constructed.

The upper curve gives the relation between voltage and current with the rotor rigidly clamped, and short-circuited ; the second curve shows the relation of no-load current to voltage, the motor running light ; the third curve gives the corresponding watts input, with motor running light.

From the curves we see that at a pressure of 202 volts per phase (*i.e.*, a terminal pressure of 350 volts with a star-connected stator) the current I_s is 532 amperes with locked rotor, the no-load current I_0 is 40 amperes, and the power absorbed at no-load is 1,680 watts

per phase. The power taken by the motor at rest with a current of 532 amperes was estimated from a measurement at a lower pressure to be 25,100 watts per phase at 202 volts. The stator resistance per phase was ·032 ohm.

From these figures,

$$\text{Cos}\,\phi'' = \frac{1{,}680}{202 \times 40} = 0{\cdot}218$$

$$\text{or } \phi'' = 77 \text{ deg. (approx.)}$$

$$\text{Cos}\,\phi' = \frac{25{,}100}{532 \times 202} = 0{\cdot}233$$

$$\text{or } \phi' = 77 \text{ deg. (approx.)}$$

The values of the power-factor happen to be practically the same in this case for both locked and no-load tests.

Referring to Fig. 218, take two lines at right angles $O\ E$ and $O\ A$ From O draw a line $O\ B^1$ making an angle of 77° with the

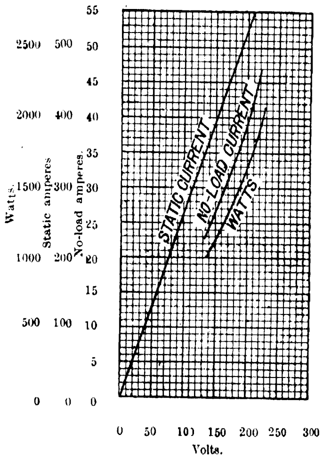
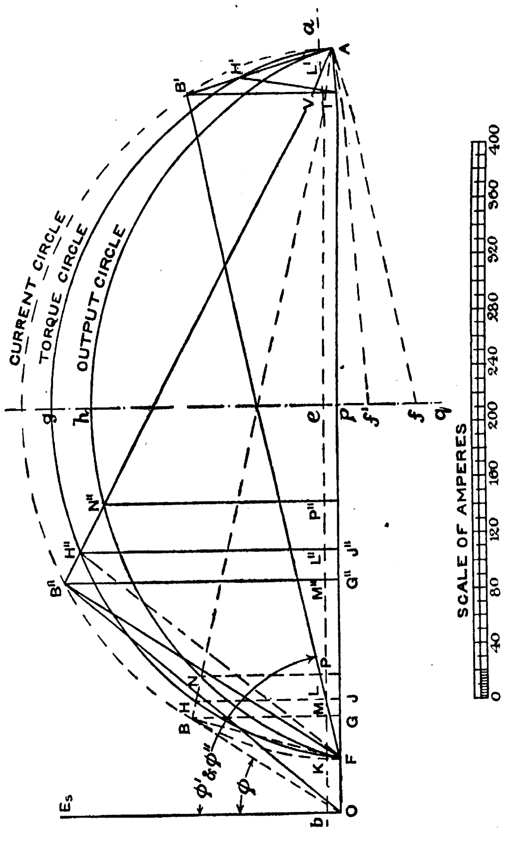


FIG. 217.—No-load Curves of Induction Motor.

vertical line $O\ E$; this line $O\ B^1$ represents then the phase of the current with locked rotor and of the no-load current. Next, select a suitable ampere scale for the diagram, say one centimetre corresponds to 20 amperes. Make, therefore, the piece $O\ K =$ 2 cms $= I_0$, and make the piece $O\ B^1 = 26{\cdot}6$ cms. $= I_s$ Now draw the semi-circle $A\ B\ F$, having its centre along the line $O\ A$ (viz., at point p), and passing through the points B^1 and K already found. Through point K draw the line $a\ b$ parallel to $O\ A$.


CURRENT CIRCLE
TORQUE CIRCLE
OUTPUT CIRCLE
SCALE OF AMPERES
0 40 80 120 160 200 240 280 320 360 400

To get the output circle, the output of the motor is zero for the point B^1, as the motor is then at rest. Therefore join this point to point A, and from the latter draw the line $A\ f$, so that the angle $B^1\ A\ f$ is 90° ; the point f so found is the centre of the "output" semicircle which can now be drawn at such a radius that it passes through the points A and F. The line $B^1\ A$ is thus a tangent to the circle $A\ N\ F$, so that the value of the output for the point B^1 corresponding to the stator current of 532 amperes is zero.

To find the torque circle, scale off the values $B^1\ A$ and $O\ A$ from the diagram, which will be found to be 6·4 and 27·5 cms. respectively ; the latter corresponds to a pressure per phase of 202 volts. Hence

$$\text{Value of vector } B^1\ A = \frac{6\cdot 4}{27\cdot 5} \times 202 = 47 \text{ volts.}$$

The copper drop per phase of the stator for the current I_s = 532 amperes is $(532 \times 0\cdot032) = 17$ volts. Hence, to get the desired point, H^1, on the torque circle, we mark off along $B^1\ A$ from the point B^1 a piece equal to $\frac{17 \times 6\cdot 4}{47} = 2\cdot 31$ cms. Hence the torque circle is fixed by finding the point H^1 ; from a suitable centre along $p\ f$ (viz., f^1) draw it through points A, H^1, and F.

Finally, to get the slip line, drop a perpendicular from the point B^1 on the radius $A\ f^1$, thus getting the line of $B^1\ T$, which turns out to be 6·25 cms. in length, T being the point of intersection with $O\ A$. This is equal to a slip of 100 per cent as the motor is not running ; at the load corresponding to the stator current $O\ B$, torque $H\ L$, and output $N\ P$, the slip is equal to the piece $V\ T$ cut off on the slip line $B^1\ T$, which scales 0·25 cm. Consequently the full-load slip is 4 per cent ; at the maximum load the motor will carry (156 b.h.p.) the slip is 16 per cent.

The scale of watts for both input and output circles is obtained as follows : Taking the condition corresponding to a stator current, $O\ B$, by measurement $B\ G$ ($= i \cos \phi$) represents 120 amps. Hence at phase voltage of 202,

Input represented by $B\ G = 3\ e\ i \cos \phi = 3 \times 120 \times 202 =$ 72·7 kw.

It happens that the motor gives its rated output when the stator current = 128 amperes, the value represented by $O\ B$ in Fig. 218 ; the line $N\ P$ in the diagram thus represents 80 b.h.p., and the value of all such lines as this, now that the value of one of them has been found, is definitely known.

Taking into account the known slip of the motor we could, from the known output, find the value of the torque vector $H\ L$, in pound-feet, and having one such value, the values for all the other lines representing torque would be definitely fixed.

The value of the torque may be calculated as on page 342, or we may avoid the use of a separate scale of torque by measuring the ordinates of the torque circle to the same scale as the ordinates

of the input and output circles and thus obtain its value in "synchronous watts."

The complete performance of the motor is thus given by the diagram for all loads.

In practice the diagram is accurate enough for all practical purposes, there being a slight error on the right side, *i.e.*, the actual performance of the motor is rather better than that given by the diagram. There are slight errors which appear to cancel one another out, except for very small motors. The diagram can be made accurate for these also, if the line of no-load losses $a\ b$ is slightly inclined, being drawn with a downward slope of about 30 per cent. from the point K to a. This is not necessary for motors greater than 5 b.h.p. output, but with smaller motors than this the relatively greater no-load losses slightly affect the accuracy of the diagram and render this empirical correction advisable.

Alternative Form of Circle Diagram.—Various modifications of the circle diagram described in the preceding sections have been developed and used. A convenient form of diagram is one in which the three circles of the original Heyland diagram are replaced by a single circle and two sloping lines. In this diagram (Fig. 218a), which may be taken as typical of those in general use, the output and torque are measured from the circle of input to two sloping lines, instead of from separate circles of output and torque to the base line. The main principles underlying the construction of this diagram are the same as those already discussed; the difference lies in the geometrical construction adopted for subtracting the rotor and stator losses from the lines of power input given by the circle of stator current, in order to give the torque exerted and power output.

The following detailed account of the construction of the diagram in Fig. 218a from the same readings as those already employed for the diagram in Fig. 218, page 344, will make clear the method adopted in its construction.

For convenience, the measured values employed in the construction are repeated below :—

No-load current 40 amps. per phase.
" watts 5,040 total.
Short-circuit current 532 amps. per phase.
" watts 75,300 total.
Resistance of stator 0·032 ohm. per phase.
Watts lost in stator resistance on short-circuit
$532^2 \times 0{\cdot}032 \times 3 = 27{,}180$ watts total.
Phase volts of motor = 202 volts.

Procedure.—Select a suitable current scale for the current vectors, say 1 cm. = 20 amps. per phase.

The scale to which the ordinates of the diagram represent power will then be given, because

20 amps. per phase represent $20 \times 202 \times 3$ watts
$= 12{,}120$ watts total (when the current is in phase with the voltage, *i.e.*, measured vertically in the diagram).

Hence the scale to which ordinates in the diagram represent power is

1 cm = 12·12 kw. total power.

Draw OE vertical and OA horizontal.

Draw $b\,a$ parallel to OA at a distance above it to represent the no-load power. This distance is $\frac{5040}{12120} = 0·416$ cm.

With centre O and radius representing the no-load current draw an arc to cut $b\,a$ in K.

This radius is $\frac{40}{20} = 2$ cm.

OK represents the no-load current per phase in magnitude and phase.

Similarly, draw a horizontal line at such a distance above OA as to represent the power due to the short-circuit current, and draw an arc from centre O and radius representing the short-circuit current to determine the point B^1, so that OB^1 represents the short-circuit current in phase and magnitude.

The height of B^1 above OA will be

$$\frac{75·3}{12·12} = 6·21 \text{ cm.},$$

while the length of the radius OB^1 is

$$\frac{532}{20} = 26·6 \text{ cm.}$$

We have now to find a semi-circle which has its diameter on $K\,a$ and passes through the points B^1 and K.

To find the centre of this circle, join KB^1 and bisect this line by a perpendicular line, cutting $K\,a$ at e. The point e is then the centre of the semi-circle to be drawn through K and B^1.

Ordinates of this semi-circle measured to the line OA (such as BG) represent the total power supplied to the motor. Ordinates measured to the line $b\,a$ (such as BL) represent the power used in the electrical circuits of the stator and rotor, *i.e.*, the total power after the no-load losses (friction and iron losses) have been subtracted.

Ordinates to the line OB^1 represent the power of the motor after subtraction of the no-load losses and the stator and rotor copper losses, *i.e.*, they represent power developed mechanically as useful output.

The torque line (which gives power supplied to the rotor, see page 339) can be obtained by finding the point H^1 which divides the total copper losses on short-circuit, represented by B^1L^1 in the ratio of stator to rotor losses.

Since the stator resistance per phase is 0·032 ohm, the stator copper loss on short-circuit is

$$532^2 \times 0·032 \times 3 = 27{,}180 \text{ watts total.}$$

This is represented by a line of length

$$\frac{27180}{12120} = 2·24 \text{ cm.}$$

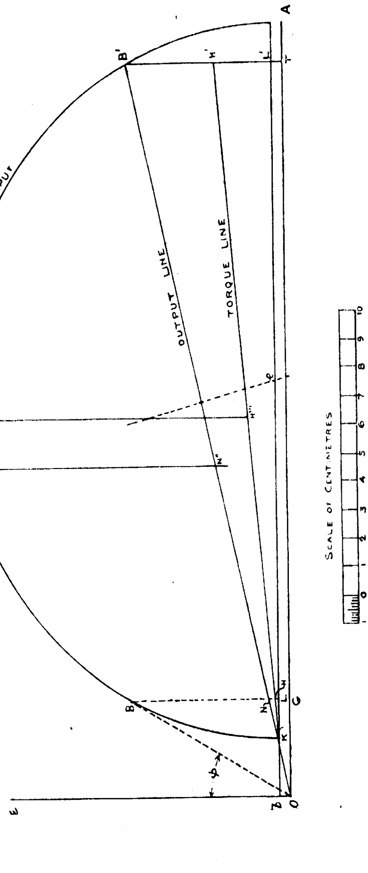


Fig. 218a.—Simplified Circle Diagram of Induction Motor.

Hence we mark off $B^1 H^1$ equal to 2·24 cm, and thus obtain the line of rotor input or torque.

Ordinates from the circle to this line give the torque of the motor in "synchronous watts" (see page 339) to the same scale of watts as that already used. The ordinates such as $H\,L$ give the power spent in rotor resistance.

If we draw a tangent to the circle parallel to the output line $K\,B^1$, this will give us the point B^{II} for maximum output.

Similarly a tangent to the circle drawn parallel to the torque line $K\,H^1$ will give the point B^{III} for maximum torque.

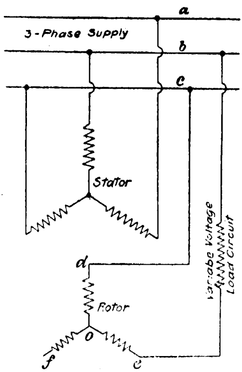


FIG. 219.—Use of Induction Motor as A.C. Booster.

The diagram is now complete, and represents the following quantities :—

- $O\,E$ phase of terminal voltage (per phase).
- $O\,B$ stator current on load (per phase).
- $K\,B$ rotor equivalent current on load (per phase).
- $O\,K$ stator no-load current (per phase).
- $O\,B^1$ stator short-circuit current (per phase).
- $L\,G$ losses on no-load (total).
- $N\,H$ stator copper losses (total).
- $H\,L$ rotor copper losses (total).
- $B\,N$ output of motor on load.
- $B\,H$ torque of motor in synchronous watts.
- $B^{II}\,N^{II}$ maximum power output.
- $B^{III}H^{III}$ maximum torque.
- $\cos\phi$ power-factor on load.

Use of Induction Motor as Voltage Regulator.—Brief reference may be made to a convenient method of regulating or

varying the voltage of an alternating-current circuit often employed for instrument calibration, &c. The stator of the motor is connected to a supply of constant voltage, whereby a constant electromotive force is induced in the rotor, which remains stationary. *The phase of this electromotive force depends entirely on the position of the rotor,* and by clamping the rotor in any position, the voltage between slip-rings may be adjusted so as to be in any relative phase to the voltage of the supply mains, and may be made to supply currents to a circuit at any phase different from that of the mains. By connecting the rotor winding in series with the mains and a load circuit, the rotor voltage may be made to increase or reduce the circuit voltage by rotating the rotor into any desired position. The connections for varying the voltage in one phase of a 3-phase circuit are indicated in Fig. 219. Evidently the regulation would be equally simple performed in all three phases.

Induction Generator.—We have seen that voltage is induced in the rotor of an induction motor as a result of the motion of the rotor conductors relatively to the rotating field. As the speed of the induction motor approaches the speed of synchronism, this relative motion becomes less, and the induced voltage of the rotor accordingly diminishes. If the speed of the rotor is artificially increased beyond the speed of synchronism, there will again be motion of the rotor conductors relatively to the field, but in the reverse direction, since the conductors will be moving faster than the field under these conditions, and will consequently be cut by the lines of the field in an opposite direction.

It follows that the direction of the induced rotor voltages (and consequently the direction of the rotor currents) of an induction motor running above synchronism is the reverse of that of a motor running below synchronism. The torque which is set up between the rotating field and the rotor therefore changes its sense as the speed rises above synchronism. Below synchronous speed, the rotating field drag the conductors round with it, but the conductors rotate more slowly than the field. Above synchronism, the field retards the conductors of the rotor, which revolve faster than the field and in opposition to its drag. Under these last conditions, the induction machine has changed from its operation as a motor into that of an *induction* generator. The rotor currents now react upon the stator windings and induce energy currents which are capable of performing useful work in the alternating circuit connected to the stator terminals.

This action of an induction machine is closely analogous to that of a shunt-wound continuous-current machine. The shunt machine, when connected to the supply circuit, will run below its "critical" speed as a motor. If accelerated to a speed above the "critical" value, the machine becomes a generator and returns power to the line from which it previously took current.

The induction generator is not self-exciting, so that it is necessary that the rotating field should be maintained from an external source. This involves the continued supply of an idle alternating current from the mains, which will be the same for the induction generator as for the induction motor. It is the energy component of the current taken by the stator from the mains which is reversed when the speed passes through synchronism; the magnetising component of this current is unaffected.

Another important point is that the frequency of the generated stator currents is independent of the speed of the rotor, both above and below synchronism. This becomes clear when it is remembered that the changes which take place in the rotor currents *opposite to any fixed point on the stator* have always the frequency of the stator supply and are independent of the speed of rotation (see page 338).

The behaviour of the induction generator may be traced on the circle diagram (for instance, Fig. 216, page 339) by continuing the circles on the lower side of the horizontal line. Both "input" and torque become reversed, while the magnetising current remains the same as before.

Cascade Connection of Induction Motors. — Induction motors are said to be connected "in cascade" when the rotor winding of one motor is connected to the stator of a second, so that the stator of the second motor is supplied with the power generated electrically in the rotor of the first. This connection is indicated in Fig. 220.

When so connected, the frequency of the currents supplied to the second motor is equal to the slip of the first.

$$\text{Speed of the first motor} = \frac{f - s_1}{p_1} = n_1 \text{ revs. per sec.} \quad . \quad . \quad (1)$$

$$\text{Speed of second motor} = \frac{s_1 - s_2}{p_2} = n_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where s_1 s_2 are the respective slips of the two motors, and p_1 p_2 the number of their pole pairs.

These speeds may have any relative value, if the motors are not mechanically coupled. If both are running unloaded, one motor may be brought to rest by applying a slight friction to its pulley; the other motor will increase its speed to near synchronism at the same time.

Under all conditions we have the relation

$$f = n_1 p_1 + n_2 p_2 + s_2.$$

The sum of the speeds of the two motors is thus approximately constant, since s_2 is small.

Suppose the two motors to be mechanically coupled by a belt, so that their relative speeds are in the ratio

$$\frac{n_1}{n_2} = k$$

Employing this relation between n_1 and n_2, we find that the speed of the first motor becomes

$$n_1 = \frac{k\,(f - s_2)}{p_1\,k + p_2}$$

while the speed of the second motor is

$$n_2 = \frac{f - s_2}{p_1\,k + p_2}$$

If the motor pulleys are equal, or if the motors are direct coupled, the speed of the set becomes (since $k = 1$)

$$n = \frac{f - s_2}{p_1 + p_2}$$

which is the speed of a motor having the same number of poles as the two machines together.

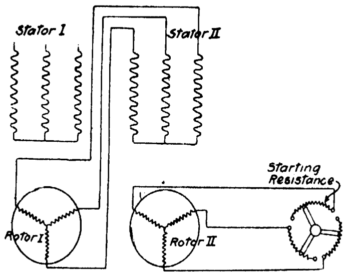


FIG. 220.—Cascade Connection of Motors.

By neglecting the losses in the motors, and consequently assuming that s_2 is zero, it is easy to express their relative outputs.

Assuming equal speeds, let w_0 = total watts supplied from line to first motor (see page 328).

In motor (I) watts generated in rotor $= w_0 \frac{s_1}{f} = w_0 \frac{p_2}{p_1 + p_2}$

This is the power given electrically to the second motor.

Consequently mechanical output of motor (II) $= w_0 \frac{p_2}{p_1 + p_2}$

" " " " (I) $= w_0 \frac{p_1}{p_1 + p_2}$

The cascade connection of induction motors has been employed in traction. It is of practical value in test rooms, in order to obtain a supply of power at a different frequency from that of the supply circuit. When two motors are mechanically coupled, it has been shown that the slip of the first one is a practically constant quantity. Its rotor may therefore be used as a source of alternating power having a frequency equal to s_1 given in the formula above. The motor connected to the supply mains, and having a second motor connected in cascade, may be looked upon as a form of frequency transformer.

By varying the relative speeds of two induction motors connected in cascade, and by changing the direction of rotation, a large range of frequencies, both above and below that of the original supply may be obtained.

The cascade connection of machines is employed in the *motor converter* for the conversion of high-pressure alternating into low-pressure continuous currents. The cascade connection in this case has the important advantage that the converter is supplied with alternating currents of a lower frequency than those of the source of supply.

The La Cour Motor Converter.—The motor converter consists of an induction motor and of a direct-current generator having a common shaft and mounted on the same bedplate. The rotor winding of the induction motor and the armature of the direct-current generator are connected in "cascade"

Fig. 221 indicates the connections of a 3-phase motor converter. S is the stator winding of the induction motor, which is connected direct to the 3-phase supply. R is the rotor winding, the phases of which are connected to equally-spaced points on the generator

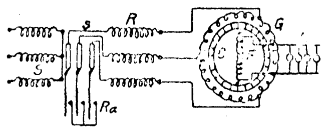


FIG. 221.—Diagram of Motor Converter.

armature G. C is the commutator, and F the field winding of the generator, which is made self-exciting. R is a 3-phase starting resistance connected to the rotor in the usual way through slip rings.

Let us first assume that both motor and generator have the same number of poles. If this is the case, the motor will run at exactly one-half the speed of synchronism, and the currents induced in its rotor and supplied to the generator armature will have a

frequency equal to one-half that of the supply circuit. These currents will serve to drive the armature of the generator, acting exactly like the armature currents of a synchronous motor, since their frequency corresponds exactly to the speed of rotation of the generator. Expressed differently we may say that the 3-phase currents induced in the rotor winding of the motor will be such as to produce a rotating field in the armature of the generator. By correctly choosing the connections between the two machines, this rotating field may be made to rotate in the opposite direction in this armature to the direction of rotation of the armature itself. Since the speed of rotation of this field will be equal to that of the armature, the field will become stationary in space, and will take up a constant position relative to the poles of the machine. This is exactly the condition which exists in the case of a synchronous motor.

The machine which we have so far called the generator is thus partly driven by the induction motor through the shaft, and made to act as an ordinary continuous-current generator, and partly acts as a synchronous rotary converter, supplied with currents induced in the rotor of the motor and having one-half the frequency of the supply circuit

In the case just assumed of an equal number of poles in motor and converter, the continuous-current machine will obtain one-half the power given to it in the form of mechanical torque through the shaft, and one-half as electrical power through the rotor currents.

If the two machines have not an equal number of poles, the speed of the set will be that given by the formula

$$n = \frac{60\,f}{p_k + p_d} \quad \text{(see page 348).}$$

where n = revolutions per minute
f = periodicity of supply.
p_k = number of pole pairs of induction motor
p_d = number of pole pairs of converter

Also the induction motor will convert the fraction $\frac{p_k}{p_k + p_d}$ of the total power supplied into mechanical power, transforming the remainder $\frac{p_d}{p_k + p_d}$ of the power into electrical power transmitted to the converter from the rotor winding (see page 353).

The starting of the motor converter is very simple, since the machine runs up to synchronism automatically. The stator is switched on to the supply circuit, and the rotor resistance is then slowly reduced. The machine gradually runs up to full speed. When this condition is reached, a voltmeter connected across the rotor slip-rings ceases to vibrate, and the rotor resistance may then be completely short-circuited. In practice, the rotor winding

has a greater number of phases (*e.g.*, nine or 12) connected to the generator armature. Three only are employed in starting, but all are short-circuited when synchronism is reached.

It is evident that, unlike the rotary converter, any ratio of transformation may be adopted in the case of a motor converter. This is accomplished by suitably choosing the ratio of stator to rotor turns in the induction motor.

The motor converter has an advantage in efficiency over the motor generator, and on circuits of high frequency has advantages in regard to commutation over the rotary converters, as well as having more easy adjustment of its output voltage.

Voltage and Current Relations in the Motor Converter.—Considering first the induction motor, we have one circumstance which does not exist in the ordinary motor. We can vary the phase relation between current and induced voltage in the motor at will by alteration of the excitation of the synchronously-running converter. Since currents in the rotor will always be represented by currents producing the same ampere-turns (in both phase and magnitude) in the stator, it follows that it is possible to bring current and voltage at the stator terminals into coincidence of phase by suitable adjustment of the converter field. If this is done, the stator takes only energy current from the line, and the magnetising current of the motor is all supplied from the rotor, *i.e.*, the rotor carries an idle current of the same number of ampere-turns as the stator winding would take under ordinary conditions of working, in order to produce the rotating field.

Let us first assume that the matters are adjusted in the manner just indicated, so that the motor is receiving current at a power-factor of unity.

The primary current per phase is then

$$I_1 = \frac{W_1}{m_1 E_1}$$

Where W_1 is the total power given to the machine ;
m_1 is the number of phases of the stator ;
E_1 is the phase voltage.

Neglecting stator copper drop,

$E_1 = 4{\cdot}44\, f_1\, f\, T_1\, F 10^{-8}$ volt.

Here f_1 is the winding factor of the stator ;
T_1 is the stator turns ;
F is the maximum flux per pole.

The currents induced in the rotor have a frequency

$$\frac{f\, p_d}{p_s + p_d}$$

while the induced voltage has the value

$$E_2 = \frac{p_d}{k\,(p + p_d)} E_1$$

k being the ratio of transformation at standstill, between stator and rotor tapping points, when open-circuited.

The rotor current has one component balancing the stator ampere-turns, viz. :—

$$I_2 = \frac{m_1}{m_2} k\, I_1 \text{ amps., } m_2 \text{ being number of rotor phases.}$$

In addition to this, the rotor carries a magnetising current I_m, giving the ampere-turns required to produce the rotating field.

Thus the total rotor current will be

$$I_2 = \sqrt{I_2{}^2 + I_m}$$

The power transmitted electrically by the rotor to the converter is

$$W_{2e} = m_2 E_2 I_2 = \frac{p}{p_s + p_d} W_1$$

The power transmitted mechanically is

$$W_2 = \frac{p}{p_s + p_d} W_1$$

As far as the first portion of the power is concerned, the motor operates as a transformer, and as regards the second part as an induction motor.

Neglecting the small loss in winding resistances, we know that the ratio between the rotor induced voltage and the commutator voltage will be given by the usual expression for the voltage ratios in a rotary converter, viz. :—

$$\frac{\text{alternating voltage between adjacent connectors}}{\text{voltage at commutator}} = \frac{\sin\frac{\pi}{m_2}}{\sqrt{2}}$$

The ratio between the alternating current supplied at each point of the direct-current armature winding, and the alternating current flowing in the armature conductors is given by

$$\frac{\text{current supplied at tapping point}}{\text{current in conductors of armature}} = \frac{1}{2 \sin\frac{\pi}{m_2}}$$

For a mesh-connected rotor, the current per phase of the rotor winding will be equal to the alternating current per phase of the winding of the direct-current armature. With star-connected rotor, the rotor current per phase is evidently the current supplied to each point of the direct-current armature.

The connection between the watt component of the alternating current and the direct current at the commutator is given from the following considerations :—

$$\text{The direct current} = I_d = \frac{W}{E_d}$$

where W is the power output, and E_d the commutator voltage.

Also, the power represented by the alternating currents supplied from the rotor to the armature has already been shown to be

$$W_{2e} = \frac{p_a}{p_s + p_a} W.$$

Hence the alternating current supplied by the rotor to the armature will produce an alternating current in the armature conductors having a value

$$I_a = \frac{p_a}{p_s + p_a} \times \frac{\sqrt{2} \,.\, I_d}{m_2 \sin \frac{\pi}{m_2}}$$

In addition to this current, there may be an idle alternating current depending upon the excitation given to the generator field magnets.

Let us suppose that it is desired that the magnetising current of the induction motor shall be completely supplied by the rotor, and that in addition a leading idle current of I_x amperes is to be supplied to the circuit in order to neutralise the idle lagging currents taken by other machines on the same circuit.

This leading current will correspond to a rotor current of $\frac{m_1}{m_2} k\, I_x$ amperes, so that the total idle current will have the value

$$I_m + \frac{m_1}{m_2} k\, I_x$$

where I_m is the rotor magnetising current.

This current will be at right angles in phase to the current I_a previously discussed, and will not produce any useful effect at the commutator. It will add to the heating of both rotor and armature without affecting the output of the set.

CHAPTER XII.

SINGLE-PHASE MOTORS.

Single-phase Induction Motor.—If a 3-phase induction motor is running while connected to the supply circuit by three wires in the usual way, and one of these wires is broken, the motor will continue to rotate and to perform its work, merely taking a somewhat greater current from the remaining wires. The current supplied to the machine under these conditions is a single-phase alternating current.

Further investigation will show that when supplied with single-phase current only, the properties of the motor are somewhat different from those of the ordinary 3-phase motor. Thus, if the motor be stopped and supplied with single-phase current while at rest, it will not start itself. If, however, it be started by hand, it will probably continue to rotate and gradually attain its normal speed. Further, it will be found that the motor may be started equally well in either direction, and will gain speed and continue to rotate with complete indifference as to its direction of rotation.

The explanation of this behaviour is to be found in the fact, to be explained immediately, that, when stationary, there is an oscillating field (instead of a rotating field) set up in the stator by the single-phase alternating current. Let Fig. 222 represent diagrammatically the stator coils of a 2-pole motor with squirrel cage rotor, the phase supplied being shown by a firm line, the idle coils being shown by dotted lines. The flux due to the winding $A\ A^1$ will pass through the rotor in such a way as to induce currents in the conductors a, b, c, in one direction, and in the conductors a^1, b^1, c^1 in the opposite direction, and these conductors may be imagined to form circuits $a\ a^1$, $b\ b^1$, $c\ c^1$, &c., through which the stator flux is threaded, and in which it will produce currents. The whole of the rotor conductors may be imagined to be divided up in this way into separate circuits, the axis of each circuit coinciding with the axis of the stator coil $A\ A^1$. These currents will thus act on the same magnetic circuit as the stator winding, and their magnetic effect will be neutralised by an increase of stator ampere-turns exactly equal and opposite to those rotor ampere-turns, in order that the stator flux may be maintained at its previous value.

If, now, the rotor in Fig. 222 is moved round fairly rapidly by hand, the conductors under $A\ A^1$ will move across the flux at $A\ A^1$, and will have currents induced in them, the strength of which will depend upon the strength of the flux and the speed of rotation. The conductors in the neighbourhood of $b\ b^1$ will move *along* the

direction of the flux, instead of across it, and will have little or no current induced in them by the rotation. The rotation will consequently produce an electromotive force in the rotor winding which is *in phase* with the flux generated by the stator current and tends to produce currents in the conductors of the rotor which form closed circuits about a horizontal axis, as in Fig. 222.

Now these currents will produce a horizontal flux which will not affect the stator winding. The currents are consequently purely magnetising currents, and will be 90° in phase behind the electromotive force producing them. The rotation of the rotor thus gives rise to an alternating magnetic flux which is at right angles to the main flux both in direction and in phase. The sense of this flux will depend on the direction of rotation of the rotor, and its magnitude upon the speed.

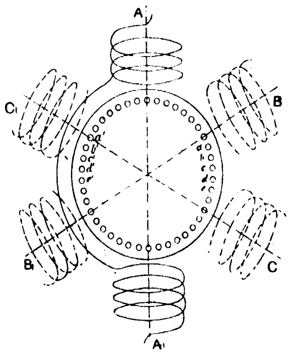


FIG. 222.—Principle of Single-phase Induction Motor.

As already explained in the discussion of the production of a rotating field, the necessary conditions for the existence of such a field are fulfilled by the formation of two alternating magnetic fluxes in two directions perpendicular to each other, the phase of the fluxes differing by a quarter period. These conditions, therefore, will exist in the rotor of the induction motor supplied with single-phase current as soon as it is rotated with sufficient speed to produce the current which forms the second field. At slow speeds the currents produced by the motion of the conductors across the stator field will be weak and the field produced by them will also be weak. The rotating field will consequently be unequal in strength and will have its maximum strength as it rotates into the direction of the axis of the stator field, and will be weakest when it has moved into the position at right angles to this. We

shall consequently obtain at first a field rotating with the same average speed as the periodicity of the supply, but varying in strength during each revolution. As the speed rises the field will become more uniform until ultimately, when the speed attains its steady value, we shall have a rotating field differing only slightly from the uniform field to be found in a 2-phase motor.

The greater current taken by the single-phase winding is due to the fact that this has to supply the whole of the magnetising current of the motor as well as the current necessary to balance the demagnetising ampere-turns of the rotor.

It is interesting to note that the rotating field will now be capable of acting on the idle windings of the stator which we assumed to be cut out of circuit. Thus, if one wire of a 3-phase motor be interrupted, while the rotor is revolving, so as to convert the motor into a single-phase motor, and if voltmeters are connected across the two idle phases, they will be found to have voltages induced in them of practically the same value as that of the supply. Further, the phase of the voltages induced in these windings by the rotating field will be the same as before, or 120° respectively in advance and behind the phase of the voltage in the winding which is still supplied with current.

Starting Devices of a Single-phase Motor.—It has been explained that a single-phase induction motor with short-circuited rotor is not self-starting, since no rotating field is set up in the stationary rotor. The various methods employed for starting such motors consist in the addition of an extra winding to the stator between the main windings, *i.e.*, so situated as to produce a field in a direction perpendicular to the field of the main winding. This extra winding must then be supplied by some means with an alternating current *out of phase* with the current of the main winding, since this is the condition for the production of a rotating field. The chief differences in the starting devices employed by different makers lies in the method of producing the phase-difference in the "starting phase." The usual arrangements consist of either an inductive resistance or condenser in the starting circuit, or a non-inductive resistance in the main circuit. By any of these means a difference in the relative inductance of the main and auxiliary circuits is produced, and a difference of phase in the currents in them is brought about.

The starting torque of a single-phase induction motor is usually low and the starting current is relatively high; it is consequently desirable whenever possible to start up under light loads only. The difference in phase produced between the currents in the main and auxiliary phases is only small, and consequently the flux perpendicular to the main flux is weak. It is not of much use to make the auxiliary winding itself highly inductive by the shape given to the slots in which it is wound, since the rotor currents will largely neutralise the leakage lines and so destroy the self-induction. Hence, it is usual to have a large choking coil in the starting circuit external to the motor, and to make the auxiliary winding itself

of only a few turns, so that its self-induction forms only a small part of that of the auxiliary circuit. In this case, it is evident that the higher the self-induction of the coil, the greater will be the proportion of the applied voltage absorbed externally to the motor. Consequently the greater the lag obtained, the smaller will be the strength of the flux.

In the case of the Heyland motor, the starting coil is wound in square holes, so as to give a large leakage and great lag, but the winding is connected directly to the mains and a very strong flux is obtained and a large starting torque. The impedance of the starting phase is thus kept smaller than that of the running phase, although the self-induction is greater, and by the special method of winding, the leakage flux is counteracted to a comparatively small extent by the rotor currents. The current is greater in the starting phase than in the main phase, but the total currrent at starting under full load does not exceed twice the normal full-load current.

The measurements to be made on single-phase motors are practically identical with those already described for polyphase motors, except that special attention must be paid to starting

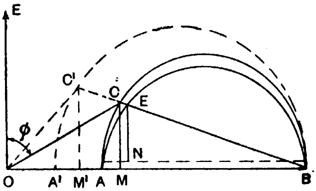


FIG. 223.—Approximate Diagram for Single-phase Induction Motor.

current and torque, as it is chiefly in this respect that different types vary. It will be found that the maximum torque which can be overcome by a single-phase motor is usually lower than that of a 3 phase or 2-phase motor of the same rated output.

CIRCLE DIAGRAMS FOR THE SINGLE-PHASE INDUCTION MOTOR.

We will first give the approximate construction, which is that most usually employed for commercial purposes, and afterwards give a diagram from which the performance of the motor may be more completely studied under all conditions of loading.

Approximate Diagram.—A horizontal line $O\ B$ is taken, and a length $O\ A^1$ set off from O, so that $O\ A^1$ represents the magnetising current which the motor would take per phase, if the stator were wound as for a 2-*phase* motor, with the same magnetic circuit flux per pole, &c., as the actual motor. To the same scale $O\ B$ is made to represent the theoretical short-circuit current per phase of the 2-phase motor, when supplied at the same voltage.

Thus $\frac{O\,A^1}{O\,B} = \sigma$ is the "coefficient of dispersion" of the motor.

If a circle were described on $B\,A^1$ (as shown dotted in Fig. 223) we should have the ordinary circle diagram for a 2-phase motor, $O\,C^1$ representing the stator current per phase, and $C^1\,A^1$ the equivalent rotor current; while $C^1\,M^1$ would be proportional to the torque (neglecting the motor losses).

In order to make the diagram show the conditions for a single-phase motor, a length $O\,A$ is set off from O equal to twice $O\,A^1$, and a semi-circle (shown in full in Fig. 223) is drawn on $B\,A$. This semi-circle then represents the locus of the primary current for the single-phase motor with sufficient accuracy for ordinary purposes within the working range of loads. The diagram may be completed by describing torque and output circles passing through B and A exactly as with a polyphase motor. The quantities derived from these circles are, however, not represented to the same scale as for the polyphase motor. Also, they can only be taken from the diagram without correction, for loads which do not greatly exceed the full load of the motor. It will be seen that the angle of lag ϕ of the current behind the electromotive force $O\,E$ is greater in the single-phase motor than in the corresponding polyphase motor, having the same value for its dispersion coefficient σ, *i.e.*, the same ratio $\frac{O\,A^1}{O\,B}$.

The value of σ is calculated in the same way for both types of motor, from the dimensions of teeth, air-gap, &c.

The scale to which the torque is represented on the completed diagram of the single-phase motor is best found as follows: Determine the power supplied to the motor for some definite value of the stator current; deduct the stator $I^2\,R$ losses corresponding to this condition of working, and also the iron and friction losses (no-load losses), and divide the remaining power by the angular velocity ($2\,\pi\,n$). This will give the torque corresponding to the conditions assumed, and may be used to determine the scale to which the line $E\,N$ and all similar lines represent the torque. $C\,M$ represents the input to the motor, and a third circle of output (not shown in the diagram) may be drawn to give the output of the single-phase motor by subtraction of the rotor copper drop from $E\,B$ in the usual way. $E\,N$ represents the "synchronous watts" of the motor to the same scale that $C\,M$ gives the input watts.

More Complete Diagram.—It is well known that the behaviour of a single-phase induction motor may be studied by imagining the oscillating flux of the stator to be replaced by two oppositely rotating fluxes, each having a constant value equal to one-half the maximum value of the alternating stator flux. The following construction, while employing a somewhat similar principle and leading to similar results, is approached from a rather different point of view.

Let the stator winding $A\ A^1$ (Fig. 224) of the single-phase motor carry a current producing $I\ T$ ampere-turns, and give rise to a field for simplicity shown as forming only two poles which have a vertical axis in Fig. 224. Imagine a winding $B\ B^1$ consisting of two equal sections to be added to the stator, and suppose further that the sections of this winding carry equal and opposite currents, I, each equal to that in the actual stator winding. This imaginary winding $B\ B^1$ is supposed to be wound on the stator so as to form a field perpendicular in space to that of the actual winding, and to be supplied with current which is at right angles in phase to the actual stator current. The addition of this winding would not in any way alter the actual conditions of working, since the added winding will carry equal and opposite ampere-turns in its two sections.

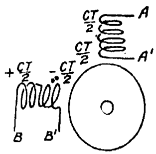


FIG. 224.—Stator of Single-phase Motor, Imaginary Winding Added.

We may now look upon the rotor as being acted upon by the ollowing stator currents :—

I. $\begin{cases} \dfrac{I\ T}{2} & \text{ampere turns in coil A.} \\ \dfrac{I\ T}{2} & \text{,, ,, ,, B perpendicular to those in A in both space and time.} \end{cases}$

II. $\begin{cases} \dfrac{I\ T}{2} & \text{ampere turns in coil } A^1. \\ -\dfrac{I\ T}{2} & \text{,, ,, ,, } B^1 \text{ perpendicular to those in } A^1 \text{ in both space and time.} \end{cases}$

Now, each of these pairs of windings A, B and A^1, B^1 will fulfil all the conditions for the production of a rotating field, exactly as in a 2-phase motor. But the direction of rotation of the field due to coils A, B will be opposite to that due to coils A^1, B^1.

The actual conditions are not in any way changed by the addition of equal and opposite ampere-turns, so that the state of things just described corresponds exactly to that in the actual motor. We may thus look upon the rotor as being acted upon by two oppositely rotating fields, each due to a 2-phase winding of $\frac{T}{2}$ turns per phase. It can be shown that the effect of each of these fields is independent of that of the other, so that we may, for greater clearness, imagine each field to act on a separate rotor.

It is not difficult to see that the current induced in the rotor by the two fields will be quite distinct when we remember that the frequencies of the currents will have quite different values. The frequency of the currents due to one field will be s cycles per second, while the frequency of the currents due to the other rotating field will be $2f - s$, where s is the positive slip in cycles per second of the rotor behind the field rotating in the same sense as the rotor and f is the frequency of supply.

We thus arrive at the result that the single-phase motor is equivalent to two 2-phase motors acting on a common shaft, having their stators connected in series, but their rotors electrically independent.

We may thus replace the single-phase motor, having stator coils A A^1 and supplied with $C\,T$ ampere-turns, by two 2-phase motors as in Fig. 225, motor I. with coils A, B, and motor II. with coils A^1, B^1 on their respective stators, motors I. and II. being supplied in series and each having a stator current per phase producing $\frac{C\,T}{2}$ ampere-turns.

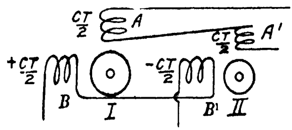


FIG. 225.—Two Polyphase Motors, Equivalent to a Single-phase Motor.

If these motors are stationary, they will both have a slip of f cycles per second, or of 100 per cent, but in opposite directions, since their fields rotate oppositely. Their torques will be equal and opposite, and the resultant torque on their common shaft will consequently be zero. If the shaft has a rotation of n revs. per second in the sense of the rotation of motor I., the slip of motor I. will be $f - n$ cycles per second, while the slip of motor II. will have become $f + n$. For any speed of rotation in either direction the algebraic sum of the slips of both motors will be $2f$.

The two motors must be considered always to take equal currents, since they are connected in series according to our supposition; but the voltage across their terminals will vary with their relative speeds. The sum of the two imaginary motor voltages forms the actual voltage of supply of the single-phase motor, which is constant. We must now consider how to ascertain the distribution of this constant voltage of supply between the individual motors.

Let E be the constant total voltage of the single-phase motor, E_1, E_2 the voltages of the two polyphase motors, and C the current per phase taken by these motors.

Further let

$$G_1 = \frac{E_1}{C}$$

and

$$G_2 = \frac{E_2}{C}$$

where G_1, G_2 may be called the total "apparent impedances" of the motors when running (this is, of course, a quite different quantity from the impedance of the motor windings).

We have then the relations

$$E = E_1 + E_2$$

also

$$E_1 = CG_1$$

and

$$E_2 = CG_2$$

Hence

$$\frac{E_1}{E_2} = \frac{G_1}{G_2}$$

so long as the motors are considered as being connected in series so as to take the same current.

The quantities G_1, G_2 will vary with the speed of the motors, but will have a definite value at any given speed. Let us now imagine the two polyphase motors to be connected in *parallel* to the supply circuit (instead of in series) and to be still operating at the previous speed, so that the values of G_1 and G_2 remain the same as before. The currents taken by the motors under the new conditions will be inversely proportional to the "apparent impedance" of each motor. We will suppose the currents to be

$$C_1 = \frac{E}{G_1} \text{ for motor I.}$$

$$C_2 = \frac{E}{G_2} \text{ for motor II.}$$

whence

$$\frac{C_1}{C_2} = \frac{G_2}{G_1}$$

or, from the previous equation,

$$\frac{E_1}{E_2} = \frac{C_2}{C_1}$$

That is to say, the voltages of the motors when connected *in series* will bear to one another the inverse ratio of the currents which the motors would take if connected *in parallel* to a common supply, while running at the same speed.

Thus, if we find (by means of a circle diagram or otherwise) the current of one of the 2-phase motors when working on the *constant voltage*, E, first with a slip $f - n$, and then with a slip $f + n$, these will be the currents denoted as C_1 and C_2 above. We can from these currents determine the voltages E_1 and E_2 at the terminals of the two motors when connected in series across this voltage, and when their rotors are running at a speed n, corresponding to a speed of n revs. per second of the single-phase motor.

If we now draw two circle diagrams, representing two

similar motors working at voltages E_1 and E_2 and having slips of $f - n$ and $f + n$, we can obtain the torque, input and output of each motor by the usual construction as applied to polyphase motors. By adding the two values of any of these quantities obtained for the two motors, with due regard to sign we obtain the torque, input, and output of the single-phase motor of which they are the equivalent.

We may now proceed to the construction of an actual diagram. A circle diagram is drawn for the single-phase motor exactly as if it had been provided with a 2-phase stator winding, having

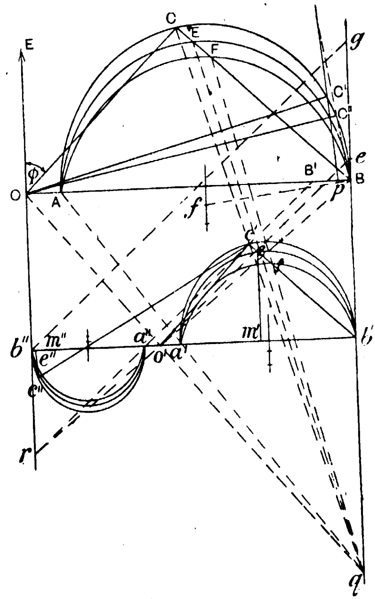


FIG. 226.—Circle Diagram for Single-phase Motor.

one-half the number of coils per phase of the actual single-phase winding, and working on the full voltage of supply. Such a diagram is shown in Fig. 226, where the coefficient of dispersion of the motor $\sigma = O\ A/O\ B$. O C^1 is the short-circuit current and C^1p drawn perpendicular to the radius $f\ B$ of the torque circle is the slip line, the length $p\ C^1$ representing a slip of 100 per cent. By dividing $p\ C^1$ into ten divisions, and continuing the line beyond C^1 marked

with the same length of subdivision as before, we obtain the means for finding the running conditions for slips of greater than 100 per cent., *i.e.*, for a backward rotation of the motor. For the purpose of a finer graduation of the slip at high speeds, a second slip line is shown on the right of the diagram, from which slips up to 10 per cent. may be read more exactly.

For the purposes of explanation of the further method of procedure, we will assume the single-phase motor to be working with a slip of 15 per cent. (choosing a large slip to make the method of construction more distinct). The current $O\ C$ corresponds to a slip of 15 per cent. in Fig. 226, since the line $B\ C$ is drawn through the 15 per cent. mark on the slip line $p\ C^1$.

Now, if the polyphase motor I. works with a slip of 15 per cent., the motor II. will work with a slip of $200 - 15 = 185$ per cent. Accordingly, the stator current corresponding to a slip of 185 per cent. is determined also, being the line $O\ C''$. The point C'' is obtained by joining B to the point 185 per cent. on the slip line, the junction line cutting the input circle at C''.

The currents which would be taken by the polyphase motors I. and II. when connected severally to the voltage of supply are represented in phase and magnitude by the lines $O\ C$, $O\ C''$; in fact, these are the currents C_1 and C_2 previously spoken of. We have now to draw the diagrams for these motors when connected *in series* across the mains, remembering that the voltages which they will receive are inversely proportional to the magnitudes of the currents just found, and that the sum of the voltages is the voltage E represented by the line $O\ B$ in the upper diagram.

Draw a horizontal line $b''\ b'$ below the diagram, Fig. 226, making $b''\ b'$ equal to $O\ B$ and placing b'' vertically under O. The line $b''\ b'$ is now to be divided in the ratio $\frac{O\ C}{O\ C''}$. This may easily be done by marking off lengths $b'\ e = O\ C''$ and $e\ g = O\ C$ vertically from b' (in the diagram half these lengths are marked off for the sake of reducing the length of the diagram). Join $g\ b''$, and draw $e\ o'$ parallel to $g\ b''$ to cut the line $b''\ b'$ in o'. The voltage E represented by $b''\ b'$ is now subdivided so that $b''\ o'$ represents the voltage of motor II. and $o'\ b'$ is the voltage of motor I.

On $b''\ o'$, $o'\ b'$ we have now to construct the circle diagrams which are to represent the performances of the two motors. In order to do this we must first divide each of these lines at points a', a'' in such a way that

$$\frac{o'\ a'}{o'\ b'} = \frac{o'\ a''}{o'\ b''} = \frac{O\ A}{O\ B} = \sigma.$$

The point a' is easily found by drawing a line $O\ o'$ and producing it to cut the vertical line through $B\ b'$ at q. Joining $A\ q$ we obtain the required point a'. The point a'' for the motor II. is similarly obtained by drawing $B\ o'$ to cut the vertical line drawn downwards from O at r, and a second line from a point B' taken so that $B\ B' = O\ A$, drawn to r to cut $b'\ o'$ in a''.

Semicircles may now be drawn on $b' a'$ and on $a'' b''$ to represent the loci of the currents taken by the motors I. and II. Drawing $o' c'$ parallel to $O C$ we have the current of motor I. Similarly, $o' c''$ drawn parallel to $O C''$ gives the current of motor II. These currents will be found to be equal, although they do not appear to be in phase with one another in the diagram, because the voltages at the motor terminals are drawn as if in phase. The resultant of these two currents (the line $c' c''$) is the current actually taken by the stator winding of the single-phase motor.

The torque and output circles may now be put into the smaller diagrams from the larger ones. This is done directly in the case of the left-hand diagram by drawing radiating lines through C E F in the large diagram to the point q to intersect the line $c' b'$ at corresponding points c' e' f', through which the circles must pass, as well as through b' and a'. The circles in the right-hand diagram are directly derived from those on the left, since lines drawn through o', which are tangential to the circles of one diagram, will also be tangential to those of the other.

The two diagrams now drawn on $b' b''$ represent the performance of the single-phase motor at the speed chosen. Thus, for example, the torque of the motor I. is given by the line $e' m'$, and that of motor II. by $e'' m''$. The torque of the single-phase motor is the difference between these, since motor II. always exerts a negative torque, acting in fact as a generator. Similarly the resultant output is given by the difference of the ordinates of the output circles of the two motors. The resultant rotor current of the single-phase motor is the resultant of the lines $a' c'$ and $a'' c''$, representing the respective rotor currents of the two motors.

It is now possible to see to what extent the original approximate diagram (Fig. 223) gives correct results. The torque of motor II. is evidently very small when the slip of motor II. is large, *i.e.*, when the slip of the single-phase motor is small. In such cases it is consequently nearly correct to neglect the negative torque and output of motor II., as is done in the construction of Fig. 223. Further, when motor I. runs synchronously, the voltage $o' b''$ of motor II. will be only the impedance drop in this motor, due to the current taken by the motor I. This small voltage is represented in Fig. 223 by the length $A^1 A$, which may also be taken as showing the magnetising current of the second motor. At synchronism it would then be found that the diagrams, Figs. 223 and 226, would give the same construction, and at speeds near synchronism the divergence in results is very small. For the stationary single-phase motor, Fig. 223 would still show a considerable positive torque, whereas in Fig. 226 the torque would be the sum of two equal and opposite torques, since the two semicircles on $b'' a''$, $b' a'$ would then be equal. Thus, for low speeds the diagram shown in Fig. 223 would give misleading results, while near synchronism it gives satisfactory information.

The simple diagram (Fig. 223) is based on the assumption of a uniform rotating field. In a single-phase motor the field only

has an approximately uniform value at synchronous speed. The field becomes elliptical with a major axis coinciding with the axis of the stator winding at lower speeds. The lower the speed the less is the strength of field measured perpendicular to the axis of the stator winding. This divergence is neglected in the construction of Fig. 223, and is accounted for by the oppositely rotating flux which forms the basis of the more complete construction given in Fig. 226.

It is to be remembered that the two small diagrams shown in Fig. 226 give the results for one value of the slip only. A series of such diagrams can, however, be rapidly constructed when the construction is once grasped.

SINGLE-PHASE COMMUTATOR MOTORS.

General.—This class of motor is growing in importance, especially for alternating-current traction work. The armature resembles that used for continuous currents. The field system is necessarily laminated, in order to reduce eddy-current losses which would result from an alternating flux, and may be arranged with definite salient poles as in continuous-current motors, or the motor may have a uniform air gap, the position of the poles then being determined by the windings alone.

Electromotive Force in Armature.—Before considering any special type of alternating-current commutator motor, it is necessary to form a clear idea of the two electromotive forces which are induced in the armature by an alternating field.

Let us consider the case of an armature exactly similar in construction to that employed in continuous-current motors, situated between two poles, in which an alternating flux is maintained, as indicated in Fig. 227. Imagine the armature to be coupled to a motor, and so made to rotate; but suppose that no current is supplied to it.

Electromotive Force of Rotation.—If at any moment the left-hand pole is of North polarity, an electromotive force will be induced in the conductors which move across the flux. The direction of this electromotive force will be the same for all conductors situated in the left-hand half of the armature $B\ A\ B^1$, which are moving downwards across the flux, and will be such as to act in the winding from B to B^1. Similarly, the conductors in the other half of the armature $B^1\ A^1\ B$ will have induced in them an electromotive force due to their moving across the flux in an upward direction, producing a resultant voltage in the winding from B to B^1. The result is the production of a difference of potential in the armature, which has a maximum value between the points B and B^1. A reversal of the field will reverse the sense of this difference of potential, which will then have a maximum positive value at B instead of B^1. By connecting a voltmeter to brushes fixed at B and B^1, we should obtain a deflection indicating the value of the voltage induced in the armature.

This voltage reverses with each reversal of the field, and consequently has a *periodicity* which depends upon that of the field, and which is independent of the speed of rotation of the armature.

The *magnitude* of the voltage induced by rotation depends on the strength of the field, and on the speed with which the conductors cut the lines of the field. With a definite maximum value of the alternating flux, the voltage will depend only on the speed of rotation of the armature, and will be directly proportional to this speed.

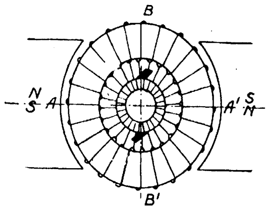


FIG. 227.

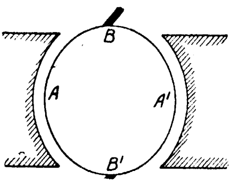


FIG. 228.

Diagrams Illustrating Single-phase Armature.

Assuming the more usual case of a drum-wound armature, the average value of this voltage will be given by the usual formula for a generator :—

$$E_{av} = \frac{n\ N\ F_{av}}{10^8} = \frac{4\ n\ \frac{T}{2}\ F_{av}}{10^8}$$

where n = revs. per second of armature.

N = active conductors on drum-wound armature.

F_{av} = average value of flux $= F\frac{2}{\pi}$

T = No. of turns on armature $= \frac{N}{2}$

Substituting F (the max. value of the flux) for F_{av}.

$$E_{av} = \frac{\frac{2}{\pi} 4\ n \frac{T}{2}\ F}{10^8}$$

or $$E = 1{\cdot}11\ E_{av} = \frac{2}{\pi}\ 4{\cdot}44\ n \frac{T}{2}\ F\ 10^{-8}$$

where E is the virtual value of the electromotive force.

Although the conductors at $A\ A^1$ are those in which the electromotive force of rotation has the highest value, no difference of

potential exists between the points A A^1 in the armature. The *axis* of the armature along which the maximum difference of potential is formed is the neutral axis B B^1. The *phase* of this induced voltage is the same as that of the flux, *i.e.*, practically* the same as that of the field current.

Electromotive Force due to Transformer Action.—If the armature is considered to be stationary, there is still an electromotive force induced in it, due to the alternate introduction and removal of the field flux. The maximum electromotive force due to this cause will be induced in those armature coils which present the greatest surface normal to the flux, *i.e.*, in the coils at B B^1. The direction of the electromotive force of all coils round the upper half of the armature in Fig. 227 will be identical. The direction of the electromotive forces in the coils of the lower half of the armature will be oppositely directed round the armature. Hence the points of maximum difference of potential in the armature will be A A^1 in this case. As with the rotational electromotive force, the axis joining the most active conductors is perpendicular to the axis of greatest difference of potential in the armature, which is now the axis of the main flux.

The *phase* of the induced voltage is 90° behind that of the flux, since the voltage is proportional to rate of change of flux.

The *frequency* of the electromotive force is evidently the same as that of the flux.

The *magnitude* of the electromotive force depends only on the value of the flux and its rate of alternation. It is therefore independent of the speed of the motor, and will have the same value whether the motor rotates or is stationary. Its virtual value will be given by the usual formula employed for transformers, viz. :—

$$E = \frac{2}{\pi} . \ 4{\cdot}44 \, f \, \frac{T}{2} \, F \ 10^{-8}$$

when $\frac{T}{2}$ = number of turns of the armature winding acting in series $= \frac{1}{2}$ total turns.

F = maximum value of the flux.

$\frac{2}{\pi}$ = a constant, introduced on account of the fact that the same number of lines do not thread through all the armature turns. With a non-uniform field or other system of armature winding a different constant would have to be employed.

* Except for lag of the flux due to hysteresis.

We may summarise the comparison between the two induced voltages as follows :—

	Electromotive Force of Rotation	Electromotive Force due to Transformer Action
Axis of most active conductors.	Parallel to field.	Normal to field.
Axis of resultant electromotive force in armature.	Normal to field.	Parallel to field.
Phase.	Same as that of field.	90° behind field.
Magnitude.	Proportional to flux and speed of motor.	Proportional to flux and frequency, independent of speed.

The ratio between the electromotive force due to rotation of the armature in an alternating flux and the electromotive force due to transformer action of the same flux is seen to be :—

$$\frac{\text{Rotation electromotive force}}{\text{Transformer electromotive force}} = \frac{n}{f}$$

The Series Motor.—The electrical circuit of a simple alternating-current series motor is identical with that of the direct-current series motor. In construction the two machines are similar, except that the whole of the magnetic circuit of the alternating machine is laminated, in order to avoid excessive eddy-current losses.

Since there is only a single electrical circuit, the same current flows in the armature and field. We may assume the flux of the field to be in phase with this current (*i.e.*, we shall neglect the slight phase difference due to iron losses).

A *torque* will be produced by the current in flowing from brush to brush, since the conductors carrying this current will be situated in the main field, and the armature currents will be in phase with this field. The torque at any moment will consequently be directly proportional to the product of the current and flux, *i.e.*, nearly proportional to (current)^2 because the flux is itself nearly proportional to the current. Evidently (as in the continuous-current motor) the currents flowing at A (Fig. 228) will tend to produce a torque in the same sense as the currents at A_1, since these currents will be flowing in opposite directions through the plane of the diagram. The torque of the series motor is constant in direction, since armature and field currents reverse simultaneously. The torque will be pulsating and unidirectional.

The electromotive forces in the armature may be enumerated shortly on the basis of the preceding section.

(1) Due to rotation in the main flux an electromotive force will be induced in phase with the flux, and acting along the axis

from brush to brush. This is the back electromotive force with which we are familiar in continuous-current motors. Its phase is not in direct opposition to the terminal voltage of the motor, but varies with the phase of the current, since it is in phase with this current.

(2) Due to transformer action of the main flux, an electromotive force is formed 90° behind the flux in phase, and acting along the axis of the field, *i.e.*, between A and A^1 in Fig. 228. Since this electromotive force does not produce any difference of potential between B and B_1, it has no effect on the circuit, or on the behaviour of the motor. These alternating differences of potential formed between the points A and A_1 do not affect the armature currents, since they balance one another in the armature winding.

(3) The armature currents will produce a reaction flux along the axis B B_1. This flux will be in phase with the armature current. In cutting this, the rotating armature conductors will have induced

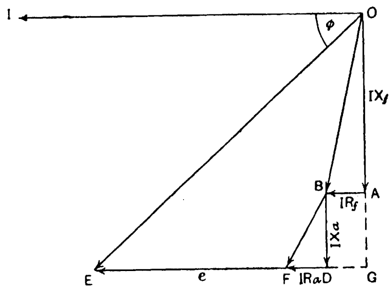


FIG. 229.—Vector Diagram of Series Motor.

in them a voltage of rotation along the axis A A_1. This voltage, like the last one, and for the same reason, will have no resultant effect on the motor circuit.

(4) The armature reaction cross flux will have a transformer action on the armature coils, producing thereby the usual back electromotive force of self-induction, 90° in phase behind the current, and acting along the axis B_1 B. Otherwise expressed, the armature will have reactance in virtue of this flux, necessitating the application of a voltage at the brushes 90° in phase in advance of the current, and equal to the product of current and reactance. The production of an alternating flux by the current flowing in a winding can always be expressed as a self-induction of the winding.

We are now in a position to enumerate the components of the applied voltage, and to show their phase relations on a diagram, on the assumption that iron losses may be neglected.

O I represents the current taken by the motor.

The voltage will have the following components (see Fig. 229) :—

$A\ B = I \times R_f$ component overcoming field resistance in phase with current.

$O\ A = I \times X_f$ component overcoming field reactance 90° in advance of current.

$D\ F = I \times R_a$ component overcoming armature resistance in phase with current.

$B\ D = I \times X_a$ component overcoming armature reactance 90° in advance of current.

$F\ E = e =$ component overcoming induced electromotive force of rotation in phase with current.

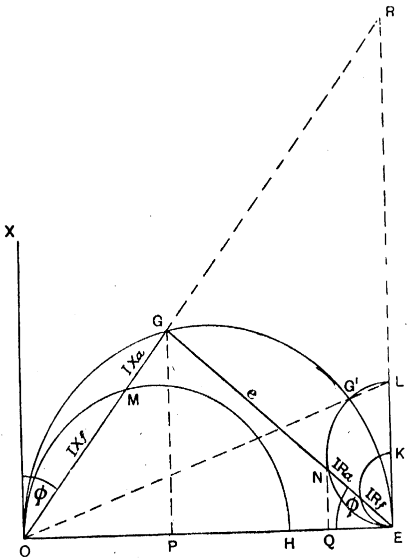


FIG. 230.—Circle Diagram of Series Motor.

These various voltages will be easily followed out in the diagram, Fig. 229. In this diagram it is evident that $O\ B$ indicates the voltage across the field, and $B\ F$ the volts across the armature when stationary. The conditions at starting (if without added resistance) would be obtained by assuming the voltage as represented by $F\ E$ to be zero. The applied voltage would then be the vector $O\ F$.

If we assume a constant applied voltage, we may invert the

diagram given in Fig. 229, so that the component voltages lie in a semicircle described on a diameter representing the applied voltage $O\,E$. The various component voltages may be separated for any value of the load by the addition of semicircles as shown in Fig. 230. The length of the diameter $O\,H$ of the semicircle which cuts off the reactance voltage $I\,X_f$ of the field, is obtained by dividing $O\,E$ in the ratio of the reactance of the field to that of the armature, so that

$$\frac{O\,H}{H\,E} = \frac{X_f}{X_a}$$

The vertical diameter $E\,L$ of the semicircle cutting off the total resistance voltage is drawn so that

$$\frac{E\,L}{O\,E} = \frac{\text{total resistance of motor}}{\text{total reactance of motor}} = \frac{R_a + R_f}{X_a + X_f}.$$

The point L may also be obtained by drawing the dotted line $O\,L$, making an angle $X\,O\,L$ with the vertical line $O\,X$ equal to the angle of lag of the current behind the terminal volts on short-circuit, and producing the line to cut the vertical line through E.

The point K divides the line $E\,L$, so that

$$\frac{E\,K}{E\,L} = \frac{R_f}{R_f + R_a}$$

$G\,P$ is proportional to the power supplied to the motor (see page 339).

$N\,Q$ is proportional to the power wasted in the resistance of the windings of both armature and field.

The difference between the ordinates $G\,P$ and $N\,Q$ is proportional to the useful output.

The speed of the motor is proportional to the length of $L\,R$*

The current is proportional to the length of $O\,G$, the power-factor being the cosine of the angle $X\,O\,G$, marked ϕ.

$O\,G_1$ is the starting current corresponding to full voltage applied.

Compensation for Armature Reaction.—It will be noticed from the preceding diagrams that there are two factors which reduce the power-factor of the motor-circuit, viz., the reactance of the field and the reactance of the armature. The field reactance is a necessary evil in all series motors, since the flux to which the reactance is due is the flux upon which the torque depends, and without which the motor could not work. The armature reactance is due to a reactance field formed along an axis such that the currents

* The speed will be directly proportional to the electromotive force of rotation and inversely proportional to the flux through the armature, so that the speed $n \propto \frac{N\,G}{G\,O}$.

Since the straight line through $L\,N$ would be parallel to $R\,O$, and since the triangles $E\,G\,R$ $O\,G\,E$ are similar, we have

$$\frac{G\,N}{G\,E} = \frac{R\,L}{R\,E}$$

$$\therefore \frac{G\,N}{R\,L} = \frac{G\,E}{R\,E} = \frac{G\,O}{E\,O}$$

Hence $R\,L = \frac{G\,N}{G\,O}\,E\,O$, which proves the proportion, since $E\,O$ is a constant.

in the armature experience no resultant torque in passing through it. It was seen on page 372 that this flux produces an electromotive force at right angle in phase to the current. This flux is of no advantage, and is therefore neutralised in commercial motors in the manner now to be explained.

The current in the armature of the motor illustrated in the diagram Fig. 227, page 370, produces a vertical flux. This action of the armature winding is illustrated by the coil marked F in Fig. 231. If the production of this field could be prevented, the winding would be non-inductive. The method actually adopted is suggested by the dotted coil in Fig. 231, having the same number of turns as the armature,* but wound in an opposite direction round the magnetic circuit followed by the reaction flux, and tending, therefore, to produce an equal and opposite field. When connected in series it will be seen that a current flowing through the two windings in Fig. 231 will produce no resultant flux in the air gap. A slight stray field will always remain, but both windings joined in series will be very nearly non-inductive.

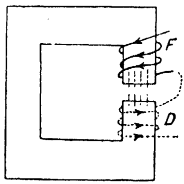


FIG. 231.—Diagram to Show Principle of Neutralising Winding.

An alternative method for preventing the armature reaction flux crossing the air gap is to make the coil D a separate circuit short-circuited upon itself. In this case any flux entering D will produce alternating currents in it, oppositely directed to those flowing in F, and thus ensuring that the resultant ampere-turns on the circuit remain practically zero (c.f., the low impedance of a transformer with short-circuited secondary). With the latter arrangement the currents in coil D will automatically vary in sympathy with the current in F (which represents the armature), so that in effect the action is almost identical with the first arrangement when the coils are connected in series.

The compensating winding employed in a series motor consists of an additional winding on the stator placed with its axis midway between the main poles and connected either in series with the main circuit or short-circuited on itself. This coil would tend to produce a vertical flux in the diagrams (Figs. 227 and 228) equal and opposite to the vertical armature reaction flux.

The effect of this neutralising winding is to raise the power

* The turns are here for simplicity assumed to be equal in number. This is not exactly the case in commercial motors.

factor of the motor by reducing or eliminating the component voltage marked $B\ D$ in Fig. 229 and $G\ M$ in Fig. 230.

Sparking at the Brushes.—One effect of the voltage induced by transformer action of the alternating field flux is of importance, owing to its influence on the commutation of the motor.

Although the voltage induced between the points $A\ A_1$ (Fig. 227) has been shown to be without effect on the behaviour of the motor, the induced voltages in individual coils of the armature at the moment when they are short-circuited at the commutator by the brushes may give rise to heavy currents. In order to illustrate this point we may refer to Fig. 232, where it is evident that the coil indicated is temporarily short-circuited by the brush,

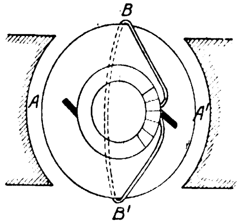


FIG. 232.—Sparking Due to Transformer Action.

and that it is in the position to enclose a large part of the alternating flux of the field. The electromotive force induced in coils whlie short-circuited like the one shown will tend to produce heavy currents, which will produce heating at the brushes and sparking when the circuit is broken as the brush leaves successive segments. This is the principal cause of the tendency to spark in a series alternating-current motor, which has to be carefully guarded against. Two features in the design of alternating-current motors are adopted to minimise the effect. The number of turns in the armature per commutator segment is made as small as possible, and a low frequency is usually adopted. Special forms of brushes have also been employed to overcome sparking.

General Characteristics of the Series Motor.—In general behaviour, as regards speed and torque, the alternating-current series motor resembles the continuous-current series motor. It is essentially a variable-speed motor, giving a high torque at low speeds, and liable to run at an excessively high speed when unloaded. It is important to notice that the torque depends on the current only, and not on the power-factor. Thus the motor exerts a high starting torque while taking a small power from the line, because of its low power-factor when starting or running at low speeds.

THE REPULSION MOTOR.

Preliminary Consideration.—We shall introduce here an illustration of a principle which will assist us in the discussion of the repulsion and compensated repulsion motor.

Let A and B in Fig. 233 represent two transformers with their primary windings connected in series to a source of constant voltage. Let the transformers be without leakage self-induction, and have negligible iron losses. Let the secondary of B be short-circuited while transformer A is on open circuit.

Under these conditions the voltage E_B across the primary of B will be very small, representing only the drop in the resistance of its windings due to the current I, which flows through the primary of both transformers. Its phase will be that of I.

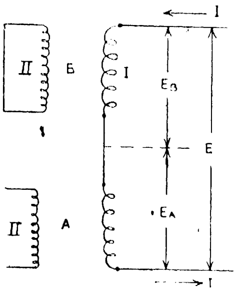


FIG. 233.

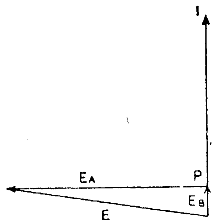


FIG. 234.

Diagram of Short-circuited and Open-circuited Transformers in Series.

The greater part of the total voltage E will be represented by the voltage E_A between the terminals of the open-circuited transformer A. The phase of E_A will be nearly 90° in advance of the current I.

Transformer B will behave practically as a non-inductive resistance of low value, the flux produced by the current in the primary winding being "neutralised" by the action of the short-circuited secondary winding.

Transformer A represents a high inductance, since the primary current in this case sets up a strong flux in the transformer core. The great divergence between the angle of phase difference between current and voltage in the two cases may also be regarded as follows: In transformer B the magnetising component of the current is very small, since only a small flux is produced; current and voltage are consequently practically in phase. The current in the primary of A is almost entirely magnetising current, and therefore at right angles in phase to the terminal volts.

The relation between the voltages and current may be seen from the phase diagram (Fig. 234).

An important point to be noted is that in the transformer B the small flux inducing the low voltage required for overcoming the secondary copper loss is practically at right angles to the current in phase, while in transformer A flux and current are in phase. In either case flux and induced voltage are mutually perpendicular.

In a transformer loaded non-inductively, the current is almost at right angles to the flux in phase, the magnetising current being relatively small. On no-load, current and flux are in phase (except for iron losses). As the load increases in such a case, the phase difference between primary current and flux increases from almost zero to almost 90° (c.f. transformer diagrams, Figs. 89 and 90, pages 159 and 160).

Action of the Repulsion Motor.—Referring to the discussion of the series motor on page 372, we have seen that if a closed-circuit armature of the usual continuous-current type is placed in an

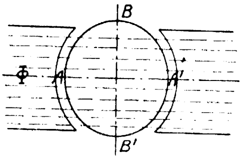


FIG. 235.

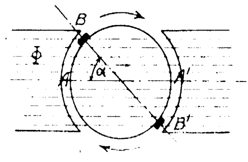


FIG. 236.

Diagrams Illustrating Principle of the Repulsion Motor.

alternating magnetic field, a voltage results along the axis $A\ A^1$ of the field due to transformer action. Brushes placed on the commutator on the axis $A\ A^1$ would be at different potentials, and if they were short-circuited a heavy current would flow through the armature and the conductor connecting the brushes. This current could produce no resultant torque on the armature, since the torque due to the currents in individual conductors will balance. For instance, the torque due to currents flowing in the armature conductors between B and A will be exactly balanced by an oppositely-directed torque due to the currents flowing from B^1 to A (see Fig. 235). If the brushes were moved into the positions $B\ B^1$ (Fig. 235) while still short-circuited and with the armature stationary, there would again be no torque, because there is no resultant electromotive force in the armature along the axis $B\ B^1$, due to transformer action, so that there will be no current between the brushes.

Now let us suppose that the brushes are moved into an intermediate position, as shown in Fig. 236. There will be an electromotive force formed between them, although this will be less than that existing between A and A^1; consequently a current will

flow through the armature and conductor joining the brushes. This current will produce a torque, although not such a large one as the same current could produce with the brushes situated along the vertical axis of the armature. In fact, the currents flowing to the brush along the path *A B* will produce a torque in an opposite direction to the current in the conductors between *B* and the top of the armature.

In order to consider what happens under these conditions, it will be convenient to imagine the actual horizontal field as replaced by two component fields, one parallel to the short-circuited brush axis and one perpendicular to this axis. We shall therefore consider the arrangement shown in Fig. 238, to represent the same conditions as Fig. 236, where the stator field is shown as split into two parts, *B* and *A*, acting respectively along and perpendicular to the brush axis. The relative strengths of the horizontal and

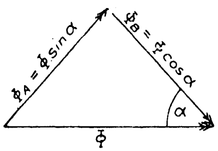


FIG. 237.

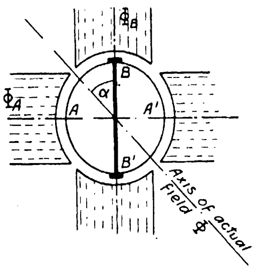


FIG. 238.

Component Fields Acting in Repulsion Motor.

vertical fields in Fig. 238 will be $\Phi_A = \Phi \sin \alpha$ and $\Phi_B = \Phi \cos \alpha$ where Φ is the strength of the actual field in Fig. 236, and α is the angle between the brush axis and the stator field. This is also indicated in the vector diagram of fluxes in Fig. 237. We must further consider that the current flowing round the magnets forming the horizontal and vertical fields is the same ; *i.e.*, we shall look upon the two sets of field windings as being connected in series. The actual stator winding is therefore considered to be divided into two coils *A* and *B*, as indicated in Fig. 239. This does not, however, mean that the phases of the two fluxes will be identical. With motor stationary, the two windings will be related to one another exactly like the primary windings of two transformers which are connected in series, one transformer having an open-circuited secondary, while the secondary of the other is short circuited (see Fig. 233, page 378). The voltages across the primary windings of these transformers, and consequently the fluxes produced by their primary currents, will be almost 90° out of phase with one another, as explained in the previous section.

At starting we have a distribution of voltage between the portions A and B of the stator winding similar to that indicated in the diagram for the two transformers already given (Figs. 233 and 234).

Referring to Fig. 238, the vertical flux $\Phi_B = \Phi \cos a$ produces an armature current lagging 90° behind the flux in phase. The resultant vertical flux is rendered very small by the action of this current, and the voltage across the primary winding B having a vertical axis is also small. Coil A forms a purely inductive circuit and gives rise to a strong horizontal flux, Φ_A, which is in phase with the current. The current flowing through the armature and short-circuited brushes is almost exactly in phase with the horizontal flux $\Phi_A = \Phi \sin \alpha$, and its action upon this flux (which will have a high value) is to produce a strong starting torque.

When the motor rotates, we have two additional voltages introduced into the circuit, due to the conductors cutting the horizontal and the vertical component fluxes shown in Fig. 238. Both voltages will be in phase with the respective fluxes.

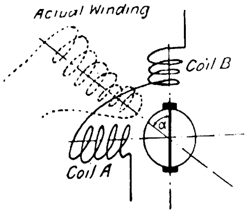


FIG. 239.—Equivalent Stator Coils of Repulsion Motor.

The cutting of the vertical component of the main flux will induce an electromotive force along the horizontal axis of the armature. This electromotive force can have no effect on the main circuit, since the electromotive force induced in the conductors of one-half of the armature will exactly balance that which is induced in the conductors of the other half, and there is no path for a current to flow between the points A, A^1, between which the difference of potential is formed.

Rotation in the horizontal component field will produce an electromotive force in the armature which will act upon the circuit formed by the short-circuiting connecting between the brushes. This electromotive force is in phase with the horizontal field (*i.e.*, in phase with the current of the stator), and being in the nature of the usual back electromotive force of rotation, must be overcome by an increase in the applied voltage acting on this circuit. The effect of this electromotive force is similar to that of introducing a non-inductive resistance between the brushes, which might be illustrated by the insertion of resistance in the secondary circuit of

transformer B in Fig. 233. The motor voltage will consequently distribute itself differently between the two component coils; the two components will assume a relative magnitude and phase represented by Fig. 240, where the full lines represent the conditions at starting, and the dotted lines the conditions when rotation has modified the voltages. There arises, due to the rotation of the motor, an increase in the voltage E_B (accompanied by a corresponding increase in vertical flux) and a decrease in voltage E_A (and decreased horizontal flux), together with a diminished current in the circuit.—It is specially important to notice the effect of this change on the phase of the current. This was originally almost identical with that of the horizontal flux, *i.e.*, vertical, in Fig. 240. The higher the speed of the motor becomes, the greater is the divergence between the phases of the current and the horizontal flux upon which it acts. As the speed of the motor increases, the power-factor at its terminals improves, while the torque falls off, due to two causes, viz., the reduction in current

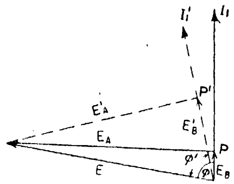


FIG. 240.—Vector Diagram of Repulsion Motor.

and the greater phase difference between current and horizontal flux. It is one of the characteristics of the repulsion motor that a rise in the power-factor is always accompanied by a decrease in torque. A consideration of the diagram (Fig. 240) for a repulsion motor supplied at constant voltage, shows that the two component voltages E_A and E_B will always lie in a semicircle described on E as diameter, since they are mutually perpendicular and have a constant resultant, so that the point P would always lie on the semicircle. In comparing the vector diagram (Fig. 240) with the diagram in Fig. 238, it is to be remembered that E_A and E_B, which are the component stator voltages, will also represent the values and phases of the voltages along two axes of the armature, E_B representing a rotor voltage along the brush axis, and E_A along an axis perpendicular thereto. The phase and magnitude are given in the vector diagram in relation to the applied terminal voltage.

Speaking generally, we may say that the characteristics of the repulsion motor resemble those of the series motor, but that the motor is suited for higher frequencies and higher terminal voltages than the simple series motor.

In the repulsion motor, as in all types of motor having a fixed axis for the rotor currents (*i.e.*, motors provided with a commutator) the frequency of the rotor currents is identical with that of the supply.

It is evident that the torque of the repulsion motor, like that of the series motor, depends upon the existence of a flux formed in the stator along an axis of the armature, which is so situated with regard to the brushes that the armature currents cannot neutralise it, *i.e.*, the armature currents cannot produce a similar but opposite flux. This flux is the horizontal component Φ_A in Fig. 238, without which the armature currents could produce no torque. Thus, like the series motor, the repulsion motor will always have a comparatively low power-factor, since a portion of the stator winding must be inductive.

In order to make it possible for the armature currents to neutralise the stator flux, the "compensated repulsion motor" was introduced.

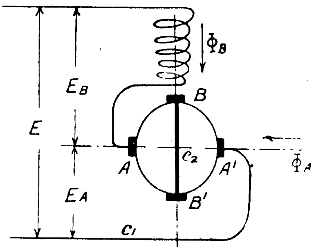


FIG. 241.—Compensated Motor.

The Compensated Repulsion Motor.—This motor is provided with an armature similar to those employed for the motors already described, but it has two brushes per pole, as indicated in the annexed diagram (Fig. 241). The short-circuited axis of the armature is that of the stator poles. The stator current is further led into the armature so as to flow through it along an axis perpendicular to this. The connections are thus the same as for a series motor, but with the addition of a pair of short-circuited brushes.

With the motor stationary, we shall have the following conditions almost identical with those of the repulsion motor—still referring to our simple 2-pole diagram :—

A small vertical flux Φ_B perpendicular in phase to the current giving rise by transformer action to a short-circuit armature current along the vertical axis in opposition of phase to the stator current.

A strong horizontal flux Φ_A in phase with the armature current most of the applied voltage being taken up by the inductive circuit of the armature between A and A^1.

Evidently the horizontal flux and short-circuit current will be in the best relative phase and direction to produce a torque.

By rotation, two additional electromotive forces are introduced, one along the vertical axis due to cutting of the horizontal flux Φ_A, and one along the horizontal axis, due to cutting Φ_B each in phase with the flux to which it is due. We thus have introduced into the circuit between A A^1 an electromotive force of rotation in phase with Φ_A, which will be in exact opposition to the electromotive force of self-induction E_A, due to the flux Φ_A formed in the circuit by transformer action (see Fig. 242). The voltage across the brushes A A^1 consequently diminishes as the speed of the motor increases.

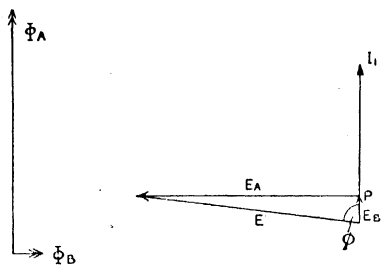


FIG. 242.—Flux and Voltage Phase Diagrams. Motor Stationary.

Along the axis B B^1 an electromotive force of rotation in phase with Φ_A is also introduced, the effect of which resembles the introduction of a non-inductive resistance into the circuit between the brushes B B^1. The electromotive force therefore increases the original difference of potential E_B across the stator winding necessary to produce the increased voltage between the brushes B B^1.

The effects just enumerated bring about a redistribution of the total applied voltage in the motor itself, the voltage E_B taken by the stator winding being increased, while the voltage E_A between the rotor brushes A A^1 is decreased. This in turn alters the phase relation between the total and component voltages. The component voltages E_A E_B are always in approximate quadrature of phase. The changes which they undergo may consequently be represented by imagining them as inscribed in a semicircle having the constant applied voltage E as diameter. The starting conditions are represented when E_B has a very small value, the point P lying to the right of the semicircle (see Fig. 242). With increasing

speed, P will move round towards the left, the reactance component of E_A continually diminishing, until it finally becomes zero when E_A consists entirely of volts overcoming the armature resistance (see Fig. 244). At this speed the vectors of current and total applied voltage coincide in phase, *i.e.*, the motor is com-

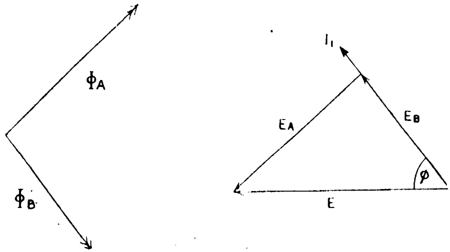


FIG. 243.—Flux and Voltage Phase Diagrams. Intermediate Speed.

pletely compensated. (The magnetising current is here neglected). This speed, at which the motor is entirely self-compensating, is the running speed of the motor.

If the turns of the armature and stator are exactly equivalent, this will be the speed of synchronism. For any other ratio of the

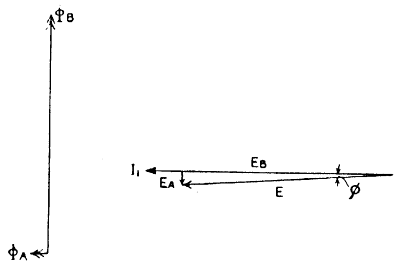


FIG. 244.—Flux and Voltage Phase Diagrams. Full Speed.

windings this speed will have another value, which will be constant for this ratio. By means of series transformers, having variable ratios of transformation, connected so that the primary winding is in series with the field and the secondary in series with the armature, it is possible to produce conditions under which the motor is self-compensating at a number of different speeds.

The changes in the magnitude and relative phases of the component voltages as the speed increases is shown by the Figs. 242-244, both flux and voltage diagrams being shown for each case.

A compensated motor will start from rest with a low power-factor, but when running at a definite speed its power-factor can be made practically unity.

The particular arrangement of connections shown in the diagrams has been varied by different makers, but the principle of neutralising a reactance voltage due to one flux by a rotational voltage induced by a second flux perpendicular in phase to the other, is a general one in all types.

CHAPTER XIII.

THE COMPOSITION OF VOLTAGE WAVES.

Equivalent Sine Waves.—It is very seldom in actual practice that the voltage and current in an alternating circuit vary strictly according to the simple harmonic law, although means are usually adopted to make divergence from this law fairly small. In the treatment of alternating problems it is usual to regard the various alternating functions as having an exact sinusoidal variation, and to treat them accordingly by vectors and formulæ based on this assumption. In certain cases this is liable to lead to serious errors; and it is then not admissible to regard the alternating waves as sinusoidal. In the majority of problems connected with generators, motors, and transformers, the treatment of current and voltages as sine functions does not lead to errors, in spite of the fact that the actual current and voltages are hardly ever truly sinusoidal. In such problems we deal with the "equivalent sine waves" of current and voltage, *i.e.*, with sine waves which have the same R.M.S., or virtual, value as the actual waves.

If two non-sinusoidal waves, representing respectively the current and voltage of a circuit, are plotted, it is not possible to see from an inspection of the curves what the phase difference between the curves is; in fact, the curves cannot be said to have a definite difference in phase at all. The fundamental curves will have a definite relative displacement, but each of the higher harmonics may have a different displacement. The phase difference cannot be taken as the angular distance between the points at which the curves pass through their minimum or maximum values.

On the other hand, the "equivalent" sine waves will have a definite phase relation. This phase relation is such that the equivalent waves give the same effective power in the circuit as the actual waves; *i.e.*, the equivalent waves have the same *mean product* as the actual waves.

Let e, i represent instantaneous values of the actual non-sinusoidal waves.

,, e_e, i_e be the virtual values of the "equivalent" sinusoidal waves.

Then $e_e = \sqrt{\text{mean value of } (e^2)}$.

$i_e = \sqrt{\text{mean value of } (i^2)}$.

Power of the circuit in watts = mean value of $(i \times e) = i_e e_e \cos\phi$,

$$\therefore \quad \cos\phi = \frac{\text{mean value of } (i \times e)}{\sqrt{\text{mean value of } (e^2) \times \text{mean value of } (i^2)}}.$$

The angle of phase difference in the circuit is the angle ϕ calculated from this relation between the current and voltage of the circuit.

It is only by putting this interpretation on the value of, and relations between, non-sinusoidal waves that simple vector diagrams and formulæ, such as have been given in the preceding chapters, have any practical value.

A typical case is that of the relation between the applied voltage and magnetising current of a transformer. Even if the applied voltage is sinusoidal, so that the flux also varies according to this law, the magnetising current is much distorted owing to hysteresis

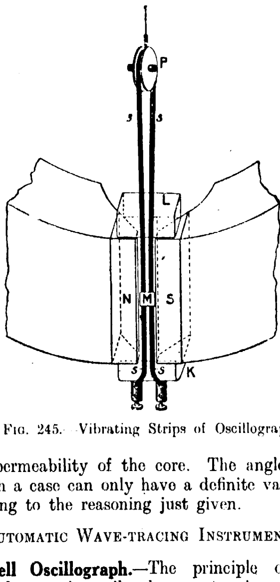


FIG. 245.—Vibrating Strips of Oscillograph.

and varying permeability of the core. The angle of lag of the current in such a case can only have a definite value when interpreted according to the reasoning just given.

AUTOMATIC WAVE-TRACING INSTRUMENTS.

The Duddell Oscillograph.—The principle of this instrument is that of a moving-coil galvanometer, in which the coil is replaced by a double strip of phosphor-bronze, stretched in such a way that the strip responds to very rapid changes of current sent through it.

Fig. 245 is a diagrammatic view of the galvanometer part of the instrument, showing the principle on which it works. In the narrow gap between the poles N, S of a powerful magnet are stretched two parallel conductors s, s formed by bending a strip of phosphor-bronze back on itself over the pulley P. A spiral

spring attached to this pulley serves to keep a uniform tension on the strips, and a guide piece L limits the length of the vibrating portion to the part actually in the magnetic field. A small mirror M bridges across the two strips as shown in the figure. The effect of passing a current through the loop formed by the strip is to cause one side of the loop to advance and the other side to recede, so that the mirror is caused to turn about a vertical axis. Each strip or leg of the loop passes through a separate gap (not shown in Fig. 245) in the magnetic circuit. The clearance between the sides of the gaps and the moving strip is very small and the whole of the "Vibrator," as this part of the instrument is called, is immersed in an oil bath, the object of the oil being to damp the movement of the strips and make the instrument dead-beat. Small fuses in the loop circuit protect this from injury in case of accidental excessive current. The fuses consist of very fine wires enclosed in glass tubes, which are held in position by spring clamps.

The beam of light reflected from the mirror M is received on a screen or photographic plate, the instantaneous value of the current being proportional to the linear displacement of the spot of light so formed. With alternating currents the spot of light oscillates to and fro as the current varies, and would thus trace a straight line. Hence, to obtain an image of the wave-form, it is necessary to traverse the photographic plate or film in a direction at right angles to the direction of movement of the spot of light. A second mirror can be interposed in the path of the beam, and this mirror caused to vibrate or rotate so as to impart to the beam of light a uniform motion proportional to time in a direction at right angles to its original plane of vibration. The spot of light will now trace out on a stationary screen or plate the time-curve of the variation of the P.D. or current, as the case may be. If the variations are periodic, as in alternating currents, then the second mirror can be synchronised, and the spot of light caused to trace out the wave-form over and over again.

In the *high-frequency type* of oscillograph, the periodic time of vibration of the strips is only $\frac{1}{10000}$ sec. The magnetic field is produced by an electromagnet wound in two sections, the ends of each section being brought out to terminals, so that when they are in series the magnet may be excited off a 220 volt direct-current circuit without using resistance. By putting the coils in parallel the excitation can be obtained off a 110 volt circuit. There is no need of an exact adjustment of the exciting current, as the magnetic circuit is saturated, so that a change of 4 per cent. more or less from the correct value produces only about 1 per cent. change in the sensibility.

The normal scale distance is 50 cm., at which distance a convenient working deflection is 2 cm. to 3 cm. on each side of the zero line, and this deflection will be obtained with a R.M.S. current through the strips of from 0·05 to 0·10 ampere, according to wave-form, &c. The maximum deflection on each side of the zero-line should not exceed 4 cm.

With the projection (*low-frequency*) *pattern,* alternating currents having frequencies from 20 to 100 per second may be examined.

The normal scale distance for projection work is 300 cm., and the deflection on each side of the zero line should not exceed 50 cm. The primary working current through the strips is 0·5 ampere.

The synchronous motor for the projection type is of the attracted-iron type, and has no moving wires or connections. As the motor must run synchronously with the wave-forms it is required to record, it should be supplied with current from the same source as the circuit under investigation. The motor can be used over a wide range of frequencies, viz., from 20 to 120 per second; when working at frequencies below 40 it is advisable to increase the moment of inertia of the armature, by means of a brass disc. The magnets of the motor are permanently connected in series, and the junction point brought out to a terminal, so that the magnets may be used either in series or in parallel to suit the voltage and frequency. The working current for frequencies between 25 and 100 per second should be 0·75 to 1·0 amperes, the magnet coils being in series. When using the motor coils in series the two extreme terminals are connected to the source of power, viz., terminals 1 and 4, numbering them from the left-hand side. To run in parallel the current leads should be connected to terminals 2 and 4, terminals 1 and 3 being connected together by a wire. In order to reduce the sparking when running up to speed, a resistance fixed on the motor frame is permanently connected between terminals 3 and 4.

On a 100-volt alternating circuit at 100 frequency, and with the two magnets in series, no resistance is required when running synchronously; but the starting is greatly facilitated if a resistance of about 50 ohms is put in series when running up to speed.

To start the synchronous motor, lift the tail of the vibrating mirror off the cam by moving the end of the lever, which moves in a horizontal plane, away from you, when you face the vibrating mirror; see that the armature is free in its bearings, and switch on the current, then lift up the end of the second lever and give the armature a touch, when it will start running. After the motor has been running thus for about half a minute it will probably be at, or near, the synchronous speed, when the lever may be depressed and the motor will pull itself into step and continue revolving synchronously. If unsuccessful in putting the motor into step the first time, raise the lever and repeat. A little practice will enable the experimenter to recognise when the speed is correct, as there are characteristic beats in the sound emitted by the motor when it is running near the synchronous speed, owing to the hum of the alternating current nearly coinciding in frequency with the noise made by the arm of the contact maker. When the motor is in step and the lever depressed, the tail of the vibrating mirror is pressed against the cam by means of the horizontal lever until the tail follows continuously the curve of the cam; the resistance

in series with the motor may now want adjusting to make the curves quite stationary on the screen.

Method of Connection of Oscillographs to the Circuits to be Investigated.—For Low Tension Circuits.—Fig. 246 shows the necessary connections. The synchronous motor is only employed for projection purposes and is omitted when a falling plate or other photographic apparatus is to be used. R_1 is a high non-inductive resistance connected across the mains in series with one of the vibrators. S_2 is a switch and f_2 the fuse (on the oscillograph) in this circuit. The resistance of R_1 in ohms should be rather more than ten times the voltage of the circuit, so that a current of a little less than 0·1 of an ampere will pass through it. The vibrator will then give the E.M.F. curve of the circuit on an open scale. (For the Projection Oscillograph, type 4, the resistance R_1 should be only twice the supply voltage since 0·5 of an ampere is required to give full scale deflection on a large screen.)

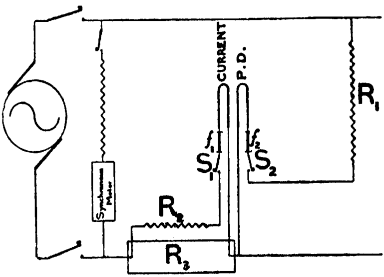


FIG. 246.—Oscillograph Connection in Low-tension Circuit.

To obtain the current wave form the shunt R_3 is connected in series with the circuit under investigation and the second vibrator is connected across this shunt. Here also f_1 is a fuse, S_1 a switch, and R_2 an adjustable resistance box. The switch S_1 is however unnecessary if an infinity plug is included in this box. The shunt R_3 should have a drop of about 1 volt across it in order to give a suitable working current through the vibrator. The resistance R_2 is not absolutely essential but it is a very great convenience in adjusting the current through the vibrator, and by its use the sensitiveness of the vibrator can be adjusted so that a round number of amperes in the shunt gives 1 mm. deflection. This adjustment is best made with direct current.

A most important point to be borne in mind when connecting the oscillograph in circuit is, that the two vibrators should be so connected to the circuit that it is impossible under any circumstances that a higher potential difference than 50 volts should exist between one vibrator and the other, or between either vibrator and the frame.

The Scheme of Connections for High Tension Circuits is made according to Fig. 247. It will be seen that the modification only applies to the vibrator which gives the E.M.F. wave, and consists in adding two more resistances R_4 and R_5.

Referring back to the Low Tension Diagram, Fig. 246, it will be seen that in case the fuse f_2 blows, or the vibrator is accidently broken, we immediately get the full supply voltage in the instrument itself. This is not permissible in high voltage work, and therefore the resistance R_5 is introduced as a permanent shunt to the oscillograph vibrator.

The resistance R_4 is an exact duplicate of R_2, being a 21 ohm plug resistance box for adjusting the sensitivity of the vibrator to an even figure.

In practice R_5 is usually a part of R_1 and in the high voltage resistances supplied by the maker, taps are brought out near one end to serve as R_5. One of these taps is usually 50 ohms distant from the end terminal and the other only 5 ohms from the end.

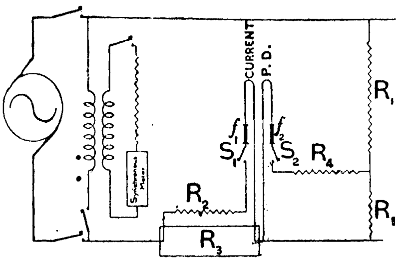


FIG. 247.—Oscillograph in High-tension Circuit.

The use of these taps is as follows. The large resistance consisting of $R_1 + R_5$ is so chosen with respect to the voltage of the circuit under investigation that the current through R_1 is about 0·1 of an ampere. It should never be more than this continuously. Then R_4 is connected to the 50 ohm tap and since the resistance of the oscillograph vibrator circuit is variable from about 5 to 26 ohms by means of R_4, current through the oscillograph can be controlled from about 0·066 to 0·091 of an ampere which gives an open wave form to a convenient scale.

If now it is desired to record large rises of E.M.F., such as may occur in cases of resonance, the height of the wave must be reduced in order to keep these rises on the plate. This is accomplished by disconnecting R_4 from the 50 ohm tap and connecting it to the 5 ohm tap, when the current through the vibrator will be from 0·05 to 0·016 of an ampere according to whether the resistance R_4 is in or out of circuit.

When instead of using the falling plate the cinematograph camera is being used it becomes necessary always to work on the

5 ohm tap since the width of the film is much less than that of the plate, and the current must therefore be less. In experiments where sudden rises of voltage are expected it is often advisable to keep R_1 as great as possible.

That end of the resistance R_1 referred to as R_5 in the diagram should be securely connected to the supply main and no switch or fuse used. A switch may, if desired, be used in series with R_1, provided it is inserted at the point where R_1 joins the supply main remote from R_5.

It will be seen that fuses f_1 and f_2 are shown in Fig. 247. Provided that the connections are always made in accordance with the diagram, and the vibrators are always shunted by R_5 and R_3 respectively, there is not much objection to the use of these fuses, but on general principles it is wise to avoid fuses in high tension work and the makers therefore supply, with each permanent magnet oscillograph, dummy fuses which can be inserted in place of the ordinary fuses when desired.

It is not usual to employ the projection type of oscillograph on a high-tension circuit. If this is done, the synchronous motor must be driven from a transformer, as shown in Fig. 247.

The remark previously made about keeping both vibrators and the frame of the instrument at approximately the same potential applies with additional emphasis in high tension work.

The Ondograph.—In principle the ondograph consists of a ballistic galvanometer, through which a condenser is discharged after having been connected to the voltage to be measured. The alternate connection of the condenser to the voltage and to the galvanometer is accomplished by a rotating commutator driven by a synchronous motor. The motor is driven from the source of current, the wave-form of which it is desired to take. The motor is provided with gearing for operating the commutator, the ratio of teeth in the gear wheels being so arranged that while the motor makes 1,000 revolutions, the commutator makes 999. It follows that the condenser is connected to the circuit $\frac{1}{1000}$ of a cycle later each time the commutator makes one revolution. The deflection of the galvanometer is thus made to vary slowly, following the variations of the voltage wave-form, and completing a complete cycle of movements every 1,000 revolutions of the motor. The galvanometer is connected to a long arm with a pen which traces out the form of the wave with ink on a revolving drum, which is operated by clockwork.

The commutator has three brushes bearing upon it, by means of which it operates the alternate charge and discharge of the condenser.

The brushes are connected with the condenser $c\ c^1$, and the galvanometer E, as indicated in the diagram, Fig. 248, $I\ I$ are the main terminals for the supply, whose wave-form is to be taken, while $G\ G^1$ are the terminals of the synchronous motor. $d\ d'\ d''$ are the brushes bearing on the commutator. As the commutator

revolves, the condenser is first put on the main circuit for an instant, then the condenser terminals are connected for a like period with the galvanometer, and the latter thus receives the discharges of the condenser and indicates the voltage at which the condenser has been charged. As this voltage increases and diminishes throughout the 1,000 revs., the galvanometer needle swings slowly back and forth and shows the voltage variations corresponding with a complete cycle. By changing the capacity of the condenser the sensitiveness of the indicator can be regulated.

In the cut G G^1 represents the terminals of the motor which also receives the current of the main circuit, and works at 110 volts, the latter being obtained, if necessary, by a regulating transformer. The terminals H H^1 are connected to the condenser

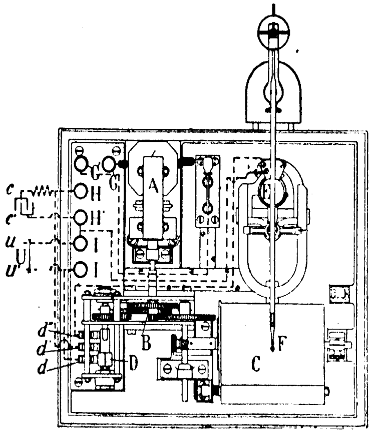


FIG. 248.—The Ondograph.

c c^1 through a resistance which is sufficient to avoid sparks at the charging brushes ; I I are the terminals of the main circuit which receive the voltage U to be taken. The synchronous motor makes a number of revolutions per minute which corresponds to one-half of the frequency of the main circuit. At D is the commutator with its three brushes d d' d''.

The indicating instrument for voltage or current waves is a galvanometer with a permanent magnet and rectangular current coil of fine wire mounted between the poles. The movable part E receives the discharges of the condenser in rapid succession, and turns slowly from one side to the other, passing through the zero position. The movable part operates a long needle which carries a pen F which swings to and fro, tracing the curve upon a revolving

cylinder. The needle is not mounted directly upon the coil of the galvanometer, but, in order to use as long a needle as possible, and thus make it move nearly in a straight line, it has its pivot fixed considerably in the rear, while the coil has a short arm which works the needle by a projection passing in a slot. The record is made upon a sheet of paper which is rolled upon the revolving cylinder. The cylinder is operated by the synchronous motor, by gearing, at a speed which makes it register three complete waves upon its circumference.

The Ondograph may also be used to study voltage variation, &c., in continuous-current machines. To accomplish this the synchronous motor is replaced by a direct drive effected by establishing an unyielding mechanical coupling between the Ondograph and the shaft of the machine to be studied.

The application of the Ondograph is not limited to periodic phenomena. It can be extended to all phenomena of short duration, provided that the phenomenon can be reproduced in such a manner that it is periodic by repetition.

Composition of Curves.—All curves of current or electromotive force may be considered to consist of a principal sine wave (called the *fundamental*), which is the regular curve to which the actual curve approximates, together with other sine waves of smaller amplitude and having frequencies differing from that of the fundamental.

The frequency of these minor waves will always be some multiple of the frequency of the fundamental, *i.e.*, of the periodicity of the supply. Further, there will be only one sine curve having a given frequency, since two sine waves of equal period may always be added together to give a single resultant sine wave. These small waves, which have each a periodicity which is a multiple of that of the fundmental, are called *harmonics*.

The general expression for a simple harmonic electromotive force may be written in the form

$$e = E \sin (2 \pi f t + \theta)$$

where e = instantaneous value of the electromotive force.
f = frequency of the alternator.
t = time counted from any given moment.
θ = phase of the voltage at moment from which time is counted.
E = maximum value of the voltage.

The instantaneous value of the harmonic which has a frequency of $2 f$ will be

$$e_2 = E_2 \sin (4 \pi f t + \theta_2)$$

where E_2 is the maximum value of this harmonic, and θ_2 is its phase at the same moment as θ is the phase of the fundamental.

The instantaneous value of the voltage composed of fundamental and harmonics will consequently be

$$e = E_1 \sin (2 \pi f t + \theta_1) + E_2 \sin (4 \pi f t + \theta_2) + E_3 \sin (6 \pi f t + \theta_3) + E_4 \sin (8 \pi f t + \theta_4) + \ldots$$

In this equation the constant E_1, E_2, &c., gives the maximum values of the corresponding harmonics.

We may write

$$\sin(2\pi f t + \theta) = \sin 2\pi f t \cos\theta + \cos 2\pi f t \sin\theta,$$

and since θ is a constant term, each of the sine functions in the equation may be written as the sum of a sine and a cosine term, and the equation then takes the form

$$e = a_1 \sin 2\pi f t + a_2 \sin 4\pi f t + a_3 \sin 6\pi f t + \ldots$$
$$+ b_1 \cos 2\pi f t + b_2 \cos 4\pi f t + b_3 \cos 6\pi f t + \ldots$$

where a_1, a_2, a_3, b_1, b_2, b_3 are constants.

This form of the equation is sometimes more convenient.

Harmonics of a Symmetrical Curve.—The wave forms produced by the electromotive force of an alternator are always symmetrical about the horizontal axis, *i.e.*, the negative half waves are exactly similar to the positive half waves. The north and south poles of an alternator are similar, and therefore generate similar electromotive forces ; but even if the poles were not exactly

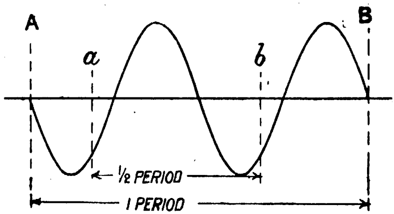


FIG. 249.—Even Harmonic

alike, the half waves would be similar, since both north and south poles contribute to the electromotive force of each half wave. From this follows the important fact that the wave-forms met with in an alternating circuit contain only "odd harmonics," that is to say, that the periodicity of any harmonic is 1, 3, or 5, &c., times the periodicity of the fundamental. That this must be so follows from the consideration that any two points on the curve which are exactly half a period apart must be equal values of the varying quantity, but having opposite signs, since they are values of the electromotive forces generated at similar positions under opposite poles. This is only true in the case of curves having a frequency which is an odd multiple of that of the fundamental. Thus, on referring to Fig. 249, if the distance between the two vertical lines A B is one full period of the fundamental, the frequency of the curve will be two per cycle of the main curve, so that the curve is the second (*i.e.*, an even) harmonic. Taking any two vertical lines such as those shown dotted at a, b, half a period apart, it is evident that the values separated by half

a period are equal, but *similar* in sign. A curve having this as one of its harmonics would, consequently, not be symmetrical in the sense of its having its negative half wave an exact repetition of its positive half wave, although it would be symmetrical in the sense that each complete wave would be similar to the preceding one.

Curves which have their positive and negative portions of dissimilar shape, like the one shown in Fig. 253, contain both odd and even harmonics.

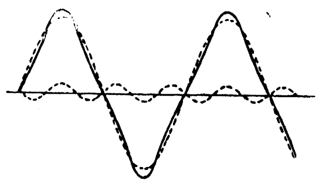


FIG. 250.—Third Harmonic in Opposition. Wave Peaked.

The expression for the harmonics of a symmetrical wave-form obtained from an alternator may consequently be written in the following form, in which the even harmonics are omitted :—

$$e = E_1 \sin (2 \pi f t + \theta_1) + E_3 \sin (6 \pi f t + \theta_3) + E_5 (10 \pi f t + \theta_5) + \ldots$$

Examples of curves containing only even harmonics are sometimes met with, as in curves of variation induced in the exciting circuit of an alternator by the armature current

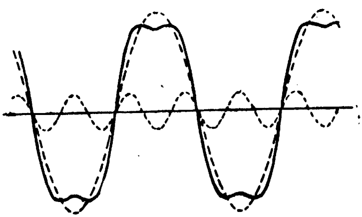


FIG. 251.—Third Harmonic in Phase. Wave Flattened.

Curves containing both odd and even harmonics are not symmetrical in any ordinary sense of the term. Such a curve is the one shown in Fig. 253.

Form of the Waves.—It is important to notice the effect of the addition of a harmonic to a fundamental wave upon the shape of the resulting curve. If the two curves are coincident in phase when the fundamental harmonic passes through its zero value, as

in Fig. 251, the resultant curve is flattened at the top. If, on the other hand, the two curves are in opposition of phase at this point (*i.e.*, they cross the axis in opposite directions, as shown in Fig. 250), the resultant curves becomes peaky. In either case the half waves are symmetrical about a vertical line. If the harmonic is intermediate in phase, like that shown in Fig. 252, the curve becomes irregular, having one side flattened and the other raised to a peak.

In analysing a curve, the number of harmonic waves which must be considered will depend on the extent to which the waveform differs from a simple sine curve and the degree of accuracy with which it is necessary to express the true shape. Usually it is sufficient to consider the third and fifth harmonic, in addition to the fundamental. This will, however, be largely governed by the number of slots per phase in which the armature is wound, the shape of the poles, &c.

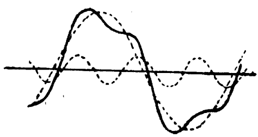


FIG. 252.—Third Harmonic Out of Phase. Wave Distorted.

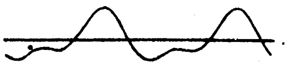


FIG. 253.—Curve containing both Odd and Even Harmonics

It is often possible to obtain an idea of the order of harmonic having the greatest frequency by counting the number of points in half a wave at which a sudden change in curvature occurs. The number of such changes gives the order of this harmonic.

Effect of Star Connection of Phases.—An important modification of the voltage wave-form at the terminals of an alternator sometimes results if the phase windings are star-connected, owing to the fact that all those harmonics disappear which have a frequency of three times (or any multiple of three times) the fundamental.

The wave-form of the voltage generated in each phase winding of a 3-phase alternator is determined by the flux distribution and by the disposition of the winding. If the alternator is mesh-connected, the voltage which is given to the line is the voltage thus determined.

If the alternator phases are connected in star, the voltage given to the line is not the same as the voltage generated in the phase winding, but the sum of two phase voltages added together with a relative phase displacement of 60° or $\frac{1}{6}$ cycle (see page 250).

By adding together two sine waves having a relative displacement of 60°, we should obtain a resultant sine wave having an amplitude of $\sqrt{3}$ times that of the component waves. This is the condition assumed in Chapter IX., where the relation between the phase voltage and the line voltage of a star-connected circuit is discussed.

If the waves of phase voltage which are added together contain a third harmonic (having three times the frequency of the fundamental), the third harmonic of one wave will be displaced by one-sixth of the period of the fundamental relatively to the third harmonic of the other wave. Now one-sixth of the fundamental period is three-sixths or one-half of the period of the harmonics. It follows that the third harmonics are added with a relative displacement of half a period or of 180° of phase difference. In other words, the two third harmonic waves will act in mutual opposition between the terminals of the alternator and will cancel one another, so far as any voltage given to the external circuit is concerned.

For the same reason, any harmonics having any multiple of three times the frequency of the fundamental will be in mutual

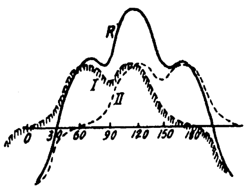


FIG. 254.

opposition between the alternator terminals. The student should convince himself of these statements by sketching curves of third and ninth harmonics and noting that points which lie one-sixth of a cycle apart have equal ordinates, but opposite sign.

It follows that the electromotive-force wave of a star-connected alternator can never contain any harmonics having a frequency which is three times, or any multiple of three times, that of the fundamental wave.

In Figs. 254 and 255 are shown the modification which may be introduced in the wave-form of an alternator by the elimination of harmonics due to star-connection of the phases.

In Fig. 254 the two dotted curves show the wave-form of the voltage which would be observed at the terminals of either of the phase windings. They each consist of a fundamental sine wave and a third harmonic. The voltage as measured between the terminals of the star-connected alternator is the sum of these curves and is shown by the full line. This is a sine curve, since the harmonics have been eliminated by the addition. Clearly, the value of the alternator terminal voltage will not now be exactly $\sqrt{3}$ times the phase voltage.

In Fig. 254 a similar addition of two-phase voltages is shown, but here the dotted curves contain a fifth harmonic, as well as a third harmonic. In the resultant curve, the fifth harmonics are added and do not cancel one another, whereas the third harmonics have again disappeared.

The general result of this elimination of all harmonics in the phase voltage which have a periodicity of 3 n times that of the fundamental is that the phase voltage and terminal voltage often have very different wave-forms, the terminal voltage usually approximating more nearly to a sine curve.

Effect of Iron Losses in Core on Wave-form.—When an alternating voltage is applied to a circuit surrounding an iron core, the relation existing between the voltage and current of the circuit is not quite so simple as has been assumed in the discussion given in earlier chapters on account of the imperfect magnetic qualities of the iron, which give rise to iron losses.

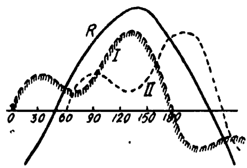


FIG. 255.

Let us suppose that a sinusoidal voltage is applied to a winding of negligible resistance acting on a magnetic circuit containing iron. The voltage will give rise to a current and the current will produce a magnetic flux, the value of which depends on the value of the reactance of the circuit. Since no voltage is absorbed by resistance, the applied voltage must be equal to the induced back voltage (or reactance voltage) *at every instant.* The back voltage is proportional to the rate of change of flux at every instant, whence it follows that, under the conditions assumed, the values of flux will be sinusoidal, like those of the applied voltage and the flux curve will differ in phase from the curve of applied voltage by a phase angle of 90°.

The flux in the magnetic circuit is thus determined entirely by the value of the applied voltage, since it will rise to such a value as to induce a back voltage equal to the applied voltage. The current taken by the winding will rise to the value necessary to produce this flux.

If the iron were a perfect magnetic medium, *i.e.*, if it had a constant permeability and if the induction in it could be varied without producing iron losses, we should have the following relation

between the applied voltage, the resulting flux, and the current in the winding :—

Applied voltage $= e = E \sin\theta$.
Resulting flux $= f = F \cos\theta$.
Current in coil $= i = I \cos\theta$.

The current and flux would thus be in phase with one another and both 90° in phase behind the applied voltage.

The permeability of iron varies with the strength of the magnetising current; also, owing to the effect of hysteresis, the ratio between the magnetising current and flux is different during periods of increasing current from that during periods of decreasing flux. The result of this behaviour of the iron is that, when a sinusoidally varying flux is induced in it, the current necessary to produce this flux, instead of varying sinusoidally like the flux, has a distorted wave-form and is no longer in phase with the flux.

Let us consider first the effect of the varying permeability of the iron upon the current. In Fig. 256 the curve on the left shows

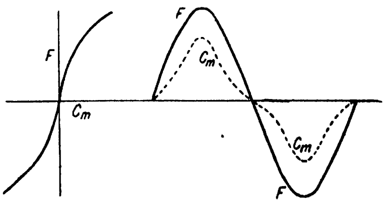


FIG. 256.—Curves of Flux and Magnetising Current. Iron without Hysteresis.

the relation between magnetising current, C_m, and flux, F. This curve would be a straight line if the permeability of the iron remained constant throughout the cycle. Curve F on the right of the figure represents the sinusoidally varying flux. The curve, C_m, of magnetising current is derived from the curve of flux by plotting the value of the current corresponding to any particular flux, as indicated in the curve on the left. It is to be noted that the curve C_m is more peaked than the wave of flux, but is in phase with it. The distorted current wave is 90° out of phase with the voltage (which would be represented by a sine curve 90° in advance of the wave of flux). It follows that the current is still an idle current, and the variation in permeability does not involve a loss of power.

In Fig. 257 is shown the further effect upon the current of hysteresis of the core of the transformer.

Again the curve on the left shows the connection between the no-load current, C_0, and the flux, F. The wave of current is shewn on the right of the figure, and is obtained as in the last case. The current C_0 may be regarded as made up of the two component

currents ; firstly, C_m, which is the true magnetising current, and which has the phase of the flux, and secondly, the current C_e introduced by the hysteresis of the circuit which has a phase 90° in advance of the flux and which is consequently an energy current in phase with the applied voltage. The two components are shown separately as dotted lines.

It will be hardly necessary to say that, with the low inductions usual in transformer cores, the variations in permeability and the effects of hysteresis will be much smaller than those shown in the diagrams.

Example of Curve Analysis.—The following examples of the analyses of a curve obtained experimentally will enable the student to analyse for himself any curve which he obtains, even if he is not able to follow the mathematics on which the methods are based. He may omit the mathematical proof if he is not able to follow it.

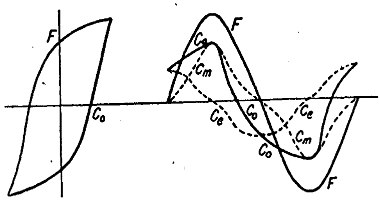


FIG. 257.—Curves of Flux and No-load Currents. Iron with Hysteresis.

Outline of Mathematical Proof of Method.—Denoting the variable quantity by e,

$$\text{Let } e = A_0 + A_1 \sin\theta + A_2 \sin 2\theta + A_3 \sin 3\theta + \ldots$$
$$+ B_1 \cos\theta + B_2 \cos 2\theta + B_3 \cos 3\theta + \ldots \quad (1)$$

Multiply both sides of the equation by $\sin\theta$, then

$$e \sin\theta = A_0 \sin\theta + A_1 \sin^2\theta + A_2 \sin 2\theta \sin\theta + A_3 \sin 3\theta \sin\theta + \ldots$$
$$+ B_1 \cos\theta \sin\theta + B_2 \cos 2\theta \sin\theta + B_3 \cos 3\theta \sin\theta + \ldots \quad (2)$$

Integrating both sides of this expression between the limits 0 and 2π, we have :—

$$\int_0^{2\pi} e \sin\theta \, d\theta = \int_0^{2\pi} A_1 \sin^2\theta \, d\theta \quad \ldots \quad (3)$$

the other terms all vanishing with the limits chosen. Hence, mean value of $e \sin\theta$ = mean value of $A_1 \sin^2\theta$.

$$= \frac{A_1}{2}$$

$$i.e., A_1 = 2 \times \text{mean value of } e \sin\theta \quad \ldots \quad (4)$$

Similarly, by multiplying equation (1) by

$$\left.\begin{matrix}\sin 2\,\theta, \sin 3\,\theta \quad . \quad . \quad . \\ \cos \theta, \cos 2\,\theta, \cos 3\,\theta\end{matrix}\right\} \text{ the value of } \left\{\begin{matrix}A_2, A_3 \quad . \quad . \\ B_1, B_2, B_3 \quad .\end{matrix}\right.$$

may be determined.

$$\text{Thus } A_2 = 2 \times \text{mean value of } e \sin 2\,\theta,$$

$$B_1 = 2 \times \text{mean value of } e \cos \theta,$$

$$B_2 = 2 \times \text{mean value of } e \cos 2\,\theta; \text{ and so on.}$$

Since all symmetrical wave-forms will consist of odd or even terms only, only half of the constants need to be determined. In any ordinary alternating circuit the wave-form will contain odd harmonics only.

Method of Analysing Curve (Summation).—Plot one-half wave of the curve to a large scale on squared paper, the horizontal scale being chosen to enable the curve to be divided conveniently into a number of equal parts. These parts may conveniently correspond to 30 of 6° each, 15 divisions of 12° each, or 12 divisions of 15° each, according to the accuracy desired.

In order to obtain the value of the fundamental curve, proceed as follows: Measure the height of the curve at each of the horizontal divisions decided upon, tabulating the values as shown in Table I., column 2, entering at the same time the angle corresponding to each ordinate in column 1. In column 3 enter the sines of the angles given in column 1, as found in any set of mathematical tables. Column 4 is then obtained by multiplying together the figures in columns 2 and 3.

Column 5 gives the cosines of the angle in column 1, and column 6 is the product of these cosines by the numbers in column 2.

Column 7 is obtained by writing down the values of the sines of 3 × angles in column 1. Multiplying these sines by the figures in column 2 we obtain column 8. Column 9 contains the values of the cosines of the angles whose sines are in column 7, and the tenth column gives the products obtained by multiplying them by column 2. These columns of figures will give the fundamental and the third harmonic. If the fifth or higher harmonics are also present, further similar columns for $\sin 5\,\theta$, $e \sin 5\,\theta$ and $\cos 5\,\theta$ $e \cos 5\cdot\theta$, etc., must be added.

After filling in the columns as described, add together all the figures in column 4 and divide by half the number of these figures, thus obtaining twice the mean value. Carry out the same operation for columns 6, 8, 10, &c. In this way the constants A_1, B_1, A_3, B_3, &c., are obtained.

The expression for the curve is then given by substituting the values found for these constants in the equation

$$e = A_1 \sin \theta + B_1 \cos \theta + A_3 \sin 3\,\theta + B_3 \cos 3\,\theta + \quad . \quad . \quad . \quad .$$

This equation may be put into a simpler form by making the following substitutions:—

$$F_1 = \sqrt{A_1^2 + B_1^2} \qquad F_3 = \sqrt{A_3^2 + B_3^2}, \text{ \&c.}$$

$$\tan \phi_1 = \frac{B_1}{A_1} \qquad \tan \phi_3 = \frac{B_3}{A_3}, \text{ \&c.}$$

The function then becomes finally :—

$$e = F_1 \sin(\theta + \phi_1) + F_3 \sin(3\,\theta + \phi_3) + \quad . \quad . \quad .$$

As an illustration of this method of analysing a curve, its application to the curve in Fig. 258 may be given.

This curve was obtained by the method described in Experiment XIV. from a small Pyke & Harris inductor alternator, having seven inductors. The armature was connected to a condenser having a capacity of 6 micro-farads in series with a single incandescent lamp. The curve was plotted from readings taken

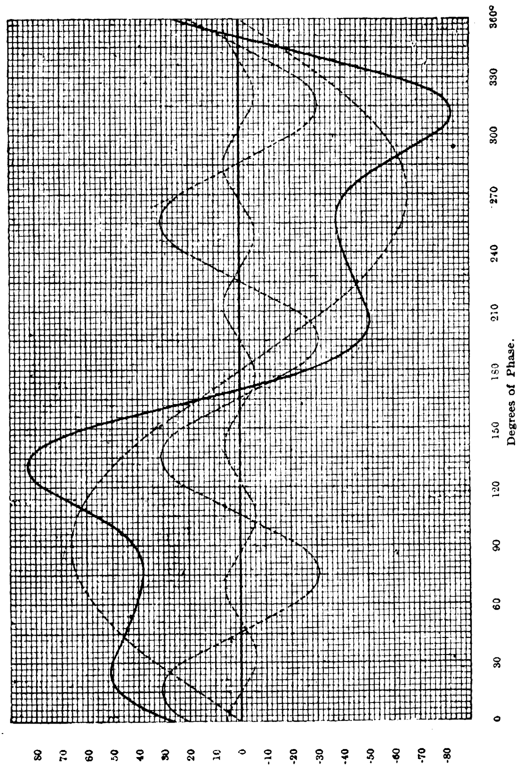


FIG. 258.—Experimentally Determined Curve and Harmonics.

at the terminals of the lamp. The R.M.S. volts of the alternator were 80, those across the lamp 51·8, and across the condenser 61·5.

Table II. shows the method adopted for entering up the observations and calculated figures. It will be seen that there are several modifications from the form of Table I.; these were made in order to enable logarithms to be used for the multiplication.

In the case of the fifth harmonic, logarithms were not used on account of the comparative simplicity of the values of sin 5 θ and cos 5 θ.

A separate column for values of the cosines was not considered necessary, since the column of logarithmic sines may be used, taking values of $90 - \theta$, instead of θ, &c.

The method of obtaining the first form of the equation given at the foot of the table will be clear from the description already given. The following calculation shows the steps for arriving at the second, simplified, form of the expression :—

As seen from the first form of the equation,

$$A_1 = 66 \qquad B_1 = -0{\cdot}073$$

$$A_1 \sin\theta + B_1 \cos\theta = \sqrt{A_1^2 + B_1^2}.\ \sin\left\{\theta + \tan^{-1}\left(\frac{B_1}{A_1}\right)\right\}$$

$$= \sqrt{66^2 + (0{\cdot}073)^2}.\ \sin\left(\theta + \tan^{-1}\frac{-{\cdot}073}{66}\right)$$

$$= 66 \sin(\theta - 0^\circ\, 4') = 66 \sin\theta, \text{ practically,}$$

$$\text{since } \tan 0^\circ\, 4' = {\cdot}00112 = \frac{{\cdot}073}{66}$$

Similarly,

$$A_3 = 22{\cdot}96 \qquad B_3 = 20{\cdot}72$$

$$A_3 \sin 3\,\theta + B_3 \cos 3\,\theta = \sqrt{A_3^2 + B_3^2}.\ \sin\left\{3\,\theta + \tan^{-1}\left(\frac{B_3}{A_3}\right)\right\}$$

$$= 30{\cdot}93 \sin(3\,\theta + 42^\circ)$$

also

$$A_5 = -1{\cdot}546 \quad B_5 = 5{\cdot}776$$

$$A_5 \sin 5\,\theta + B_5 \cos 5\,\theta = \sqrt{A_5^2 + B_5^2}.\ \sin\left\{5\,\theta + \tan^{-1}\left(\frac{B_5}{A_5}\right)\right\}$$

$$= 5{\cdot}97 \sin(5\,\theta + 105^\circ).$$

A method of analysis which, although not quite so simple, is less laborious to carry out, is the following :—

Analysis by Method of Superposition.—Again, a short mathematical summary of the principles of this method of analysis will be given, followed by a detailed description of the method of carrying out the calculation in practice. The non-mathematical reader should be able to carry out an analysis from the description

TABLE I.

1	2	3	4	5	6	7	8	9	10
θ	e	Sin θ	e sin θ	Cos θ	e cos θ	Sin 3 θ	e sin 3 θ	Cos 3 θ	e cos 3 θ
	2 Mean $= A_0$ $= 0$ for 360°		2 Mean $= A_1$		2 Mean $= B_1$		2 Mean $= A_3$		2 Mean $= B_3$

TABLE II.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
θ	$f(\theta)$	$\log f(\theta)$	$L \sin \theta$	$\log f(\theta) \sin \theta$	$f(\theta) \sin \theta$	$\log f(\theta) \cos \theta$	$f(\theta) \cos \theta$	$L \sin 3\theta$	$\log f(\theta) \sin 3\theta$	$f(\theta) \sin 3\theta$	$\log f(\theta) \cos 3\theta$	$f(\theta) \cos 3\theta$	$\sin 5\theta$	$f(\theta) \sin \theta$	$f(\theta) \cos 5\theta$
0	26·5	1·4232	$-\infty$	$-\infty$	0	1·4232	26·50	$-\infty$	$-\infty$	0	1·4232	26·50	0	0	26·50
6	37·9	1·5786	9·0192	0·5978	3·96	1·5762	39·69	9·4900	1·0686	11·71	1·5568	36·04	·5	18·95	32·82
12	45·6	1·6590	9·3779	0·9769	9·48	1·6494	44·61	9·7692	1·4282	26·80	1·5670	36·90	·866	39·49	22·80
18	49·6	1·6955	9·4900	1·1855	15·33	1·6737	47·18	9·9080	1·6035	40·14	1·4647	29·16	1·000	49·60	0
24	51·0	1·7076	9·6093	1·3169	20·74	1·6683	46·59	9·9782	1·6858	48·51	1·1976	15·76	·866	44·17	− 25·50
30	50·2	1·7007	9·6990	1·3997	25·10	1·6382	43·47	10·0000	1·7007	50·20	$-\infty$	0	·5	25·10	− 43·47
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—
162	25·3	1·4031	9·4900	0·8931	7·82	1·3813	-24·06	9·9080	1·3111	20·46	1·1723	−14·87	1·00	25·30	0
168	6·2	0·7924	9·3179	0·1103	1·29	0·7828	− 6·06	9·7692	0·5616	3·64	0·7004	− 5·02	·866	5·37	− 3·10
174	−11·5	1·0607	9·0192	0·0799	−1·20	1·0583	+11·44	9·4900	0·5507	-3·55	1·0389	10·93	·5	-5·75	9·96
					Sum of 30 terms = 990·00 $2 \times \text{mean} = \frac{990{\cdot}00}{15} = 66{\cdot}00 = A_1$		Sum of 30 terms = −1·09 $2 \times \text{mean} = \frac{-1{\cdot}09}{15} = -0{\cdot}073 = B_1$			Sum of 30 terms = 344·43 $2 \times \text{mean} = \frac{344{\cdot}43}{15} = 22{\cdot}96 = A_3$		Sum of 30 terms = 310·79 $2 \times \text{mean} = \frac{310{\cdot}79}{15} = 20{\cdot}72 = B_3$		Sum of 30 terms = −23·19 $2 \times \text{mean} = \frac{-23{\cdot}19}{15} = -1{\cdot}546 = A_5$	Sum of 30 terms = 86·65 $2 \times \text{mean} = \frac{86{\cdot}65}{15} = 5{\cdot}776 = B_5$

Equation to curve : $f(\theta) = 66 \sin \theta - {\cdot}073 \cos \theta + 22{\cdot}96 \sin 3\theta + 20{\cdot}72 \cos 3\theta - 1{\cdot}546 \sin 5\theta + 5{\cdot}776 \cos 5\theta$

or $f(\theta) = 66 \sin \theta + 30{\cdot}93 \sin (3\theta + 42^\circ) + 5{\cdot}97 \sin (5\theta + 105^\circ)$.

and illustration of the method even if he is obliged to omit the mathematical introduction.

. **Lemma.**—Let $f\ (a, \beta)$

$$= \sin a + \sin (a + \beta) + \sin (a + 2\beta) + \ldots + \sin (a + \overline{n-1}\,\beta)$$

$$= \frac{\sin \frac{1}{2}\beta}{\sin \frac{1}{2}\beta}\left\{ \sin a + \sin (a + \beta) + \sin (a + 2\beta) + \ldots + \sin (a + \overline{n-1}\,\beta) \right\}$$

$$= \frac{1}{\sin \frac{1}{2}\beta}\,\frac{1}{2}\left\{ \begin{array}{l} \cos (a - \frac{1}{2}\beta) - \cos (a + \frac{1}{2}\beta) + \cos (a + \frac{1}{2}\beta) - \cos \\ (a + \frac{3}{2}\beta) + \ldots \cos (a + \overline{n-1}\,\beta - \frac{1}{2}\beta) - \\ \cos (a + \overline{n-1}\,\beta + \frac{1}{2}\beta) \end{array} \right\}$$

$$= \frac{1}{2 \sin \frac{1}{2}\beta}\left\{ \cos (a - \frac{1}{2}\beta) - \cos (a + \overline{n-1}\,\beta + \frac{1}{2}\beta) \right\}$$

$$= \frac{1}{2 \sin \frac{1}{2}\beta} \times 2 \sin \left(a + \frac{n-1}{2}\beta \right) \sin \left(\frac{n-1}{2}\beta + \frac{1}{2}\beta \right)$$

$$= \frac{\sin \frac{n\beta}{2} \cdot \sin \left(a + \frac{n-1}{2}\beta \right)}{\sin \frac{1}{2}\beta}$$

Now if $\sin \frac{n}{2}\beta = 0$, and $\sin \frac{\beta}{2}$ does not equal 0, then $f\ (a, \beta) = 0$.

But in the special case when $\frac{\beta}{2} = \pi$, $\beta = 2\pi$

and the original expression becomes

$$f\ (a, \beta) = \sin a + \sin (a + 2\pi) + \sin (a + 4\pi) + \ldots + \sin (a + \overline{n-1}\,2\pi)$$
$$= \sin a + \sin a + \sin a + \ldots$$
$$= n \sin a$$

i.e., an expression of the form

$\sin a + \sin (a + \beta) + \sin (a + 2\beta) + \ldots \sin (a + \overline{n-1}\,\beta)$

becomes equal to

$n \sin a$ when $\beta = 2\pi$.

Let the curve to be analysed be written in the following form :-

$$f\ (\theta) = y_1 = a_1 \sin (\theta + \alpha_1) + a_2 \sin 2\ (\theta + \alpha_2) + a_3 \sin 3\ (\theta + \alpha_3) + \ldots$$

Then we may write :—

$$f\left(\theta + \frac{2\pi}{n} \right) = y_2 = a_1 \sin \left(\theta + \alpha_1 + \frac{2\pi}{n} \right) + a_2 \sin 2 \left(\theta + \alpha_2 + \frac{2\pi}{n} \right) + a_3 \sin 3 \left(\theta + \alpha_3 + \frac{2\pi}{n} \right) + \ldots$$

Similarly,

$$f\left(\theta + \frac{4\pi}{n} \right) = y_3 = a_1 \sin \left(\theta + \alpha_1 + \frac{4\pi}{n} \right) + a_2 \sin 2 \left(\theta + \alpha_2 + \frac{4\pi}{n} \right) + a_3 \sin 3 \left(\theta + \alpha_3 + \frac{4\pi}{n} \right) + \ldots$$

and so on.

Adding these quantities together,

$$y_1 + y_2 + y_3 + \ldots = a_1 \left\{ \sin (\theta + \alpha_1) + \sin \left(\theta + \alpha_1 + \frac{2\pi}{n} \right) + \sin \left(\theta + \alpha_1 + \frac{4\pi}{n} \right) + \ldots \right\}$$
$$+ a_2 \left\{ \sin 2 (\theta + \alpha_2) + \sin 2 \left(\theta + \alpha_2 + \frac{2\pi}{n} \right) + \sin 2 \left(\theta + \alpha_2 + \frac{4\pi}{n} \right) + \ldots \right\}$$
$$+ a_3 \left\{ \sin 3 (\theta + \alpha_3) + \ldots \right\}$$

The expression thus consists of a number of terms of the form :—

$$a_m \left\{ \sin m \left(\theta + \alpha_m \right) + \sin m \left(\theta + \alpha_m + \frac{2\pi}{n} \right) + \sin m \left(\theta + \alpha_m + \frac{4\pi}{n} \right) + \ldots \right\}$$

By the previously-given lemma each of the expressions within the brackets will be zero, unless

$$\sin \frac{2\pi m}{2n} = 0$$

$$\text{i.e., unless } \frac{m}{n} \text{ is an integer,}$$

in which case the general term

$$= a_m \, n \sin m \, (\theta + a_m).$$

Let $n = 3$; then since $\frac{m}{n}$ must be an integer, all the terms will cancel except those for which m is a multiple of 3, *i.e.*, all harmonics will vanish except the third, sixth, ninth, &c., and

$$y_1 + y_2 + y_3 = 3 \, a_3 \sin 3 \, (\theta + \alpha_3) + 3 \, a_6 \sin 6 \, (\theta + \alpha_6) + 3 \, a_9 \sin 9 \, (\theta + \alpha_9) + \ldots$$

If the curve has positive and negative half-waves which are similar (see page 396), even harmonics will not exist, hence the term containing $\sin 6 \, (\theta + a_6)$ may be neglected. Further harmonics above the fifth are seldom present of sufficient amplitude to be worth determination. Hence we have for practical purposes

$$y_1 + y_2 + y_3 = 3 \, a_3 \sin 3 \, (\theta + \alpha_3).$$

By choosing the value of θ such that $3 \, (\theta + \alpha_3) = 90°$, we can determine a_3 from observed values of y_1, y_2, and y_3. The method of doing this is as follows :—

From the curve, measure the value of the ordinate for the angle $\theta = \frac{90}{n} = 30°$. Call this y_1.

Similarly measure the ordinate at $\theta + \frac{2\pi}{n} = 30° + 120° = 150°$

Call this y_2.

also the ordinate at $\theta + \frac{4\pi}{n} = 30° + 240° = 270°$

Call this y_3.

Add the three measured ordinates together and divide by $n = 3$. The result is the coefficient of $\sin n\,\theta = \sin 3\,\theta$. Similarly, by starting from the angle O, proceed to obtain the coefficient of $\cos n\,\theta = \cos 3\,\theta$.

The analysis may be further simplified by separating sine and cosine terms, as follows :—

Let

$$f(\theta) = Y_1 = a_0 + a_1 \sin \theta + a_2 \sin 2\,\theta + a_3 \sin 3\,\theta + a_4 \sin 4\,\theta + \dots$$
$$+ b_1 \cos \theta + b_2 \cos 2\,\theta + b_3 \cos 3\,\theta + b_4 \cos 4\,\theta + \dots$$

$$f(2\pi - \theta) = Y_2 = a_0 + a_1 \sin(2\pi - \theta) + a_2 \sin 2(2\pi - \theta) + a_3 \sin 3(2\pi - \theta) + \dots$$
$$+ b_1 \cos(2\pi - \theta) + b_2 \cos 2(2\pi - \theta) + b_3 \cos 3(2\pi - \theta) + \dots$$
$$= a_0 - a_1 \sin \theta - a_2 \sin 2\,\theta - a_3 \sin 3\,\theta \dots$$
$$+ b_1 \cos \theta + b_2 \cos 2\,\theta + b_3 \cos 3\,\theta + \dots$$

$$\therefore Y_1 + Y_2 = 2(a_0 + b_1 \cos \theta + b_2 \cos 2\,\theta + b_3 \cos 3\,\theta + \dots$$
$$Y_1 - Y_2 = 2(a_1 \sin \theta + a_2 \sin 2\,\theta + a_3 \sin 3\,\theta + \dots$$

Thus $\frac{Y_1 + Y_2}{2}$ will consist of the cosine terms of the original expression alone, and $\frac{Y_1 - Y_2}{2}$ will be the sine terms.

Method of Procedure.—Plot the curve to a large scale, as for the previous method of analysis.

Draw up a table (see Table III.) with the angles showing the subdivisions which it is proposed to take in the first column. In the second column enter the measured values of the curve corresponding to these values. In column 3 put the values of the curve for the angle (360° – angle in first column). In column 4 enter the difference between the numbers in columns 2 and 3. Column 5 is obtained by dividing the figures in column 4 by two. Column 6 is the sum of the readings entered in columns 2 and 3, while column 7 is the half values of these numbers.

The coefficients of the cosine terms will be obtained from column 7, while those of the sine terms will be got from column 5.

In order to determine the coefficient for the nth harmonic, proceed as follows :—

For the sine term, divide the complete period of the curve into n parts, taking the angle $\frac{\pi}{2n}$ as the starting point, since this makes $\sin \theta = 1$.

Measure the ordinates of the curve for the angles $\frac{\pi}{2n}, \frac{\pi}{2n} + \frac{2\pi}{n}, \frac{\pi}{2n} + \frac{4\pi}{n}$, &c. Add these ordinates together and divide by n. The result is the required coefficient.

For the cosine term, divide the curve into n parts, starting from the angle 0 (making $\cos\theta = 1$).

Measure the ordinates for the angle $0, \frac{2\pi}{n}, \frac{4\pi}{n}$ &c. Add them together and divide by n.

Having obtained one harmonic (*e.g.*, the third), subtract its corresponding ordinates from the original curve, and so obtain a new curve which is the original curve—third harmonic.

The coefficients were worked out as follows .

TABLE III.

1	2	3	4	5	6	7
θ	$f(\theta) = Y_1$	$f(30°-\theta) \doteq Y_2$	Difference $Y_1 - Y_2$	½ Difference $\frac{Y_1 - Y_2}{2}$	Sum $Y_1 + Y_2$	½ Sum $\frac{Y_1 + Y_2}{2}$
0	26·5	26·5	0	0	53·0	26·5
6	37·9	11·5	26·4	13·2	49·4	24·7
12	45·6	− 6·2	51·8	25·9	39·4	19·7
18	49·6	−25·3	74·9	37·45	24·3	12·15
24	51·0	−44·0	95·0	47·5	7·0	3·5
30	50·2	−60·2	110·4	55·2	−10·0	− 5·0
—	—	—	—	—	—	—
—	—	—	—	—	—	—
162	25·3	−49·6	74·9	37·45	−24·3	−12·15
168	6·2	−45·6	51·8	25·9	−39·4	−19·7
174	−11·5	−37·9	26·4	13·2	−49·4	−24·7

NOTE.—In the construction of this table it will be seen from the use made of the figures that not all the figures require to be filled in in all the columns. In the present example only 14 out of 30 values of θ are actually required for the harmonics plotted in Fig. 258. In testing, for the higher harmonics, however, a greater number of values was required. It is best, therefore, to put down the first column complete, and to fill in the further columns as required.

Unless this curve is found on trial to be a simple sine curve, determine the next higher harmonic (say, fifth), and again subtract its ordinates from the residual curve Proceed thus until only the fundamental is left. In practice the third and fifth (and occasionally the seventh) harmonics will be the only ones to be determined.

3rd Harmonic.

Cosine Term.		Sine Term.	
Angle.	$\frac{1}{2}$ Sum.	Angle.	$\frac{1}{2}$ Difference.
0°	26·5	30°	55·2
120°	17·8	150°	55·2
240°	17·8	270°	−41·4
	3)62·1		3)69·0
	$20{\cdot}7 = b_3$.		$23{\cdot}0 = a_3$.

The figures given (Table III.) are taken from the same curve (Fig. 258) as that from which Table II. was derived.

To confirm the result a check reading was taken in each case, thus :—

$$\theta = 12. \qquad 3\ \theta = 36.$$

Angle.	$\frac{1}{2}$ Sum.	Angle.	$\frac{1}{2}$ Difference.
12°	19·7	12°	25·9
132°	19·7	132°	63·8
252°	11·0	252°	−49·2
	3)50·4		3)40·5
	$16{\cdot}8 = b_3 \cos 3\ \theta$		$13{\cdot}5 = a_3 \sin\ 3\ \theta$
	$= b_3 \cos 36°$		$= a_3 \sin\ 36°$

Thus we get $\cos\ 36° = \dfrac{16{\cdot}8}{20{\cdot}7} = {\cdot}8116$

and $\sin 36° = \dfrac{13{\cdot}5}{23} = {\cdot}587.$

Now from Tables we see value of $\cos 36° = {\cdot}8192$ and $\sin 36° = {\cdot}5878$.

As both of these values are within 1 per cent of being correct we may consider the result satisfactory, considering that the readings from the curve were only taken to first place of decimals.

5th Harmonic.

Cosine Term.		Sine Term.	
Angle.	$\frac{1}{2}$ Sum.	Angle.	$\frac{1}{2}$ Difference.
0°	26·5	18°	37·45
72°	−11·0	90°	41·4
144°	12·2	162°	37·45
216°	12·2	234°	−62·1
288°	−11·0	306°	−62·1
	5)28·9 $5{\cdot}78 = b_5$		3)− 7·9 $-1{\cdot}58 = a_5$

A check reading was again taken as before, and found to confirm these results.

First Harmonic (Fundamental) Cosine Terms.

When $\theta = 0$ $\quad 3\,\theta = 0 \quad 5\,\theta = 0$

$\text{Cos}\ \theta = 1 \quad \text{Cos}\ 3\,\theta = 1 \quad \text{Cos}\ 5\,\theta = 1$

$$b_1 \cos\theta + b_3 \cos 3\,\theta + b_5 \cos 5\,\theta = 26{\cdot}5$$

$$\therefore\ b_1 + b_3 + b_5 = 26{\cdot}5$$

i.e., $b_1 + 20{\cdot}7 + 5{\cdot}78 = 26{\cdot}5$ and $b_1 = {\cdot}02$.

This may be assumed $= 0$, since readings are only taken to first place of decimals. Hence $b_1 = 0$.

Sine Term.

When $\theta = 90° \quad 3\,\theta = 270° \quad 5\,\theta = 450°$

$\text{Sin}\ \theta = 1 \quad \text{Sin}\ 3\,\theta = -1 \quad \text{Sin}\ 5\,\theta = 1$

$$a_1 \sin\theta + a_3 \sin 3\,\theta + a_5 \sin 5\,\theta = 41{\cdot}4$$

i.e., $a_1 - a_3 + a_5 = 41{\cdot}4$

and $a_1 - 23{\cdot}0 - 1{\cdot}58 = 41{\cdot}4$

$\therefore\ a_1 = 66{\cdot}0.$

We have now obtained the equation to the curve in the following form :—

$$f(\theta) = 66 \sin\theta + 23{\cdot}0 \sin 3\,\theta + 20{\cdot}7 \cos 3\,\theta - 1{\cdot}58 \sin 5\,\theta + 5{\cdot}78 \cos 5\,\theta.$$

This may be simplified as previously described.

Thus :—

$$a_3 \sin 3\,\theta + b_3 \cos 3\,\theta = \sqrt{a_3^2 + b_3^2}\sin\left\{3\,\theta + \tan^{-1}\left(\frac{b_3}{a}\right)\right\}$$

$$= 31 \sin(3\,\theta + 42°)$$

$$a_5 \sin 5\,\theta + b_5 \cos 5\,\theta = \sqrt{a_5^2 + b_5^2}\sin\left\{5\,\theta + \tan^{-1}\left(\frac{b_5}{a_5}\right)\right\}$$

$$= 6 \sin(5\,\theta + 105)$$

and our equation thus becomes

$$f(\theta) = 66 \sin \theta + 31 \sin (3\,\theta + 42^\circ) + 6 \sin (5\,\theta + 105^\circ),$$

which agrees with the expression obtained by the method of summation (page 403).

Exactly the same operation as just described for the determination of the third and fifth harmonics was carried out for the seventh, ninth, and eleventh harmonics, by dividing the curve into 7, 9, and 11 parts. In each case the values of the coefficients was found to be zero, or practically zero, showing that these harmonics were not present.

A single check reading has been taken above to confirm the result of the calculation of each harmonic. It would be preferable in most cases to take sufficient points to enable a curve to be plotted from them. If the curve is found to be a sine curve, the readings are shown to be trustworthy. If this is not so, a mean curve should be plotted through the points obtained.

R.M.S. Value.—The R.M.S. or virtual value of the curve, whose equation has been determined, can be obtained from the maximum values of the component waves of which it is formed.

Thus, if the equation has been obtained in the form

$$e = E_1 \sin (\theta + \phi_1) + E_3 \sin (3\,\theta + \phi_3) + E_5 \sin (5\,\theta + \phi_5) + \ldots$$

where e is the instantaneous value of the quantity for any value of θ,

then E_1, E_3, E_5, &c., are the maximum values of the respective harmonics, and the R.M.S. value is given by the expression

$$e_{\text{virt.}} = \sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{2}}$$

In the curve analysed above we have

$$E_1 = 66,\ E_3 = 31,\ E_5 = 6,$$

and the virtual voltage is consequently

$$e_{\text{virt.}} = \sqrt{\frac{66^2 + 31^2 + 6^2}{2}} = \sqrt{\frac{5353}{2}} = 51{\cdot}73 \text{ volts.}$$

As mentioned on page 405, the actual value registered by the voltmeter was 51·8 volts.

CHAPTER XIV.

Symbolic Notation.

Purpose of the Symbolic or Algebraic Notation—In calculations on alternating vectors, representing voltages, currents or fluxes it is necessary to take account of the phase relationships, as well as magnitudes, of these alternating quantities. Further, it is necessary to be able to determine the effects which the constants of the circuit (resistance, reactance, etc.) have upon the magnitude and phase of the alternating currents and voltages.

All these relationships can be shown graphically by vector diagrams. From such diagrams the magnitudes of the quantities and their phase differences can be ascertained by actual measurement or by geometrical calculation.

Figs. 9 and 10, p. 31, and numerous subsequent diagrams illustrate this procedure.

The symbolic notation has been developed in order to simplify such calculations and make it possible to deal with them by algebraic processes. Naturally, it is in the more complicated circuits that this algebraic method of calculation is found to be the most advantageous.

The main idea underlying the symbolic notation is the representation in algebraic form of the vectors in terms of their two components, which are respectively parallel and perpendicular to some convenient axis. All vector quantities parallel to this axis are in phase with one another, and can be treated by simple arithmetical rules to give a resultant in the same phase. Similarly, components perpendicular to the chosen axis can be added or subtracted to give a resultant having a corresponding phase. The results of these two operations can be combined to give a numerical result having a phase which is easily calculated.

Symbol j.—Vectors (or components of vectors) which are perpendicular to the axis of reference are indicated by having the letter j written in front of them, indicating that they are "multiplied" by the symbol j.

Multiplication by j does not alter the magnitude of the vector, but indicates that the vector has been rotated through an angle of 90° in a counter-clockwise direction.

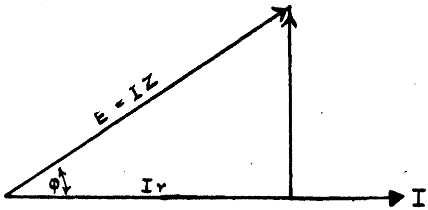


FIG. 259.
Component Voltages. Current taken as Axis of Reference.

For example, let I in Fig. 259 be the vector of current flowing in an inductive circuit of resistance r ohms and reactance x ohms. If we choose the current vector as our axis of reference, the two components of the applied voltage, E, will be Ir along the axis of reference and jIx perpendicular to this axis, where the symbol j indicates that the voltage Ix is rotated 90° in a counter-clockwise direction, and is, therefore, 90 degrees in advance of the current.

Every time a vector is multiplied by j it is rotated 90°. This process may be repeated; that is, a vector already containing j may be again multiplied by j to produce a further rotation of 90° counter-clockwise. This would cause the vector to be rotated through 180° from the axis so that it would again be along the axis of reference, but in a negative direction.

Consequently, multiplication by $j \times j$ (usually written j^2) produces the same result as if the vector had been reversed or multiplied by -1.

It is for this reason that we write

$$j^2 = -1.$$
$$\therefore j = \sqrt{-1}.$$

$\sqrt{-1}$ is an "imaginary" number and has no numerical signification; it merely indicates geometrical rotation.

Multiplication by j^3, that is, multiplication by j three times, produces the same result as multiplication by $-j$, and brings the vector into a position 90° behind the original axis in phase.

It is important to remember that real and imaginary quantities can never be added arithmetically. Real quantities can be added arithmetically and imaginary quantities can similarly be added, but real and imaginary quantities can only be combined by taking into account the 90° phase-difference between them. The signs + or — connecting real and imaginary quantities must always be understood to mean *geometric* addition or subtraction.

Impedance.—If a current of I amperes flows in a circuit having a resistance r ohms and reactance x ohms, the voltage, E, required to maintain the current will be the geometric sum of the two components:—

Ir in phase with the current, and

jIx perpendicular to this in phase.

This may be expressed conveniently in symbolic form :—

$$E = Ir + jIx = I\ (r + jx) \text{.....................} (1).$$

Here $r + jx$ is the impedance of the circuit, expressed as a complex number, that is, a number consisting of both real and imaginary parts.

The convenience of writing the impedance in this form is that it enables us to multiply the current vector by the impedance and obtain the separate component voltages. From these it is easy to find the phase of the voltage, which is

$$\tan^{-1}\left(\frac{\text{imaginary part}}{\text{real part}}\right)$$

If the current is taken as the axis of reference, this angle will be the angle of lag (or lead) of the circuit and the cosine of this angle will be the power factor.

The numerical value of the impedance can always be obtained by adding the squares of its two components and taking the square-root of this sum (see p. 39).

Impedance $= z = \sqrt{r^2 + x^2}$ ohms.

Convention as to Symbols.—In order to make a distinction between the *numerical* expression for the impedance (as a certain number of ohms without distinguishing between resistance and reactance) and the expression of impedance as a *complex number* (in which resistance and reactance are shown separately), it is usual to adopt the convention of writing a capital letter Z to represent the complex number and a small z to indicate the numerical value of the whole impedance. Thus,

$$Z = r + jx \text{ ....} (2)$$

$$z = \sqrt{r^2 + x^2} \text{ ....} (3)$$

Similarly, a vector of current or voltage is indicated by a capital letter, I or E, while the numerical value (in amps. or volts) is indicated by the small letter, i or e.

We should thus write*

$$E = IZ = I\ (r + jx) \text{ ....} (4)$$

which gives the complete relationship between the vectors.

Or, stated numerically,

$$iz = e = i\ \sqrt{r^2 + x^2} \text{ ....} (5).$$

If I is taken along the axis of reference, $I = i$.

* By some writers a vector is further distinguished by adding a dot above or below the capital letter. This has not been done in the present chapter. It must be remembered that I and E are always vectors, whereas Z is a complex number and *not* a vector.

It is to be remembered that the signs + and — used in vector notation indicate *geometric* addition and subtraction. Thus, whenever the quantity being dealt with is represented by a capital letter, these signs must be interpreted geometrically. An equation giving arithmetical values only must have small letters throughout.

The reactance x is given by $2\pi fL - \frac{1}{2\pi fC}$ (see p. 107). This may be either negative or positive. The current lags behind the voltage, if it is positive, and leads the voltage if the reactance x has a negative value.

Example 1. —A current of 5 amperes flows in a circuit having a resistance of 5 ohms and reactance 8 ohms. Find the applied voltage and power-factor of the circuit.

As we are free to choose the axis of reference, it is best to take it in phase with the current. We then have the voltage

$$E = iZ = i\,(r + jx) = 5\,(5 + j8).$$
$$= 25 + j40.$$

The numerical value of this voltage is

$$e = \sqrt{25^2 + 40^2} = 47 \cdot 1 \text{ volts.}$$

Impedance $= z = \sqrt{r^2 + x^2} = \sqrt{5^2 + 8^2} = 9{\cdot}4$ ohms.

$$\text{Power-factor } \cos\phi = \frac{\text{resistance}}{\text{impedance}} = \frac{5}{9{\cdot}4} = 0{\cdot}53.$$

Example 2. A current $I = 30 + j12$ flows in a circuit having a resistance of 5 ohms and reactance 8 ohms. Find the applied voltage and power-factor.

In this case, the phase of the current is not parallel with the axis of reference.

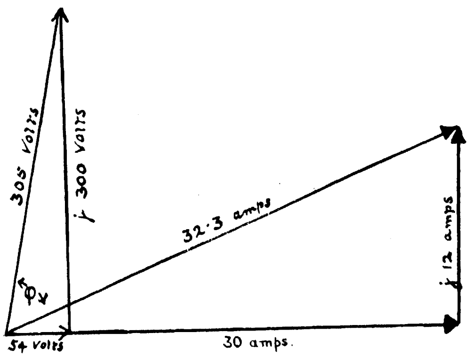


FIG. 260.
Vector Diagram for Example 2.

The vector diagram for the circuit is given in Fig. 260, where the axis of reference is taken as horizontal.

The applied voltage

$$E = IZ = I\ (r + jx) = (30 + j12)\ (5 + j8).$$
$$= 150 + j\ (60 + 240) + j^2 96.$$
$$= 150 + j300 - 96.$$
$$= 54 + j300.$$

Numerically, $e = \sqrt{54^2 + 300^2}.$

$$= 305 \text{ volts}.$$

The power-factor of the circuit is the same as in the previous example :

$$\cos\phi = \frac{\text{resistance}}{\text{impedance}} = \frac{5}{9{\cdot}4} = 0{\cdot}53.$$

The power-factor depends on the constant values of resistance and reactance in the circuit, which determine the phase difference between current and voltage. It is unaffected by the absolute values of current and voltage or by their phase in relation to the reference axis.

Fig. 260 illustrates this example, in which neither vector is in phase with the reference axis. The simpler case, of Example 1, would be illustrated by a diagram such as Fig. 259.

Admittance. If the applied voltage of a circuit is given and we wish to determine the current, we use the relationship in vectorial form.

$$I = \frac{E}{Z} = \frac{E}{r + jx} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (6)$$

When we have, as in this case, a fraction with a complex number in the denominator, we must "rationalize" the fraction, that is, we must get rid of the imaginary number in the denominator. This is done by multiplying both numerator and denominator by a factor which will make the denominator real.

For a denominator of the form $r + jx$, this factor is $r - jx$. For an expression of the form $r - jx$, the factor will be $r + jx$

In order to rationalize the current vector in equation (6), we therefore proceed as follows :—

$$I = \frac{E}{r + jx} = \frac{E\ (r - jx)}{(r + jx)\ (r - jx)} = \frac{E\ (r - jx)}{r^2 - (jx)^2} = \frac{E\ (r - jx)}{r^2 + x^2} \quad \dots (7)$$

This gives us the two components of the current

$$\frac{Er}{r^2 + x^2}$$

along the axis of reference, and

$$\frac{-jEx}{r^2+x^2}$$

rotated through 90° in a clockwise direction, that is, lagging 90°, as indicated by the factor $-j$.

Because of their frequent occurrence in circuit calculations, special names and symbols are employed for the inverse of the impedance and its components. All are measured in mhos., the inverse of the ohm.

$$\text{Admittance} = Y = \frac{1}{Z} = \frac{r}{r^2+x^2} - j\frac{x}{r^2+x^2} \quad \ldots(8)$$

$\dfrac{r}{r^2+x^2}$ is called conductance and is denoted by the symbol g.

$\dfrac{x}{r^2+x^2}$ is called susceptance and is denoted by the symbol b.

Thus we write the admittance of a circuit

$$Y = g - jb \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

or numerically (in mhos.)

$$y = \sqrt{g^2+b^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(10)$$

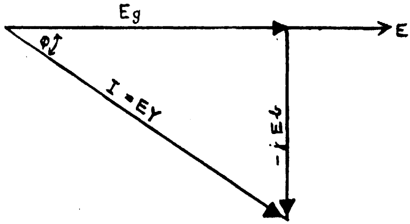


FIG. 261.

Component Currents. Voltage taken as Axis of Reference.

Fig. 261 shows the diagram of currents referred to the voltage as axis of reference.

In it the component currents are obtained by multiplying the voltage by the conductance and susceptance.

Fig. 261 should be compared with Fig. 259, in which the component voltages are obtained by multiplying the current by the resistance and voltage.

Both diagrams give the same information, but in Fig. 259 the current is the convenient standard of reference, so that this form would be useful in series circuits, where the same current flows through several impedances.

Fig. 261 would be more useful for parallel circuits, where one voltage is applied to several branches.

It may be noted that Fig. 259 could be taken as a diagram of impedance if each side were divided by I and the arrow-heads were removed (c.f. Fig. 16, p. 40). Similarly, Fig. 261 could be regarded as a diagram of admittance (on a scale of mhos.) when divided by E. The two diagrams of impedance and admittance are similar in shape for a given circuit.

Example 3.—An electromotive force $50+j75$ is applied to a circuit having a resistance of 10 ohms and reactance -6 ohms. It is required to find the current, its phase and the power-factor of the circuit.

We proceed to find the symbolic expression for the current by using the formula

$$I = EY = E(g - jb).$$

From the constants of the circuit

$$g = \frac{r}{r^2 + x^2} = \frac{10}{136}$$

$$b = \frac{x}{r^2 + x^2} = \frac{-6}{136}$$

Hence

$$I = (50 + j\,75)\left\{\frac{10}{136} + j\frac{6}{136}\right\}$$

$$= \frac{1}{136}\left\{500 + j\,(750 + 300) + j^2\,450\right\}$$

$$= \frac{1}{136}\left\{500 + j\,1050 - 450\right\}$$

$$= \frac{1}{136}(50 + j\,1050)$$

$$= 0{\cdot}368 + j\,7{\cdot}73.$$

The numerical value of the current is

$$\sqrt{0{\cdot}368^2 + 7{\cdot}73^2} = 7{\cdot}7 \text{ amps. approx.}$$

The phase of the current is

$$\tan^{-1}\frac{7{\cdot}73}{0{\cdot}368} = \tan^{-1} 21 = 87° \text{ approx.}$$

The positive sign of the second term in the expression for the current shows that this angle is in advance of the axis of reference.

The power-factor of the circuit is

$$\cos\phi = \frac{\text{resistance}}{\text{impedance}} \text{ or } \frac{\text{conductance}}{\text{admittance}}$$

Its value is therefore $\sqrt{\dfrac{10}{136}} = 0{\cdot}86.$

Addition of Vectors.—Vectors expressed in symbolic form are added by adding separately the real parts and the imaginary parts, to give the real and imaginary components of the resultant vector.

Impedances and admittances, represented by complex numbers, are added by the same procedure. In circuits where a single current flows through more than one impedance in series, it is often necessary to find the total impedance by addition of the impedances. On the other hand, when a circuit is composed of impedances forming parallel circuits, it becomes necessary to add the admittances of the branch circuits in order to obtain the admittance of the whole circuit. This procedure is like that adopted in resistance calculations on direct-current circuits containing resistances in series or in parallel. The application to alternating circuits is illustrated in examples given later.

Series Circuit.—When a current flows through impedances connected in series, we can calculate the total applied voltage by adding together the voltages spent in each impedance. Usually, it is more convenient to calculate the total impedance and then to multiply the current by this number to obtain the applied voltage. This is illustrated as follows :—

Let the circuit consist of impedances $Z_1 = r_1 + jx_1$, $Z_2 = r_2 + jx_2$, $Z_3 = r_3 + jx_3$.

The total impedance is

$$Z = Z_1 + Z_2 + Z_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$= r_1 + r_2 + r_3 + j\,(x_1 + x_2 + x_3) \quad \ldots\ldots\ldots\ldots (12)$$

Expressed numerically in ohms the impedance

$$z = \sqrt{(r_1 + r_2 + r_3)^2 + (x_1 + x_2 + x_3)^2} \quad \ldots\ldots (13)$$

The applied voltage will be

$E = IZ$, where I is the current in the circuit.

Numerically

$$e = iz \text{ volts.}$$

The phase of the resultant voltage relative to the current is

$$\tan^{-1} \frac{\text{reactance}}{\text{resistance}} = \tan^{-1} \frac{x_1 + x_2 + x_3}{r_1 + r_2 + r_3} \quad \ldots(14)$$

The power-factor is $\cos\phi = \dfrac{\text{resistance}}{\text{impedance}}$

$$= \frac{r_1 + r_2 + r_3}{\sqrt{(r_1 + r_2 + r_3)^2 + (x_1 + x_2 + x_3)^2}} \quad \ldots\ldots(15)$$

determine and add the separate branch currents; but it is usually simpler to add the admittances and thus obtain a single equivalent admittance for the network. When multiplied by the applied voltage this admittance gives the total current in the circuit.

Suppose the circuit consists of a number of parallel branches having impedances of

$$Z_1 = r_1 + jx_1, \quad Z_2 = r_2 + jx_2, \quad Z_3 = r_3 + jx_3.$$

The admittances of these circuits are (see equation (8).)

$$Y_1 = \frac{1}{Z_1} = \frac{r_1 - jx_1}{r_1^2 + x_1^2} = g_1 - jb_1 \quad \ldots\ldots(16)$$

and similarly for the other branches.

The total joint admittance for the parallel circuits is :—

$$Y = g_1 + g_2 + g_3 - j\,(b_1 + b_2 + b_3) \quad \ldots\ldots(17)$$

and hence the total current is given by

$$I = EY \quad \ldots\ldots(18)$$

or numerically

$$i = ey \text{ amps.} \quad \ldots\ldots(19)$$

where $y = \sqrt{(g_1 + g_2 + g_3)^2 + (b_1 + b_2 + b_3)^2}$

The phase of the total current in relation to the applied voltage is

$$\phi = \tan^{-1} \frac{\text{susceptance}}{\text{conductance}} = \tan^{-1} \frac{b}{g}$$

$$= \tan^{-1} \frac{b_1 + b_2 + b_3}{g_1 + g_2 + g_3} \quad \ldots\ldots(20)$$

The overall power-factor of the circuit is

$$\cos\phi = \frac{\text{conductance}}{\text{admittance}} = \frac{g}{y} = \frac{g_1+g_2+g_3}{\sqrt{(g_1+g_2+g_3)^2+(b_1+b_2+b_3)^2}} \quad (21)$$

The valuesof ϕ and $\cos\phi$ depend on the circuit constants only, and are independent of the choice made of the axis of reference·

Example 4.—Two coils are placed as a load on a 50-cycle alternator whose P.D. is kept constant at 100 volts. The resistance of each coil is 2 ohms, and their co-efficients of self-induction are 32 and 57 millihenries respectively. Find the current in the circuits and the power-factor of the load (a) when the coils are connected in series, (b) when they are connected in parallel.

The reactance of the first coil

$$x_1 = 2\pi f L_1 = 2\pi\ 50\ 0{\cdot}032 = 10 \text{ ohms.}$$

Similarly the reactance of the second coil

$$x_2 = 2\pi f L_2 = 2\pi\ 50\ 0{\cdot}057 = 18 \text{ ohms.}$$

The impedances of the coils are

$$Z_1 = r_1 + jx_1 = 2 + j10.$$
$$Z_2 = r_2 + jx_2 = 2 + j18.$$

(a) When the coils are in series their combined impedance is

$$Z = Z_1 + Z_2 = 4 + j28.$$

This has a numerical value of

$$z = \sqrt{16+784} = 28{\cdot}2 \text{ ohms.}$$

The value of the current is therefore

$$i = \frac{e}{z} = \frac{100}{28{\cdot}2} = 3{\cdot}54 \text{ amps.}$$

$$\text{Power-factor} = \cos\phi = \frac{r_1+r_2}{z} = \frac{4}{28{\cdot}2} = 0{\cdot}142.$$

(b) When the coils are in parallel we take the admittances of the two circuits, just as we previously took the impedances.

For the first circuit

$$\text{Admittance} = Y_1 = g_1 - jb_1.$$

$$g_1 = \frac{r_1}{r_1^2+x_1^2} = \frac{2}{4+100} = \frac{2}{104} = 0{\cdot}0192 \text{ mho.}$$

$$b_1 = \frac{x_1}{r_1^2 + x_1^2} = \frac{10}{104} = 0{\cdot}096 \text{ mho.}$$

Similarly

$$g_2 = \frac{r_2}{r_2^2 + x_2^2} = \frac{2}{4 + 324} = \frac{2}{328} = 0{\cdot}0061 \text{ mho.}$$

$$b_2 = \frac{r_2}{r_2^2 + x_2^2} = \frac{18}{328} = 0{\cdot}0548 \text{ mho.}$$

The joint admittance of the two circuits

$$Y = Y_1 + Y_2 = g_1 - jb_1 + g_2 - jb_2 = g_1 + g_2 - j\,(b_1 + b_2).$$
$$= 0{\cdot}0192 + 0{\cdot}0061 - j\,(0{\cdot}096 + 0{\cdot}0548).$$
$$= 0{\cdot}0253 - j\,0{\cdot}1508.$$

Thus the joint admittance is numerically

$$y = \sqrt{0{\cdot}0253^2 + 0{\cdot}1508^2} = \sqrt{0{\cdot}0232} = 0{\cdot}1525 \text{ mho.}$$

The total current of the parallel paths

$$i = ey = 100 \times 0{\cdot}1525 = 15{\cdot}25 \text{ amps.}$$

The joint power-factor is

$$\cos\phi = \frac{g_1 + g_2}{y} = \frac{0{\cdot}0253}{0{\cdot}1525} = 0{\cdot}1665.$$

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